A Multi-Asset Approach to Inflation Hedging for a U.S. Investor

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Various experts, here and abroad, believe that the immediate postwar inflationary climate has now been converted into an epoch of price stability. One hopes this cheerful diagnosis is correct. However, a careful survey of the behavior of prices and costs shows that our recent stability in the wholesale price index has come in a period of admittedly high unemployment and slackness in our economy. For this reason it is premature to believe that the restoration of high employment will no longer involve problems concerning the stability of prices.

—Paul Samuelson,
Letter to John F. Kennedy, 1961

In order to alleviate some of the effects of the financial crisis of 2007–2008, the Federal Reserve as well as other central banks have lowered interest rates to close to zero and have been using other mechanisms to supply the markets with liquidity. One of the worries that market participants have is that this will eventually lead to higher inflation. Although the short-term numbers suggest we are in a period of deflation, driven in part by the reduction in housing values, transportation, and commodity prices, a danger still exists that inflation may return.[1] What should the average investor do to hedge against inflation or what is the best way to achieve a positive real return in the presence of inflation?

Despite the importance of this question to investors, relatively little quantitative research aimed at answering this question exists (Mukherji [2003] and Chua [1999]) for several reasons. First, any attempt to maximize real returns is effectively the same as maximizing nominal returns. A lot of research has addressed this question. Second, very little theoretical work has focused on which asset classes and in what proportion would be good inflation hedges. Third, it could be difficult to find statistical models that provide a good hedge against inflation in all periods (for example, high inflationary and low inflationary periods).

A small literature has been dedicated to using asset returns to forecast macroeconomic variables, such as inflation, and these studies have found some forecasting ability by the markets (Adrangi et al. [1999], Bodie [1976], Lamont [2001], Campbell and Ammer [1993], Cozier and Rahman [1988], Fama [1975], Hayes [1999], and Titman and Warga [1989]). Our article approaches the issue from a different perspective.

First, we do not attempt to forecast inflation. Rather, we attempt to discover which assets contemporaneously provide the best hedge against inflation. Second, we use asset

class returns and attempt to find the relation between inflation and the asset classes over a variety of historical investment periods beginning in 1901. Third, we seek the optimal portfolio that minimizes the tracking error of the portfolio versus inflation while providing a given real return (the portfolio approach).

We find that an investor who is looking for a reasonable positive real return of 4.5% while minimizing the downside with respect to inflation will have an allocation that consists primarily of short-term bonds, longer-term bonds, some gold, some oil, and some emerging market equities. The weight of gold and oil together is less than 10% of the portfolio. We also find that TIPS are only slightly effective for protecting against inflation conditional on an investor using a group of asset classes. The out-of-sample performance of the real return optimizations are quite promising, providing an emulative inflation protection strategy for U.S. investors.

The article is organized as follows. We first discuss the data used in our study and then we discuss our methodology for creating inflation-hedged portfolios. We next discuss both the in-sample and out-of-sample results for various historical time periods and lastly we conclude.

DATA

Data for our study were obtained from multiple sources including Global Financial Data (GFD), Bloomberg L.P., National Association of Real Estate Investment Trusts (NAREIT), and Factset Research Systems (FRS). Central bank interest rates, one-month CD rates, commercial paper yields, commodity indices, commodity prices, consumer price indices, stock indices, total return stock indices, and wholesale price indices are from GFD. Equity sector returns for each country and exchange rates are from Factset. Data on inflation-protected fixed-income securities are from Bloomberg and data on Lehman TIPS indices are from FactSet. Real estate investment trust returns are from NAREIT. The equity index we use is the S&P 500 Total Return Index, the small-cap index is the Russell 2000 Index, the value index is the Russell 3000 Value Index, the growth index is the Russell 3000 Growth Index, and the sector indices are the FactSet Aggregate Indices: Consumer Durables, Consumer Non-Durables, Energy Minerals, Finance, Health Services, Industrial Services, Technology Services, Non-Energy Minerals, Communications, and Utilities. Other asset classes are the following. Bill Govt. is the USA Total Return Commercial/T-Bill Index, 10-Yr. Govt. is the USA 10-Year Government Bond Total Return Index, 30-Yr Govt. is the USA 30-Year Government Bond Return Index, Corp. Bond is the Dow Jones Corporate Bond Return Index, HY Bond is the Merrill Lynch High Yield Bond Return Index, IP Bond is the Lehman Brothers TIPS Index, Commodity is the Dow Jones-AIG Commodity Index, Oil is the West Texas Intermediate Spot Oil Price (US$/barrel), Gold is the Gold Spot Price-London PM Fixing (US$/ounce), Wheat is the CBOT Wheat Futures Price,[2] FX1 is the U.S. dollar to Japanese yen, FX2 is the U.S. dollar to Euro, FX3 is the U.S. dollar to British pound, One-Month CD is the USA one-month CD Rate, Real Estate is the FTSE NAREIT U.S. Real Estate Index, World Equities is the GFD World Return Index, World Bonds is the GFD Global USD Total Return Government Bond Index, Emerging Equities is the GFD Emerging Markets Return Index, and Emerging Bonds is JP Morgan Emerging Bond Index.

Wherever possible, we use total return indices. All index values are transformed into percentage change for comparability with the inflation series. For the purposes of this article, the rate of inflation is reported as the year-over-year (YoY) change in the CPI index. The year-over-year inflation number reduces seasonal and cyclical effects present in inflation numbers. To remain consistent with this convention, all of the asset returns in this article are computed on a year-over-year basis. Thus, the S&P 500 return is the return of the index over the last year.

Exhibit 1 lists the mean returns of key variables over three different time periods beginning January 1930, January 1970, and January 1990. All end in May 2009 for the United States. The key variables are CPI, PPI, broad equities (stock indices), government bonds, corporate bonds, high-yield bonds, IP (inflation-protected bonds), real estate, gold, silver, oil, and wheat. All are in U.S. dollars. The number of observations with valid data are also noted for each time period.

The correlations of oil and gold to the YoY change in CPI from 1990 to 2009 are 0.16 and 0.48, respectively. This is consistent with the general notion that gold and, perhaps, oil might serve as good inflation hedges for investors. The relationship between PPI and CPI appears to have become weaker as we moved from the 1970s dataset to the 1990s dataset.

EXHIBIT 1

Nominal Returns of Asset Classes and Correlations with Inflation for Various Time Periods

Start Period	No. of Obs.	Inflation CPI	Inflation PPI	Equities Main	Fixed Income Govt.	Fixed Income Corp.	Fixed Income High Yield	Fixed Income Infl. Prot.	Real Estate Main	Commodities Gold	Commodities Silver	Commodities Oil	Commodities Wheat
Nominal Returns													
1930	953.00	3.32	–	11.13	5.69	7.05	–	–	–	6.67	8.03	7.47	–
1970	473.00	4.63	4.03	11.01	8.85	9.76	–	–	–	11.33	11.06	14.24	7.71
1990	233.00	2.88	2.28	9.72	8.21	7.88	8.10	–	11.22	5.35	6.70	11.66	5.89
Correlations													
1930	953.00	1.00	–	0.05	-0.09	0.03	–	–	–	0.30	0.28	0.36	–
1970	473.00	1.00	0.91	-0.12	-0.33	-0.34	–	–	–	0.44	0.35	0.47	0.13
1990	233.00	1.00	0.80	-0.05	0.03	0.02	-0.02	–	0.05	0.16	-0.04	0.48	0.04

Note: This exhibit reports mean returns and correlations to CPI for various asset classes from the beginning of the listed period until May 2009. Means are reported in percentage terms.

CONSTRUCTION OF INFLATION HEDGES

Theoretical Considerations

Surprisingly, very little theoretical work has been done on the types of asset classes that might be natural hedges against inflation. Brennan and Xia [2000] presented a simple theoretical model that the optimal strategy consists of investments in cash, equities, and a single nominal bond with an optimally chosen maturity. With the exception of the work of Brennan and Xia much of the research has been empirical.

One strand of the literature has focused on how stock returns relate to inflation, but disagreement exists on the direction of the relationship. This literature includes work on money illusion and taxes (Modigliani and Cohn [1979], Hendershott [1981], and Summers [1981]). A small amount of papers have been dedicated to studying the relationship between inflation and inflation hedging within the context of the stock market, including Patel and Zeckhauser [1987] and Reilly et al. [1970].

Another strand of empirical literature has focused on the relationship between commodities and inflation. Early research that shows the importance of commodities for hedging inflation includes the work of Greer [1978] and Bodie [1983]. Froot [1995] found that, contrary to perceived wisdom, real estate indices provide relatively weak hedges for diversified portfolios, while commodities with a high energy component enhance the return–risk trade-off. Anson [1998] showed that adding commodity futures to a standard stock and bond portfolio can move the efficient frontier outward. Strongin and Petsch [1997] showed that a commodity index does well relative to stocks and bonds during periods of rising inflation. Ankrim and Hansel [1993] found that adding the Goldman Sachs Commodity Index (GSCI) improved the return–risk charactistics of a bond and stock portfolio. Schneeweis and Spurgin [1997] and Halpern and Warsager [1998] found that energy-based commodity index products provide better returns than traditional asset classes during periods of high inflation. Gorton and Rouwenhorst [2006] also showed that commodity futures returns are more highly correlated with inflation than stock and bond returns, especially over longer horizons of one to five years.

In the context of inflation-protected securities, in particular, TIPS, a series of papers has studied the impacts of these instruments on investment portfolios (Mamun and Visaltanachoti [2006], Roll [2004], and Siegel and Waring[2004]). The empirical papers closest to ours that attempt to study this relationship on a broader set of asset classes include Attie and Roache [2009] and Nishat and Mustafa [2008].

Of course, the popular news continually reinforces the common perception that gold is a good hedge against inflation. Several reasons are commonly used to explain this. First, if gold is believed to be a good hedge against inflation, then its price will rise as demand rises with expected inflation. Second, for a U.S. investor, if inflation is accompanied by the depreciation of the U.S. dollar, then the price of a fixed asset in terms of U.S. dollars will rise as the dollar depreciates. This might not be true for

investors of other countries. Third, if other commodities are a hedge against inflation, and if inflation is supply-side driven, as inputs to production rise so does inflation. With this type of inflation, commodity prices can be expected to rise along with CPI, or consumer inflation.

Short-term government bonds or short-term bank deposits might also serve as a good hedge. The argument is that cash equivalents' fixed interest rate over a short period of time will have expected inflation built into it. To the extent that there is unexpected inflation, the rates will adjust for the next period of investment. Because each period of investment is so short, the investor has a better chance of keeping up with both expected and unexpected inflation, even if with a delay. Inflation-protected bonds are viewed as good investments against inflation. In fact, several advisors have noticed a massive flood of money into TIPS-related funds in 2009. These instruments offer a real return to investors. Thus, if inflation is higher than expected, TIPS adjust their interest payments to keep the investor's real return at the expected level. A principle of all fixed-income instruments is that their prices are affected by movements in interest rates. Thus, if an inflation-protected security is held to maturity, it will provide the desired real return, but its actual return in any subperiod may be very volatile and fluctuate quite dramatically as interest rates change. Thus, depending on an investor's goals and investment horizon, inflation-protected bonds may not be the best inflation-protected investment.

The stock market measures the present discounted value of dividends or profits. To the extent that inflation pushes all prices up, one might expect that future dividends will be bid up in anticipation of future profits rising in nominal terms so that stock prices will keep pace with inflation. Certain types of inflation may hinder the functioning of the economy and/or cause firms to reduce their margins, which then might offset this effect. Certain equity sectors of the economy might do better than others depending on the source of the inflation.

Real estate has always been thought of as a good hedge against inflation, because as prices of goods in the economy rise, so will the price of a relatively scarce good, land.

Data Considerations

We would prefer to use the longest possible historical data horizon in order to capture as many inflationary periods as possible. In practice, this is very difficult because most of our main asset classes lack history prior to 1970. Thus, we have chosen to use several historical data horizons. The first time period (starting January 1901) was selected to evaluate the long-term relationship between the variables selected and inflation. The world has changed considerably since 1901, so shorter time periods were also selected to determine if there have been structural changes in the relationships. The second time period (starting January 1930) was selected because it covers the deflationary period from the inception of the Great Depression of the 1930s. Over the 39-month period from January 1930 to March 1933, U.S. CPI fell over 26%. The third time period (starting January 1970) was selected to coincide with the move away from the gold standard and fixed exchange rate system in place since the end of World War II to the floating exchange rate system that was instituted in the 1971–1973 period. This period also contains the stagflation that occurred in the late 1970s. The fourth time period (starting January 1990) was selected to measure the impact of the end of the Cold War and the emergence of Eastern Europe as important contributors to the global economy. Among all the major asset classes, we use all relevant data for any country, provided that the data existed at the beginning of the starting period.

Another issue is the multicollinearity of some of our asset classes because of the large number that we use. In particular, we use the Dow Jones Commodity Index that contains a weighting of roughly 17% in oil, 5.3% in gold, 2.2% in silver, and 3.8% in wheat.[3] Of the four, only oil has a substantial correlation with the index. This is important when interpreting the results of our study. In particular, more emphasis should be placed on the commodity asset class as a whole rather than the exact point estimates to each subcategory within the commodity asset class.

Generating Real Return Portfolios

Because most investors are interested in a combination of assets that provides a sufficient real return while protecting the portfolio on the downside against inflation, we chose to create a set of mean-variance-optimized portfolios.

In order to make the objective function of an investor related to inflation, we need to specify one that incorporates an investor's preoccupation with inflation. One such objective function would be to characterize an investor as wanting to maximize real return subject

to some minimization of the nominal return deviation from inflation. That is,

$$\min\left|V\left(r_{P,t,t+k}-\pi_{t,t+k}\right)\right| \quad \text{s.t.} \quad r_{P,t,t+k}-\pi_{t,t+k}=\tilde{\mu}_P \qquad (1)$$

where $r_{P,t,t+k}$ is the return of the investor's portfolio from time t to $t+k$, $\pi_{t,t+k}$ is the inflation rate from time t to $t+k$, and $\tilde{\mu}_P$ is the real return target of the portfolio.[4] We will want to select the group of assets for the investor that achieves this goal. Thus, the problem can be rewritten as

$$\min_{w_i}\left|V\left(\sum_{i=1}^{N} w_{i,t} r_{i,t,t+k}-\pi_{t,t+k}\right)\right|$$
$$\text{s.t.} \quad \left(\sum_{i=1}^{N} w_{i,t} r_{i,t,t+k}-\pi_{t,t+k}\right)=\tilde{\mu}_P \qquad (2)$$

We can write this in matrix notation as[5]

$$\min_{\boldsymbol{w}} \boldsymbol{w}'\Sigma\boldsymbol{w}-2\boldsymbol{w}'\boldsymbol{\gamma} \quad \text{s.t.} \quad \boldsymbol{w}'\mu=\tilde{\mu}_P+\pi_{t,t+k} \qquad (3)$$

where $\boldsymbol{\gamma}$ is an N-dimensional vector of the covariances between individual asset returns and the inflation rate over the horizon from t to $t+k$, Σ is the variance-covariance matrix of returns of the asset classes and inflation, and $\boldsymbol{w}$ represents the weights of the portfolio of asset classes,

$$\boldsymbol{\gamma}=\begin{bmatrix} C(r_1,\pi_{t,t+k}) \\ \vdots \\ C(r_N,\pi_{t,t+k}) \end{bmatrix} \qquad (4)$$

In addition, constraints are added to prohibit short selling of asset classes and that the portfolio weights sum to one. This is the same objective function as performing a mean-variance analysis on real returns.

The investment horizon is also critically important. An investor may care little or not at all about whether his portfolio beats inflation over a monthly horizon, but may be very concerned that it beats inflation over a 10- or 15-year horizon. For example, an investor who needs to protect his assets from inflation over the next 5 years might very well just purchase 5-year maturity inflation-protected bonds, while an investor who wants to maximize real return with some protection from inflation over 20 years might choose to invest in a balance of short-term Treasuries, equities, and commodities. An investor may consider many investment horizons, however, the longer the investment horizon, the fewer data points we need in order to measure the out-of-sample effectiveness of a particular investing strategy. In this article, we use a 12-month horizon to evaluate performance, that is, we attempt to hedge the investor's exposure to rolling 12-month inflation.

Finally, we have to consider the estimation horizon. For this study, we estimate the in-sample optimizations from the beginning of the period (1901, 1930, 1970, or 1990) to May 2009. For the out-of-sample optimizations, we estimate from the beginning of the period (1901, 1930, 1970, or 1990) plus five additional years. This estimate is used to construct portfolios for the next month. We then expand the estimation window forward by one month and re-estimate and form new portfolios. We continue this process until the very last month of April 2009.

EMPIRICAL RESULTS

In-Sample Results

Optimal weights. Exhibit 2 contains the results from the in-sample optimizations. The exhibit shows the portfolios with minimum tracking error to inflation for a given annual target return of 0.5%, 2.5%, or 4.5%. For periods in which such a return cannot be met, we chose a target return as close as possible to our target return. We chose to optimize over four time periods using the full set of variables for each time period. A "–" indicates that a particular variable was not available for the optimization, while a zero indicates the variable was available, but received a weight of zero. For this discussion, we focus on the target real return of 4.5% a year. In the 1901 period there were few asset classes. A portfolio of 37.4% equities, 6.5% Treasury bills, 39.5% 10-year Treasury bonds, and 16.6% oil accomplished this goal. The portfolio had a large tracking error of 10.2% and a return–risk profile of 0.44. For the 1930 period, the target return was achieved with 9.2% equities, 9.7% Treasury bills, 54.3% corporate bonds, 9.4% oil, 1% gold, 2.1% silver, and 14.2% emerging equities. For the 1970 period, the appropriate allocation was 26.6% corporate bonds, 6.1% oil, 3.9% gold, 54% one-month CDs, and 10% emerging equities. The tracking error of the position was 4.3% a year with a 1.05 return–risk ratio. And the allocation that earned the target rate in the 1990 period was 7.2% consumer staples stocks, 1% energy stocks, 0.1% technology stocks, 21.8%

EXHIBIT 2

Asset Class Real Return Optimizations for the United States

	In-Sample Weights												Out-of-Sample Weights	
	1901–			1930–			1970–			1990–			1930–	1970–
	Real Return Target													
Asset Class	0.75	2.5	4.5	0.55	2.5	4.5	0.5	2.5	4.5	0.5	2.5	4.5	4.5	4.5
Equity Index	0	17.1	37.4	0	5.1	9.2	0	0	0	0	0	0	15.0	1.1
Small Cap	–	–	–	–	–	–	–	–	–	0	0	0	–	–
Value	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Growth	–	–	–	–	–	–	–	–	–	–	–	–	–	–
S.CD	–	–	–	–	–	–	–	–	–	0	0	0	–	–
S.CS	–	–	–	–	–	–	–	–	–	0	0	7.2	–	–
S.Energy	–	–	–	–	–	–	–	–	–	0	0.9	1	–	–
S.Finance	–	–	–	–	–	–	–	–	–	0	0	0	–	–
S.Health	–	–	–	–	–	–	–	–	–	0	1	0	–	–
S.Industrials	–	–	–	–	–	–	–	–	–	0	0	0	–	–
S.IT	–	–	–	–	–	–	–	–	–	0	0	0.1	–	–
S.Materials	–	–	–	–	–	–	–	–	–	0	0	0	–	–
S.Telcomm	–	–	–	–	–	–	–	–	–	0	0	0	–	–
S.Utility	–	–	–	–	–	–	–	–	–	0	0	0	–	–
Bill Govt.	100	58.5	6.5	100	56	9.7	82	0	0	72	0	0	8.9	0.0
10-Yr. Govt.	0	13.7	39.5	0	0	0.2	0	0	0	0	0	0	7.7	0.0
30-Yr. Govt.	0	0	0	0	0	0	0	0	0	0	5.8	21.8	0.2	0.0
Corp. Bond	–	–	–	0	23.8	54.3	0	0	26.6	0	1.9	3.4	45.3	16.0
HY Bond	–	–	–	–	–	–	–	–	–	0	0	0	–	–
IP Bond	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Commodity	–	–	–	–	–	–	2.4	0	0	2.8	0	0	–	0.0
Oil	0	9.1	16.6	0	5.5	9.4	0.8	1.9	6.1	0	2	4.9	5.4	6.0
Gold	0	0	0	0	1.2	1	0	3.2	3.9	0	5	5	0.4	3.1
Silver	0	1.6	0	0	1.7	2.1	0.3	0	0	0	0	0	3.2	0.0
Wheat	–	–	–	–	–	–	0	0	0	0	0	0	–	0.0
FX1	–	–	–	–	–	–	0	1	0	10.8	3.2	0	–	0.0
FX2	–	–	–	–	–	–	0	1.5	0	0	0.3	0	–	0.0
FX3	–	–	–	–	–	–	14.6	0	0	14.4	0	0	–	0.0
One-Month CD	–	–	–	–	–	–	0	90.5	53.7	0	77.9	53.1	–	62.7
Real Estate	–	–	–	–	–	–	–	–	–	0	1.7	0.2	–	–
World Equities	–	–	–	0	0	0	0	0	0	0	0	0	4.0	0.0
World Bonds	–	–	–	0	0	0	0	0	0	0	0	0	0.0	0.0
Emerging Equities	–	–	–	0	6.7	14.2	0	1.9	9.6	0	0.3	3.3	10.1	11.1
Emerging Bonds	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Tracking Error	4.9	6.1	10.2	4.1	5.1	8.3	2.4	2	4.3	1.9	1.2	2.7	6.83	3.92
Return–Risk	0.15	0.41	0.44	0.13	0.49	0.54	0.21	1.25	1.05	0.26	2.09	1.67	0.52	0.86

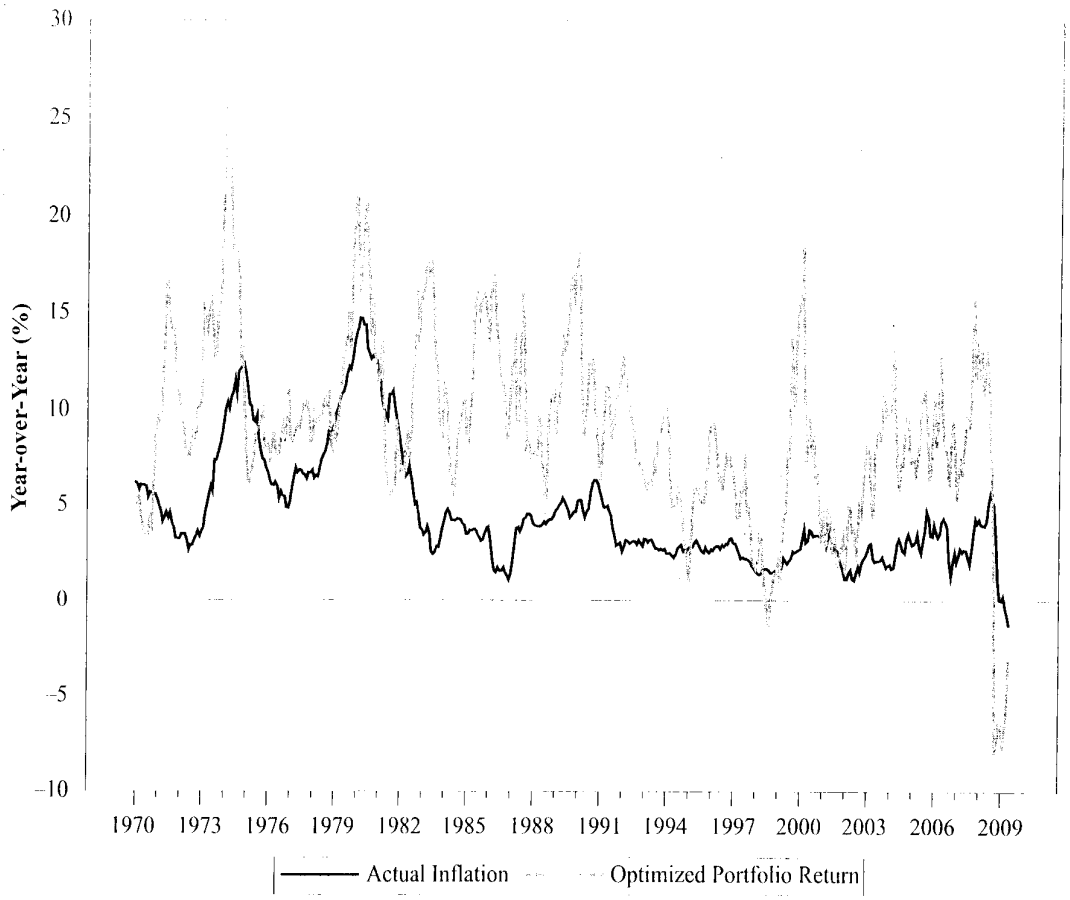
Note: The asset class weights are the weights from an optimization to minimize the portfolio return monthly tracking error with inflation, while producing the desired real return target listed. The weights of the asset classes are constrained to be greater than 0 (i.e., no shorting) and the weights must sum to 1 (i.e., fully invested). Mathematically, the optimization is $\min_w \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} - 2\boldsymbol{w}'\boldsymbol{\gamma}$ *s.t.* $\boldsymbol{w}'\boldsymbol{\mu} = \tilde{\mu}_p + \pi_{t,t+k}$. *The "–" represents asset classes whose returns did not exist at the starting period, while "0.0" indicates an asset class whose data existed at the starting period, but the optimization led to a zero weighting. "Tracking Error" represents the realized annualized tracking error of the portfolio versus inflation and "Return–Risk" represents the realized real return of the portfolio divided by the realized tracking error. Optimal portfolios are estimated from January of the starting period until May 2009.*

30-year Treasury bonds, 3.4% corporate bonds, 4.9% oil, 5% gold, 53.1% one-month CDs, 0.2% real estate, and 3.3% emerging equities. The optimized portfolio returns and actual inflation are depicted in Exhibit 3 for the period January 1970–May 2009.

A very consistent theme in all of the optimizations is that a large part of the portfolio should be held in short-term fixed-income instruments, either Treasury bills or USD one-month CDs. Another fairly common theme is that an allocation of less than 10% should be held in gold and oil. Also a good part of the portfolio should be held in a combination of 10-year and 30-year government bonds and corporate bonds. Perhaps the most surprising result is the small role that equity plays in the allocations.

EXHIBIT 3

U.S. Optimized Portfolio Returns versus Inflation (Year-over-Year), January 1970–May 2009



For many practitioners this may seem counterintuitive and contrary to a commonly held belief that equities are a good hedge against inflation. This is not as counterintuitive as it first seems, however, given our objective function. Our goal is to earn a target real return while minimizing the deviation from inflation over any given yearly horizon. Thus, for a 4.5% real return, there are other combinations of asset classes that achieve this goal while providing the portfolio with much less risk than equities. For example, corporate bonds provide a very high return with lower risk than equities.

Performance. The performance of these allocations is what ultimately matters to investors. Exhibit 4 shows the performance of the optimization analysis over the entire period. The performance statistics for the in-sample optimizations for the 1930 and 1970 periods are located in the top panel of the exhibit. For 1970, the mean yearly real return was 4.50% and the standard deviation of real returns was 4.30%.[9] The worst real return of this allocation was −11.69% for the year ending October 2008. This model produced a negative real return in 13.11% of the periods as reported in the column "Number $\leq \pi$." The mean absolute error (MAE) of this allocation, that is, the absolute return of the portfolio minus inflation averaged over the whole period, was 5.12%.

We produced three benchmarks that an investor might be more familiar with. One is an all-equity allocation, another is an all-bond allocation, and the third is an allocation of 50% bonds and 50% equities. The allocations are shown in the in-sample optimization results in Exhibit 4. The all-equity portfolio had a higher mean return of 6.38%, however, the increase in return came at a cost. In particular, this portfolio had a much larger volatility around inflation of 18.43%. The worst year-over-year real return for the all-equity portfolio was −50.89% in the year ending September 1974. In 31.71% of the periods, the all-equity portfolio produced a negative real

return; the MAE is more than three times as large as the optimal allocation.

In contrast, the 100% bond allocation leads to a slightly lower mean real return of 4.22%, but surprisingly a much higher volatility around inflation of 11.85%, a worst year-over-year return of –27.57%, and a negative real return 38.69% of the time. The 50% equity and 50% bond compromise provides a better mean real return of 5.30%, but a much higher volatility of 12.56% and a worst case return of –31.51%, and it underperforms inflation 36.58% of the time.

Thus, the optimized 1970 allocation produced a portfolio with a reasonable real return versus standard alternatives, but with a much lower downside with respect to inflation.

Out-of-Sample Results

Optimal weights. Exhibit 2 contains the average out-of-sample optimal weights for the 1930 and 1970 periods. The average allocation from the out-of-sample optimizations for 1970 is 1% equities, 16% corporate bonds, 6% oil, 3% gold, 63% one-month CDs, and 11% emerging equities. The results for the 1930 period are somewhat similar, but due to the absence of a one-month CD, more weight was distributed to the other

EXHIBIT 4

In-Sample and Out-of-Sample Real Return Portfolio Performance

Country	Period	Target Return	Mean	Standard Deviation	Max Value	Max Date	Min Value	Min Date	Number ≤ π	MAE	RMSE	No. of Obs.
In-Sample Optimizations												
Optimal Portfolio	1930	4.50	4.50	8.34	45.34	1933:06	-24.54	1947:05	24.66	7.23	9.47	953.00
Optimal Portfolio	1970	4.50	4.50	4.30	15.43	2000:02	–11.69	2008:10	13.11	5.12	6.22	473.00
100% Equity	1930	–	7.81	21.90	167.23	1933:06	–57.92	1932:06	33.37	17.90	23.24	953.00
100% Equity	1970	–	6.38	18.43	58.60	1983:06	–50.89	1974:09	31.71	16.03	19.49	473.00
50% Equity–50% Bond	1930	–	5.09	12.78	90.90	1933:06	–31.51	1974:09	34.84	10.45	13.75	953.00
50% Equity–50% Bond	1970	–	5.30	12.56	48.10	1983:06	31.51	1974:09	36.58	10.62	13.62	473.00
100% Bond	1930	–	2.37	9.66	45.88	1986:03	–27.57	1980:03	39.35	7.07	9.95	953.00
100% Bond	1970	–	4.22	11.85	45.88	1986:03	–27.57	1980:03	38.69	9.31	12.56	473.00
Out-of-Sample Optimizations												
Optimal Portfolio	1935	4.50	3.52	6.83	8.49	2009:05	-10.97	1980:03	40.87	1.45	1.99	893.00
Optimal Portfolio	1975	4.50	3.38	3.92	4.81	2009:05	–5.03	2008:10	38.01	0.87	1.16	413.00
100% Equity	1935	–	7.58	16.07	25.39	1938:06	24.51	1938:03	40.65	3.54	4.68	893.00
100% Equity	1975	–	7.81	15.53	12.83	1987:01	–21.80	1987:10	40.92	3.46	4.52	413.00
50% Equity–50% Bond	1935	–	4.70	9.38	12.72	1938:06	–12.41	1940:05	42.78	2.09	2.73	893.00
50% Equity–50% Bond	1975	–	6.20	9.85	9.48	1982:10	–7.95	1987:10	42.62	2.23	2.89	413.00
100% Bond	1935	–	1.81	7.34	13.22	1980:04	–10.46	1980:02	45.46	1.42	2.12	893.00
100% Bond	1975	–	4.59	9.59	13.22	1980:04	–10.46	1980:02	42.13	2.06	2.79	413.00
In-Sample and Out-of-Sample Portfolio with and without TIPS												
Optimal w/TIPS IS	1998:02	4.50	4.50	2.28	9.81	2008:02	-3.62	2008:10	1.47	4.57	5.04	136.00
Optimal w/TIPS OS	2003:02	4.50	2.46	4.93	4.16	2008:12	–5.60	2008:10	36.84	1.07	1.43	76.00
Optimal w/o TIPS IS	1998:02	4.50	4.50	2.28	9.82	2008:02	-3.54	2008:10	1.47	4.57	5.04	136.00
Optimal w/o TIPS OS	2003:02	4.50	2.60	4.78	3.82	2008:12	–4.85	2008:10	38.16	1.05	1.39	76.00
100% TIPS	1998:02	–	4.31	6.33	6.00	2008:12	–7.68	2008:10	38.97	1.30	1.85	136.00
100% TIPS	2003:02	–	2.89	7.71	6.00	2008:12	–7.68	2008:10	43.42	1.63	2.22	76.00

Notes: The exhibit reports in-sample and out-of-sample statistics from the optimization models reported in Exhibits 2 and 5. The out-of-sample optimizations were performed on an expanding window that begins five years after the beginning period listed. Optimal weights are chosen and used to create a portfolio that is used for the next period's hedging portfolio. "Period" is the set of variables and the time period used in the optimization, "Target Return" represents the target real return that the portfolio was optimized to achieve while minimizing the tracking error with inflation, "Mean" is the monthly average of the difference between year-over-year inflation and the model's return, "Standard Deviation" is the standard deviation of the difference in the model's value and year-over-year inflation, "Max" and "Min" are the largest (smallest) monthly difference between year-over-year inflation and the model's value, "Number ≤ π" is the percentage of months in which the model's value is lower than the year-over-year inflation for that month, "MAE" is the mean absolute error of the errors, "RMSE" is the root mean squared error between model values and year-over-year inflation, and "No. of Obs." is the number of monthly observations used to compute the performance statistics. All values are multiplied by 1000 except for the number of observations.

asset classes, especially bills, 10-year Treasury bonds, and corporate bonds.

Performance. The performance of the out-of-sample regressions and optimizations is reported in Exhibit 4. First, we note that the target real return of 4.5% was not achieved. Instead, a real return of 3.38% with a volatility of 3.92% was obtained. The worst performance in any given month was −5.03%. The percentage of months with negative real returns was higher than the in-sample case at 38%.

Comparing this out-of-sample performance to more common allocations shows that a 100% equity position has a 7.81% average return over the same period. The all-equity allocation underperforms in the worst case by 22% and underperforms inflation 41% of the time.

Given how difficult it is to perform well out of sample, these results are quite encouraging. They suggest that the inflation-hedging methodology might be implementable by the investor to achieve a desired target real return while minimizing downside exposure to inflation.[7]

TIPS Results

Earlier, we discussed the pros and cons of using inflation-protected bonds in a portfolio to hedge against inflation. Treasury Inflation-Protected Securities (TIPS) were introduced in the United States in March 1997. We obtained return data from the Lehman TIPS Index from February 1998 to May 2009. This TIPS index has a duration of approximately four years. In order to examine what role TIPS might play in our investor's universe, we performed optimizations from February 1998 to May 2009, including the TIPS total return index. The results are displayed in Exhibit 5. Columns (1) and (2) include TIPS in the analysis for the in-sample and out-of-sample optimizations, respectively. Columns (3) and (4) do not include TIPS for in-sample and out-of-sample optimizations, respectively.

The in-sample optimal allocation does not put a large weight on TIPS. In fact, the weight is a mere 1.6%. The dynamic out-of-sample optimization, however, puts a large weight on TIPS; ultimately, it is important to compare the performance of the two.

Exhibit 4 reports the performance of the various models with TIPS and without TIPS in the portfolio. As would be expected, virtually no difference in results exists between the in-sample TIPS and no TIPS, but this was to be expected given the small weight on TIPS. For the

EXHIBIT 5

In-Sample and Out-of-Sample Inflation-Optimized Portfolios with and without TIPS, February 1998–May 2009

	With TIPS		Without TIPS	
Asset Classes	**(1)**	**(2)**	**(3)**	**(4)**
Equity Index	0.00	0.00	0.0	0.00
Small Cap	0.00	0.00	0.0	0.00
Value	0.00	0.00	0.0	0.00
Growth	0.00	0.00	−0.0	0.00
S.CD	0.00	0.47	−0.0	0.40
S.CS	0.00	0.54	0.0	0.00
S.Energy	4.90	4.27	4.9	2.88
S.Finance	0.00	0.00	−0.0	0.00
S.Health	0.00	0.00	−0.0	0.00
S.Industrials	0.00	0.00	0.0	0.00
S.IT	1.60	3.11	1.5	1.59
S.Materials	0.00	0.00	−0.0	0.00
S.Telcomm	0.00	0.00	0.0	0.00
S.Utility	0.00	0.00	−0.0	0.00
Bill Govt.	0.00	0.00	0.0	0.00
10-Yr. Govt.	14.70	7.49	16.2	21.00
30-Yr. Govt.	15.90	1.27	15.3	0.88
Corp. Bond	0.00	0.00	0.0	12.70
HY Bond	0.00	0.00	0.0	0.00
IP Bond	1.60	36.33	–	–
Commodity	0.00	0.00	−0.0	0.00
Oil	2.00	1.21	2.1	4.34
Gold	10.3	4.09	10.2	4.01
Silver	0.00	0.00	0.0	0.00
Wheat	0.00	0.00	−0.0	0.00
FX1	0.00	0.00	0.0	0.00
FX2	0.00	0.00	0.0	0.00
FX3	0.00	0.00	0.0	0.00
One-Month CD	43.30	38.99	43.5	44.06
Real Estate	0.00	1.81	0.0	4.70
World Equities	0.00	0.00	0.0	0.00
World Bonds	0.00	0.24	−0.0	2.68
Emerging Equities	0.00	0.00	−0.0	0.51
Emerging Bonds	6.00	0.20	6.2	0.24
Target Return	4.50	4.50	4.50	4.50
Realized Return	4.50	2.46	4.50	2.60
Tracking Error	2.28	4.93	2.28	4.78
Return–Risk	1.98	0.50	1.98	0.54

Note: The exhibit contains the optimizations from February 1998 to May 2009 including the TIPS total return index and without it. Columns (1) and (2) contain the in-sample and out-of-sample optimization weights, respectively. Columns (3) and (4) contain the same excluding the TIPS total return index. "–" indicates that a variable was not available for the time period of estimation. All of the in-sample estimates are from February 1998 because one year is needed to compute the year-over-year return, and the out-of-sample results are from February 2003, because five years are used before first estimates are formed. "Target Return" represents the target return of the optimization used in order to create the optimal portfolio, "Tracking Error" represent the realized annualized tracking error, and "Return–Risk" represent the realized real return of the portfolio divided by the realized tracking error.

out-of-sample results, the average real return of the TIPS model is 2.46% versus 2.60% for the no-TIPS model, and the volatility around inflation is also higher at 4.93% with TIPS versus 4.78% without TIPS. The evidence suggests that a strong benefit is not derived from including TIPS in the portfolio.

Finally, the lower panel of Exhibit 4 reports the returns to purchasing the Lehman Brothers TIP Index as a stand-alone asset over the same periods. For the longer period, February 1998–May 2009, the average real return of TIPS is 4.31% compared to 4.5% for the optimizated portfolio without TIPS. The standard deviation of the optimized allocation is half as much as TIPS alone (2.28% versus 6.33%, respectively). In this period, TIPS underperforms inflation in 39% of the months, while the optimized portfolio underperforms inflation in only 1.47% of the months. The TIPS worst monthly underperformance is –7.68%, which is twice that of the optimized portfolio.

For the shorter period, February 2003–May 2009, we compare the TIPS performance to the out-of-sample optimized portfolio without TIPS. The mean real return of the TIPS is 2.89% versus 2.60% for the optimized portfolio. The standard deviation is higher at 7.71% versus 4.78%. TIPS underperforms inflation in 43% of the months, while the portfolio does so only 38% of the time. The optimized portfolio also has a smaller worst case monthly real return.

The results seem to indicate that an optimized portfolio is better at providing a real return while minimizing downside risk than simply purchasing TIPS.

Horizon Length and Higher Target Returns

The main analysis of our article considers an investor attempting to mimick or outperform the year-over-year inflation rate. It could be argued that most investors have a slightly longer horizon. Although not reported here, but contained in a supplemental appendix available upon request, we considered two other horizons: a 5-year (60-month) and a 10-year (120 month) horizon. We found that the qualitative nature of the results did not change. An investor would choose an asset mix of 50% Treasury bills or one-month CDs, 31% 10-year bonds, around 8% in oil and gold, and some real estate for the 5-year horizon to achieve a 23% total real return over the period, or 4.5% annualized. For the 10-year horizon, an investor would choose about 8.3% equities, 52% Treasury bills, 2.1% 30-year Treasury bonds, 16.1% general commodities, 15% oil/silver/gold, and about 5.7% real estate for a return of 45%, or 4.5% annualized.

We also ran the optimization for a target real return of 6.38% over the 1970 period, which would be in line with the historical average real return of U.S. equities over this period. This portfolio had a higher exposure to equity-related asset classes, but still very little equities. In particular, it consisted of 53% corporate bonds, 11% oil, 4% gold, 15% one-month CDs, and 17% emerging equities.

Downside Risk and Short-Sales Results

We mentioned earlier that alternative optimizations might be considered. In particular, we performed the optimization allowing for short selling of asset classes and minimizing downside risk rather than tracking error. Although not reported here, but available upon request, the results are qualitatively very similar to the other optimizations.

Inflationary versus Deflationary Episodes

An investment advisor or portfolio manager who believes it is possible to forecast inflation might be interested in understanding what asset classes are optimal during inflationary and deflationary episodes rather than

EXHIBIT 6

Bai and Perron Structural Breaks in U.S. Inflation Series

Beginning Year	Ending Year	Average Inflation (%)
1901:01	1916:07	2.22
1916:08	1920:10	16.44
1920:11	1922:09	–7.98
1922:10	1930:08	0.09
1930:09	1933:05	–9.04
1933:06	1941:04	0.95
1941:05	1946:05	5.24
1946:06	1948:07	12.60
1948:08	1973:04	2.44
1973:05	1982:06	9.12
1982:07	2009:05	3.12

Note: Bai and Perron (2003) tests were constructed with a constant as the regressor on the year-over-year inflation rate allowing for a maximum of 10 breaks from January 1901 to May 2009.

over an entire economic cycle. In this section, we investigate this question. We use the Bai and Perron [2003] methodology to find structural breaks with just the constant as a regressor on the year-over-year U.S. inflation series allowing up to 10 breaks in the data. Since 1901, this procedure identified 10 structural changes in the inflation series. These breaks are listed in Exhibit 6.

For these 10 periods, we ran the portfolio optimization for three of the longest inflationary periods and the only two deflationary periods in order to achieve a target real return of 4.5%. The optimal portfolio weights are presented in Exhibit 7.

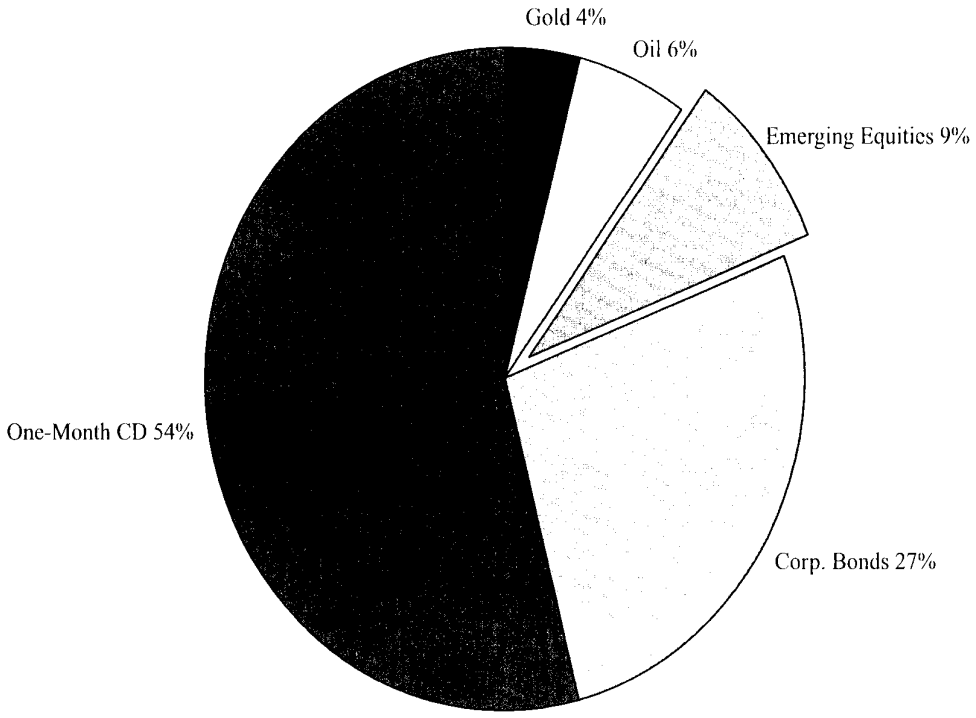
For the high-inflation period of August 1916–October 1920, the best portfolio consisted of 17% equities,

EXHIBIT 7
Optimal Portfolios in Inflationary and Deflationary Episodes

		Equities			Commodities				Cash	Bonds		Real Estate
		Equity Index	Emerging Equities	World Equities	Oil	Silver	Gold	Wheat	One-Month CD	T-Bill	World Bonds	REIT
Inflationary Periods												
1916:08	1920:10	16.57	–	–	63.33	20.10	0	–	–	0	–	–
1946:06	1948:07	0	0	0	0	18.21	18.02	63.77	–	0	0	–
1973:05	1982:06	0	10.54	0	9.28	0	0	0	61.72	0	0	17.57
Deflationary Periods												
1920:11	1922:09	0	–	–	18.31	2.92	78.75	–	–	0	–	–
1930:09	1933:05	0	0	7.14	23.11	0	0	0	–	32.09	37.66	–

Note: In-sample optimal portfolios created over specific historical period that coincides with either high inflation or deflation.

EXHIBIT 8
U.S. Investment Allocation with Average 4.5% Real Return



63% oil, and 20% silver. For the inflationary period June 1946–July 1948, the best portfolio consisted of 18% silver, 18% gold, and 64% wheat. For the most recent inflationary episode of May 1973–June 1982, the optimal allocation consisted of 11% emerging equities, 9% oil, 61% one-month CDs, and 18% real estate. This allocation is not too different from optimizations reported for the entire period.[8]

For the two deflationary periods, the results were similar in spirit. For the period November 1920–September 1922, the optimal portfolio consisted of 18% oil, 3% silver, and 79% gold. For the period September 1930–May 1933, the optimal portfolio consisted of 7% world equities, 23% oil, 32% Treasury bills, and 38% world bonds—essentially, 70% fixed income, oil, and a small amount of equities.[9]

For some inflationary periods, commodities, especially gold and oil, are very important, but contrary to common perception, equities play a very small role in a portfolio designed to minimize tracking error with respect to inflation while providing a reasonable real return.

CONCLUSION

In this article, we examined the possibility of hedging inflation by using a set of available asset classes from the perspective of a U.S. investor. We analyzed different sets of time periods as far back as 1901. We constructed these portfolios by mean-variance optimization of real returns.

We found that for an investor who wishes to achieve a given real return while minimizing the downside deviation of the portfolios return due to inflation in any given period, the best allocation usually did not include equities. In fact, the best allocation consisted of some combination of Treasury bills or one-month CDs, Treasury bonds, some gold, some oil, and some emerging market equities. We also found that although gold is important in providing a real return while minimizing the downside risk from inflation, the optimal allocation ranges only from 5% to 10% of the portfolio's value. The optimal allocation to oil is also small at about 5% of the portfolio. Only during specific high-inflationary periods—but not over longer investment horizons—do gold and oil play a more important role in the asset mix.

We found that although inflation-protected bonds may be good instruments for hedging inflation when owned from issuance to maturity, they have higher volatility than a portfolio of assets and seem to be less important when combined with other asset classes.

We found that the portfolio approach did well in sample and out of sample. The average realized real returns were positive and close to their target real returns and were achieved with low tracking error.

Future research might consider altering the methods of the optimization technique including rolling windows and different optimization algorithms. Further research might also consider higher target real returns. One might also want to examine other investment periods in which more asset classes were available to hedge inflation.

Based on our analysis, the optimal allocation for a U.S. investor with a 4.5% real return target would be that shown in Exhibit 8.

ENDNOTES

The authors would like to thank Julius Taranto for research assistance, John Barwick, Brad Case, Pierangelo De Pace, Michele Gambera, John Jurewitz, Jerry Hausman, Stephen Marks, Hans Palmer, Sofia Ramos, Dirk Schilling, Chris Sims, Gary Smith, Slavi Slavov, Alexander Subbotin, Bryan Taylor, Robert Whitelaw, David Fogel, Adam Patti, participants at the IMCA annual conference, participants of the FMA Europe in Hamburg, and participants of the EFMA in Denmark.

[1]As Milton Friedman stated in 1963, "Inflation is always and everywhere a monetary phenomenon."

[2]The Dow Jones commodity index is a rolling index in that it rolls over short-term futures contracts. The wheat variable is the return from rolling the near-term futures contract forward. Other research has found that the return of the future differs from that of the spot due to the roll yield and the collateral yield in the commodity markets. We recognize this, but feel it would not change the nature of our results. At most, it might raise the weight for the oil commodity because the historical roll yield in the oil market has been higher. Erb and Harvey [2006, p.80] argued the possibility of a zero roll return in the future, because there is no guarantee of consistent backwardation in the future.

[3]Erb and Harvey [2006].

[4]In cases where return distributions depart from normality, it may be more accurate to specify the investor's optimization problem in terms of minimizing downside risk, rather than variance. One such choice to measure downside

risk is semi-variance, and thus the investor's objective function becomes

$$\min_{w_i} \frac{1}{T}\sum_{t=1}^{T}\left[\min\left(\sum_{i=1}^{N} w_{i,t} r_{i,t+k} - \pi_{t,t+k}, 0\right)\right]^2 \quad \text{s.t.} \quad \left(\sum_{i=1}^{N} w_{i,t} r_{i,t+k} - \pi_{t,t+k}\right) = \bar{\mu}_p \tag{6}$$

[5]See Chincarini and Kim [2006].

[6]This is also the tracking error with respect to inflation.

[7]Further research might consider less frequent re-optimizing intervals, examining other periods of study, and using a rolling window to optimize rather than continually adding data to the window. We also think a method of using a constrained regression may provide more useful information and perform even better out of sample.

[8]More research is needed to examine whether it was the causation of the inflation that led to different weightings or if it was the availability of additional hedging instruments.

[9]During these specific deflationary periods, a 4.5% real return was achieved with these allocations, but, in fact, this point was not an efficient point. A much higher return could have been achieved with a lower or similar level of risk. In fact, for the 1920–1922 period, a better portfolio would have achieved a 10.25% real return with a tracking error of 3.1% over the period. It would have consisted of 87% Treasury bills and 13% oil. For the 1930–1933 period, a more efficient portfolio would have produced an average real return of 10% with a tracking error of 1.2%. This allocation would have consisted of 91% Treasury bills, 7% world bonds, 1% equities, and 1% corporate bonds.

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