The Advantages of Tax-Managed Investing

Avoiding the drag.

Ludwig Chincarini and Daehwan Kim

For simplicity and tractability, financial models often assume a frictionless world without taxes or transaction costs. While advances in on-line portfolio trading have reduced transaction costs for retail investors almost to the vanishing point, taxes remain a significant barrier to achieving the dream of the frictionless world. In fact, the same cost-saving technology can also help even passive investors reduce their taxes through tax-motivated investment strategies.

The goal of our study is to explore the impact of tax strategies on after-tax investment returns. We look at different strategies an investor might adopt to reduce taxes, including the selling of losers to offset capital gains. We also examine strategies that avoid dividends to reduce income tax. All the strategies we examine are realistic and easily implementable using popular on-line brokerage services or separately managed accounts.

To illustrate how much taxes matter, even to a passive index investor, consider one of the most tax-efficient passive vehicles thus far, the Vanguard Index 500 Fund. A taxable investor who bought the Vanguard 500 fund in 1976 and sold it in 2000 would have given up over 43% of the terminal value in taxes.

These tax losses come from three areas. The first is from the taxable events generated by fund turnover that occur as a a result of investor cash inflows and outflows, as well as stocks entering and leaving the index. For example, the Vanguard 500 had a 6% turnover in 1999. If you could achieve the same gross return with a much lower turnover, you would be unambiguously better off due to savings in taxes.[1]

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The second source of tax losses is taxes on dividends. The third source of tax losses is capital gains taxes on the assumed sell at the end.

We begin by discussing the theoretical case of tax management and quantifying the extent to which tax effects can hurt investor returns in an idealized setting. There are two types of tax effects. The first occurs when capital gains are realized as short-term instead of long-term gains. The second effect occurs due to forgone earnings from the premature payment of taxes. We measure these two effects separately.

To show the effectiveness of tax-aware portfolio management using real data in real-world situations, we compare the returns to two types of investors, the tax-aware investor (the *t-smart* investor) and the non-tax-aware investor (the *naive* investor). Because the two types of investors could have substantially different portfolios, which could confound any inferences, we attempt to keep their portfolios the same by keeping observable portfolio characteristics as similar as possible. Two portfolios with the same characteristics should have the same expected returns in the spirit of Daniel and Titman [1997].

We make sure that the portfolio of the naive investor and the portfolio of the t-smart investor have the same characteristics by letting the t-smart investor sell "loser" stocks (i.e., stocks that have capital losses) and replace them with characteristically matched stocks. While the t-smart investor's portfolio has the same characteristics as the naive investor's portfolio, the t-smart investor can manage to keep the realized capital gains below a certain level, thus reducing immediate tax payments.

The after-tax returns of the two investors may differ for more reasons than tax effects. The naive investor trades only to track index changes or to generate cash outflow, but the t-smart investor sells losers and replaces them. Thus, if there is positive serial correlation in returns as in Lo and MacKinlay [1990], the t-smart investor will benefit from selling losers and replacing them with new stocks, because the losers would have kept on losing. On the other hand, a reversal effect that results in negative serial correlation would penalize a strategy of selling the losers.

Regardless of the ex post returns, one can nevertheless compute the *effective tax rate* for both investors and determine the degree to which postponing taxes helps the t-smart investor. The effective tax rate is defined as the ratio of the total tax payment to the total capital gains. The tax benefit from tax-efficient investing is computed by comparing the effective tax rate of the naive investor and the effective tax rate of the t-smart investor.

TAXES AND ACTIVE PORTFOLIO MANAGEMENT

Many studies document the effects of taxes on investors' returns.[2] To demonstrate the impact that taxes can have on returns, we examine the impact of taxes on Vanguard's popular S&P 500 index fund. The Vanguard Index 500 Fund is the largest mutual fund in the world with over $100 billion in assets (as of April 30, 2000). As an index fund, it buys and sells stocks only to the degree that the S&P 500 stocks change. These changes are infrequent, so the Vanguard 500 turnover rate was 6% in 1999 compared to an average turnover rate of all domestic equity mutual funds of 70% and of actively managed mutual funds of 77%.[3]

We consider the tax effects to an investor who invests $10,000 in the Vanguard 500 fund in August 1976 and holds the investment until the end of 1999. Without taxes and fees, the investment would have grown to $308,200. In the "ideal" state, the investor keeps 100% of the investment's value, which is calculated from the paper performance of the S&P 500 index alone.

EXHIBIT 1
VANGUARD INVESTMENT PIE BEFORE LIQUIDATION (GAINS AND DIVIDENDS)

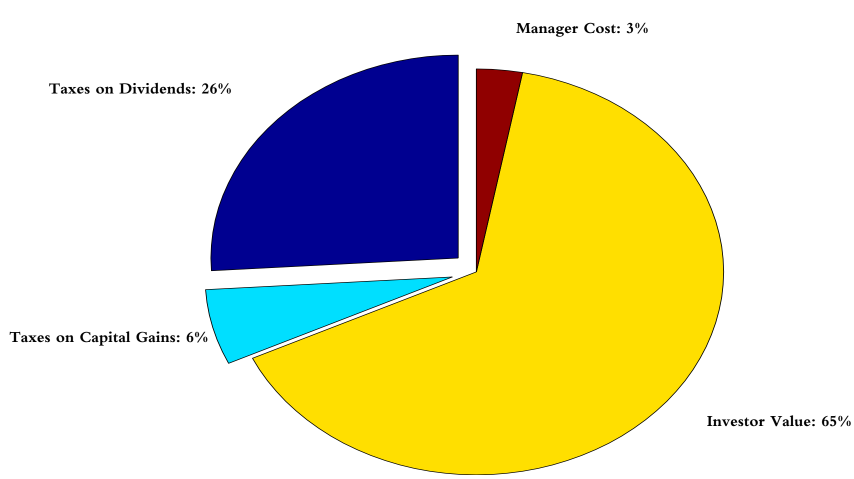
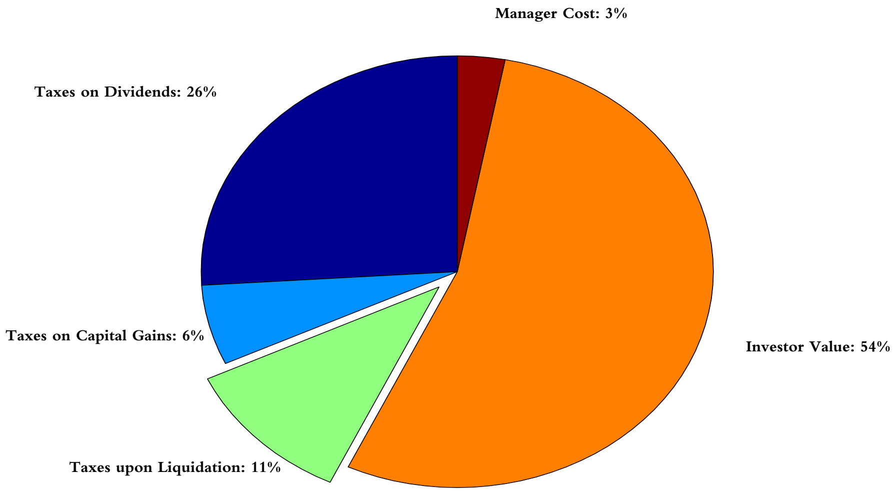


EXHIBIT 2
VANGUARD INVESTMENT PIE AFTER LIQUIDATION



At a capital gains rate of 20% and an income tax rate of 31%, the investment would grow to only \$200,500, however, assuming that the investor did not liquidate the portfolio at the terminal date.[4] Thus, a taxable investor would have lost 32%—almost a third — of the total investment to taxes, even in the low-turnover Vanguard index fund. Upon closer inspection, one finds that about a fourth of the tax drag, 26% of that 32% paid in taxes, is due to dividends paid on the underlying stocks (see Exhibit 1).

Even this is not the entire story, because we do not assume liquidation of the investor's holding at the end of the study. If we consider liquidation at the end of the horizon, the original \$10,000 grew to only \$167,700. Exhibit 2 shows the investment pie after liquidation. Now, of the total theoretical accumulation, 43% goes to taxes, 3% disappears in management fees, and 54% is left for the investor. Thus, even for a seemingly long 25-year period, the index fund suffers significant tax drag.

Tax-managed funds are certainly a step in the right direction, but they still present problems. Investors in tax-managed funds are not totally immune to capital gains taxes; these funds can still force an investor to pay taxes sooner than desired. The reason is that funds—unlike individually held stock—are not tailored to one particular individual's tax situation. For example, the fund may be forced to liquidate certain positions to meet shareholder redemptions if there are large net outflows from the fund. Even if investors intend to hold on, they will be exposed to taxes.

How much extra return does one obtain from tax management, and how much can one reduce the amount of pie that taxes swallow? We investigate techniques to manage taxes efficiently, and we try to examine what the real savings from tax management could be. In a world of negligible trading costs, an index fund is certainly not as tax-efficient an answer as a tax-managed portfolio that is owned by the individual investor. But how much can an investor actually gain from realistic tax-motivated strategies?

THEORY BEHIND TAX MANAGEMENT

Despite the claims of enormous savings from efficient tax management, it may be useful to pinpoint exactly where the savings from tax management really come from. One can represent the tax benefits by using a recursive equation to show how taxes affect returns year-to-year.

In this simplified setting, it is assumed that there is a constant long-term capital gains rate, τ_l; a constant personal income tax rate, τ_s; a constant return on the portfolio, r; a constant risk-free rate of interest, r_f; and a constant proportion of the return that is realized by short-term, l_s, or long-term gains, l_l. The value at the end of j periods for the investment is:[5]

$$V_{t+j} = V_t \left(1 + r(1 - \underbrace{(l_s\tau_s + l_l\tau_l)}_{\psi})\right)^j \qquad (1)$$

where $0 \leq l_s + l_l \leq 1$. One can observe that when taxes are not realized (i.e., $l_s = l_l = 0$) or when taxes do not exist (i.e., $\tau_s = \tau_l = 0$), the final value of the fund ignoring taxes is $V_{t+j} = V_t(1 + r)^j$.

This terminal value in Equation (1) does not consider the post-liquidation value of the investment. In order to compute the post-liquidation value of the investment, one needs to compute the basis, B_t, of the investment at any given time. This basis determines the amount of appreciation of the investment still subject to taxes. The basis, B_t, at any given time is given by:

$$B_{t+j} = B_t \left(1 + \left[(1-\tau_s)l_s + (1-\tau_l)l_l\right] r\right) \tag{2}$$

It is important when comparing return on investments to consider the post-liquidation value of the fund. Otherwise the analysis biases the results in favor of the investor who postpones taxes.[6]

The post-liquidation value of the fund, $\tilde{V}_{t+j}$, after j periods, is:[7]

$$\tilde{V}_{t+j} = V_t \Big(\left[1 + r(1-\psi)\right]^j (1-\tau_l) \cdot + \left[1 + (l_s + l_l - \psi)r\right]^j \tau_l \Big) \tag{3}$$

These equations enable us to identify the benefits of managing the taxes of one's investment. There are two benefits: short-term versus long-term taxes and forgone earnings.

Short-Term versus Long-Term Gains

The first benefit of tax management comes from the trade-off between l_s and l_l. Since long-term gains are usually taxed at a lower rate than short-term gains or dividends ($\tau_l < \tau_s$), and returns in this example are assumed to be equivalent, the investor should always minimize the possibility of short-term capital gains. This will unambiguously improve the after-tax return. The amount of that improvement is exactly the difference between τ_s and τ_l.

For example, suppose that the return in any given year is 10%, that the long-term capital gains rate is 20%, and that the short-term capital gains rate is 31%. Then the investor gains 11% of the realized return by postponing capital gains. In this simple case, it amounts to 1.1% of the total return. Of course, dividends are also taxed at τ_s, so part of l_s comes from dividends. Reducing one's dividend payments is also beneficial from a tax perspective.[8]

Forgone Earnings

The second benefit is the forgone earnings benefit. By postponing taxes on an investment, one is able to realize compounded growth on the taxed amount for the remaining years in the investment horizon. This can be great or small depending on the compounding rate used. From Equation (3), the return benefit one can obtain from avoiding the forgone earnings (*fe*) over j years with an annual rate of realization of l_l is given by:

$$r_{fe} = \frac{\tilde{V}^0_{t+j} - \tilde{V}^{l_l}_{t+j}}{V_t} \tag{4}$$

where $\tilde{V}^0_{t+j} = \tilde{V}_{t+j}$ with $l_l = 0$ and $\tilde{V}^{l_l}_{t+j} = \tilde{V}_{t+j}$ with a specified value of l_l. Intuitively, r_{fe} is essentially the return difference between someone who totally postpones all capital gains at a uniform tax rate of τ_l and someone who realizes the tax on the return each year at rate l_l at tax rate τ_l.[9]

An alternative estimate of the forgone earnings effect can be obtained by using the opportunity cost of cash. That is, rather than using the internal return of the investment, one can use the return on the cash assuming that the investor pays taxes from a cash account. This avoids the problem that the forgone earnings calculation is too dependent on any specific investment. In this case, the benefit one can obtain from avoiding the forgone earnings is:[10]

$$\tilde{r}_{fe} = r\tau_l l_l (1-\tau_l^*) \times \left((1+r_f)\delta^{j-2} \left[\frac{1 - \left(\frac{1+r_f}{\delta}\right)^{j-1}}{\left(1 - \frac{1+r_f}{\delta}\right)} \right] - \left[\frac{1-\delta^{j-1}}{1-\delta} \right] \right) \tag{5}$$

where τ_l^* is set to τ_l if the investor puts the tax payments in a risk-free investment and to 0 if the investor borrows to pay the taxes at rate r_f, and $\delta = 1 + r(1 - \tau_l l_l)$.[11] This methodology has the advantage that the forgone earnings effect is not tarnished by the ex post returns of a specific investment in a specific time period. Whether the ex post returns of a specific investment over a specific time period are positive or negative, the forgone earnings effect should be positive. Henceforth we compute forgone earnings using the second method, or the risk-free rate method with τ_l^* set to zero.

Measures of Tax Drag

To obtain an idea of the magnitude of the tax and forgone earnings effects, we construct three measures of the effect of forgone earnings on investment returns. The first measure is the *effective tax rate*, which essentially measures the amount of taxes one pays on forgone earnings losses. We measure it using the risk-free rate of interest as the forgone earnings rate:

$$\tau_e = \frac{V^*_{t+j} - \tilde{V}_{t+j}}{V^*_{t+j} - V_t} \qquad (6)$$

where $V^*_{t+j} = \tilde{V}_{t+j} + [\tilde{r}_{fe} + r\tau_l l_l \,(1\text{-}\delta^j/1\text{-}\delta)]\ V_t$, and $\tilde{V}_{t+j}$ is calculated with the specified value of l_l. This effective tax rate is the final value that would have been obtained by the investor with zero foregone earnings (V^*_{t+j}) minus the actual investor's terminal value ($\tilde{V}_{t+j}$) divided by the total appreciation of the investment had there been zero foregone earnings ($V^*_{t+j} - V_t$). This computes the effective tax rate, given the forgone earnings effect. Thus, for someone who delays all appreciation for all *j* periods, the effective tax rate is appropriately τ_l.[12]

Another measure of the detrimental effects of forgone earnings on one's investment is to determine what percent of one's final investment pie is "given away" as taxes. Thus, if your final investment is worth $11,000 had you had *zero forgone earnings* and $10,800 due to foregone earnings on the taxes that you paid, then $200/$11,000 or 1.81% of the terminal pie would have been given away.

The formula for this measure is:

$$\tau_p = \frac{\tilde{r}_{fe} V_t}{V^*_{t+j}} \qquad (7)$$

This is the dollar value of forgone earnings over the investment period divided by the total terminal value of the investment in the absence of forgone earnings and taxes.

The third way investors may wish to think of tax effects on investment value is to consider the percent of the initial investment that is lost due to taxes. For example, if you start with $10,000 and the effect of forgone earnings is 162%, this would mean that the investor's returns could have been 162% higher had the negative effect of forgone earnings on taxes not been present. Thus, you would have had $16,200 more.

The formula for this measure is:

$$\tau_i = \tilde{r}_{fe} \qquad (8)$$

The corresponding measures that isolate forgone earnings and consider only the trade-off between short-term and long-term taxes are:

$$\tau^s_e = \frac{V^0_{t+j} - \hat{V}^{l_s}_{t+j}}{V^0_{t+j} - V_t} \qquad (9)$$

$$\tau^s_p = \frac{\hat{V}^0_{t+j} - \hat{V}^{l_s}_{t+j}}{\hat{V}^0_{t+j}} \qquad (10)$$

$$\tau^s_i = \frac{V^0_{t+j} - \hat{V}^{l_s}_{t+j}}{V_t} \qquad (11)$$

where $\hat{V}_{t+j} = V_t + (V^0_{t+j} - V_t)\,[(1-\tau_s)l_s + (1-\tau_l)(1-l_s)]$. The effective tax rate, τ^s_e, is just the difference between the short-term and long-term rates. It is the terminal value in the absence of short-term capital gain taxes (V^0_{t+j}) minus the terminal value with short-term capital gains taxes, not including forgone earnings effects ($\hat{V}^{l_s}_{t+j}$).

Theoretical Results

Exhibit 3 shows the various tax drag measures for investors who realize varying degrees of long-term gains from 20% per year to 100% per year. For an investor with a ten-year horizon, one can see that in the extreme case of $l_l = 1$, not deferring gains causes the effective tax rate to be 24.10%. One can also see that forgoing earnings removes 3.10% of the final pie the investor could have had by deferring gains.

It may seem counter-intuitive that the effective tax rate can be higher than the statutory tax rate of 20%, but this is because the forgone earnings of an investment grow at the risk-free rate of 6% per year. Suppose an investor invests $10,000 at a rate of 10%. After one year, the investor's portfolio will be valued at $11,000. But this investor realizes all gains every year (i.e., $l_l = 1$). Thus, $200 is paid in taxes and the remaining $10,800 is invested at 10%. At the end of year 2, the investor's before-tax value is $11,880, resulting in another $216 in taxes. The investor also loses $12 in forgone earnings from interest on the original $200 paid in the first year. Had there been no taxes, the investor's pre-tax end-of-year 2 value would have been $12,092, making the effective tax rate: $\tau_e = (12{,}092 - 11{,}664)/2{,}092 = 20.46\%$.[13]

This is true even though the investor faces a 20% tax rate in every period. The investor's pre-tax earnings would have been $2,092 in the absence of taxes, and $428 is paid in taxes effectively due to the forgone earnings at the risk-free rate.[14]

EXHIBIT 3
THEORETICAL COSTS OF FORGONE EARNINGS FROM TAXES*

	$l_l = 0.2$			$l_l = 0.4$			$l_l = 0.6$			$l_l = 0.8$			$l_l = 1$		
Horizon	τ_e	τ_p	τ_i	τ_e	τ_p	τ_i	τ_e	τ_p	τ_i	τ_e	τ_p	τ_i	τ_e	τ_p	τ_i
1	4.76	0.00	0.00	9.09	0.00	0.00	13.04	0.00	0.00	16.67	0.00	0.00	20.00	0.00	0.00
2	4.90	0.02	0.02	9.35	0.04	0.05	13.39	0.06	0.07	17.09	0.08	0.10	20.46	0.10	0.12
3	5.04	0.06	0.08	9.60	0.12	0.15	13.75	0.17	0.23	17.51	0.23	0.30	20.92	0.28	0.38
4	5.17	0.11	0.16	9.86	0.23	0.32	14.10	0.33	0.48	17.93	0.44	0.63	21.38	0.54	0.79
5	5.31	0.19	0.28	10.12	0.36	0.56	14.46	0.54	0.83	18.35	0.70	1.11	21.83	0.86	1.38
6	5.45	0.27	0.44	10.38	0.53	0.88	14.81	0.77	1.31	18.78	1.01	1.74	22.29	1.23	2.16
7	5.59	0.36	0.65	10.64	0.71	1.29	15.17	1.04	1.93	19.20	1.36	2.56	22.75	1.65	3.17
8	5.73	0.47	0.92	10.90	0.92	1.82	15.53	1.34	2.70	19.62	1.73	3.58	23.20	2.10	4.43
9	5.87	0.58	1.24	11.16	1.14	2.46	15.88	1.66	3.65	20.05	2.14	4.83	23.65	2.59	5.98
10	6.00	0.71	1.64	11.42	1.37	3.24	16.24	1.99	4.80	20.47	2.57	6.34	24.10	3.10	7.83
11	6.14	0.84	2.11	11.68	1.62	4.17	16.60	2.35	6.17	20.89	3.02	8.13	24.55	3.63	10.04
12	6.28	0.97	2.67	11.93	1.88	5.27	16.95	2.72	7.80	21.31	3.49	10.25	25.00	4.18	12.64
13	6.41	1.11	3.33	12.19	2.14	6.56	17.30	3.10	9.70	21.73	3.97	12.73	25.44	4.74	15.68
14	6.55	1.25	4.11	12.45	2.41	8.07	17.65	3.48	11.91	22.14	4.45	15.62	25.88	5.31	19.20
15	6.68	1.39	5.00	12.70	2.69	9.83	18.00	3.88	14.47	22.55	4.95	18.95	26.31	5.89	23.26
16	6.82	1.54	6.04	12.95	2.97	11.85	18.35	4.28	17.42	22.96	5.45	22.77	26.74	6.47	27.91
17	6.95	1.69	7.24	13.20	3.25	14.17	18.69	4.68	20.80	23.37	5.96	27.15	27.17	7.06	33.22
18	7.08	1.83	8.61	13.44	3.54	16.83	19.04	5.09	24.66	23.78	6.46	32.14	27.59	7.64	39.27
19	7.21	1.98	10.18	13.69	3.82	19.86	19.37	5.49	29.06	24.18	6.97	37.80	28.01	8.23	46.12
20	7.33	2.13	11.97	13.93	4.10	23.31	19.71	5.90	34.05	24.58	7.48	44.22	28.42	8.81	53.86
21	7.46	2.28	14.00	14.17	4.39	27.22	20.04	6.30	39.69	24.97	7.98	51.47	28.83	9.39	62.58
22	7.58	2.42	16.30	14.40	4.67	31.64	20.37	6.70	46.06	25.36	8.48	59.63	29.24	9.96	72.39
23	7.71	2.57	18.91	14.64	4.95	36.63	20.69	7.10	53.24	25.74	8.98	68.80	29.64	10.52	83.38
24	7.83	2.71	21.85	14.86	5.22	42.26	21.01	7.49	61.31	26.13	9.47	79.10	30.03	11.08	95.70
25	7.94	2.85	25.17	15.09	5.49	48.59	21.33	7.88	70.36	26.50	9.96	90.62	30.42	11.64	109.45

**Values computed from the theoretical equations. The value for r is 12%, consistent with the historical return on the S&P 500. The value for r_f is 6%, consistent with current short-term government rates in the U.S. The value of τ_l^* is set to 0 for the case of borrowing to pay taxes. All calculations are based upon forgone earnings; no short/long effects are present. Calculations for the $l_l = 0$ case are not included, since the differences do not exist.*

This theoretical excursion illustrates two concepts. The first is that computing forgone earnings at the underlying investment rate may exaggerate the numbers. Despite this, when forgone earnings are computed at the risk-free rate, the effective tax rate and the amount of the final value given to Uncle Sam are still high. As the horizon lengthens to 25 years, these numbers become even more important. For the typical return on the S&P 500 over 25 years, tax drag from forgone earnings can amount to a higher effective tax rate of 10.42%. In the extreme case, and calculating forgone earnings at the underlying investment rate, the effective tax rate can approach 100% as the number of years approaches infinity.[15]

We may not live that long, but this seems a very high price to pay for not deferring taxes.

Exhibit 4 illustrates the theoretical costs of taking short-term gains or paying taxes on dividends as compared to taking only long-term gains at a lower tax rate. The effective tax rates are very clear across the board. As the long-term rate is τ_l and the short-term rate is τ_s, the effective rate will always be between the two, depending on the extent of short-term gains versus long-term gains taken.

Considering again an investor with a 10-year horizon, we see that 8.63% of final value can be eaten away by short-term over long-term gain effects, ignoring forgone earnings completely. The damage rises to 12.75% for an investor with a 25-year investment horizon.[16]

EMPIRICAL ANALYSIS

The real investment environment is much more complex. We compare the naive investor and the tax-smart investor under three different cases that reflect a variety of realistic situations.

In the first case, both investors are buy-and-hold index investors with no need to spend any of the investment. The naive investor buys an index, and holds it until the liquidation time, rebalancing only in the event of changes in the index. The t-smart investor starts with the same portfolio as the naive investor, but if necessary sells loser stocks and replaces them with characteristically matched stocks to keep the realized capital gains at a minimum. For indexes we use sector indexes only for reasons to be discussed later.

In the second case, both the naive investor and the t-smart investor are dividend-income investors; i.e., they buy dividend-paying stocks to extract dividend income from their portfolios. The naive investor buys a portfolio of dividend-paying stocks, and simply holds it until the liquidation time. The t-smart investor again buys the same portfolio as the naive investor, and again if necessary sells loser stocks and replaces them with characteristically matched stocks to keep the realized capital gains at a minimum and to offset dividend income. We repeat this analysis many times.

EXHIBIT 4
THEORETICAL COSTS OF SHORT-TERM TAXES*

Horizon	$l_s = 0.2$ τ_e	τ_p	τ_i	$l_s = 0.4$ τ_e	τ_p	τ_i	$l_s = 0.6$ τ_e	τ_p	τ_i	$l_s = 0.8$ τ_e	τ_p	τ_i	$l_s = 1$ τ_e	τ_p	τ_i
1	22.20	0.24	0.26	24.40	0.48	0.53	26.60	0.72	0.79	28.80	0.96	1.06	31.00	1.20	1.32
2	22.20	0.47	0.56	24.40	0.93	1.12	26.60	1.40	1.68	28.80	1.86	2.24	31.00	2.33	2.80
3	22.20	0.67	0.89	24.40	1.35	1.78	26.60	2.02	2.67	28.80	2.69	3.56	31.00	3.36	4.45
4	22.20	0.86	1.26	24.40	1.73	2.52	26.60	2.59	3.79	28.80	3.46	5.05	31.00	4.32	6.31
5	22.20	1.04	1.68	24.40	2.08	3.35	26.60	3.13	5.03	28.80	4.17	6.71	31.00	5.21	8.39
6	22.20	1.20	2.14	24.40	2.41	4.28	26.60	3.61	6.43	28.80	4.82	8.57	31.00	6.02	10.71
7	22.20	1.35	2.66	24.40	2.71	5.33	26.60	4.06	7.99	28.80	5.41	10.65	31.00	6.77	13.32
8	22.20	1.49	3.25	24.40	2.98	6.49	26.60	4.47	9.74	28.80	5.96	12.99	31.00	7.44	16.24
9	22.20	1.61	3.90	24.40	3.23	7.80	26.60	4.84	11.70	28.80	6.45	15.60	31.00	8.06	19.50
10	22.20	1.73	4.63	24.40	3.45	9.27	26.60	5.18	13.90	28.80	6.90	18.53	31.00	8.63	23.16
11	22.20	1.83	5.45	24.40	3.66	10.91	26.60	5.48	16.36	28.80	7.31	21.81	31.00	9.14	27.26
12	22.20	1.92	6.37	24.40	3.84	12.74	26.60	5.76	19.11	28.80	7.68	25.48	31.00	9.60	31.86
13	22.20	2.00	7.40	24.40	4.01	14.80	26.60	6.01	22.20	28.80	8.02	29.60	31.00	10.02	37.00
14	22.20	2.08	8.55	24.40	4.16	17.10	26.60	6.24	25.65	28.80	8.32	34.21	31.00	10.40	42.76
15	22.20	2.15	9.84	24.40	4.30	19.68	26.60	6.45	29.53	28.80	8.60	39.37	31.00	10.75	49.21
16	22.20	2.21	11.29	24.40	4.42	22.57	26.60	6.63	33.86	28.80	8.84	45.15	31.00	11.06	56.43
17	22.20	2.27	12.91	24.40	4.53	25.81	26.60	6.80	38.72	28.80	9.07	51.62	31.00	11.33	64.53
18	22.20	2.32	14.72	24.40	4.63	29.44	26.60	6.95	44.15	28.80	9.27	58.87	31.00	11.59	73.59
19	22.20	2.36	16.75	24.40	4.72	33.50	26.60	7.09	50.24	28.80	9.45	66.99	31.00	11.81	83.74
20	22.20	2.40	19.02	24.40	4.81	38.04	26.60	7.21	57.07	28.80	9.61	76.09	31.00	12.01	95.11
21	22.20	2.44	21.57	24.40	4.88	43.14	26.60	7.32	64.71	28.80	9.76	86.27	31.00	12.20	107.84
22	22.20	2.47	24.42	24.40	4.94	48.84	26.60	7.42	73.26	28.80	9.89	97.68	31.00	12.36	122.10
23	22.20	2.50	27.62	24.40	5.00	55.23	26.60	7.50	82.85	28.80	10.00	110.46	31.00	12.50	138.08
24	22.20	2.53	31.19	24.40	5.05	62.39	26.60	7.58	93.58	28.80	10.11	124.77	31.00	12.64	155.96
25	22.20	2.55	35.20	24.40	5.10	70.40	26.60	7.65	105.60	28.80	10.20	140.80	31.00	12.75	176.00

**Values are all computed from the theoretical equations. The value for r is 12%, consistent with the historical return on the S&P 500. Forgone earnings are completely removed; only the short- versus long-terms gains effect is accounted for. Calculations for the $l_s = 0$ case are not included, since the differences do not exist.*

In the third case, both the naive investor and the t-smart investor start by buying non dividend-paying stocks. They then extract cash flow by creating synthetic dividends every period. The naive investor sells a little bit of every stock to generate needed cash. The t-smart investor sells loser stocks to raise cash. The analysis is repeated many times for different rates of cash withdrawal.

In all situations, the t-smart investor matches stocks using two characteristics, size (ME) and book-to-market ratio (BM). All the stocks in the universe are ranked by ME and BM, and each stock i is assigned an ME percentile, s_i, $0 \le s_i \le 1$, and an MB percentile, q_i, $0 \le q_i \le 1$. For any two stocks i and j, the characteristic score is determined by the norm $d(i, j) = |s_i - s_j| + |q_i - q_j|$. If $d(i, j)$ is low enough, we say that stock i and stock j are characteristically matched.

When the investor realizes capital gains or receives dividends, we assume that the investor pays taxes by borrowing at a fixed interest rate. When the investor realizes capital losses, we assume that the investor uses the capital loss to reduce other taxable income.[17]

While realized capital gains are taxed at the rate of 20% or 31%, depending on whether the gain is long-term or short-term, other taxable income from which capital loss should be deducted is always taxed at the income tax rate of 31%. Thus, there is no need to distinguish between long-term capital losses and short-term capital losses. Furthermore, taxes do not affect the portfolio until liquidation. That is, no money flows out of the portfolio to pay taxes.

The before-tax price return ($r_{p,b}$) is calculated as the ratio of the before-tax final value of the portfolio (V^0_{t+j}) to the initial value of the portfolio (V_t), while the after-tax price return ($r_{p,a}$) is calculated as the ratio of the after-tax final value of the portfolio ($\widetilde{V}_{t+j}$) to the initial value of the portfolio. That is:

$$r_{p,b} = \frac{V^0_{t+j}}{V_t} - 1 \tag{12}$$

and

$$r_{p,a} = \frac{\widetilde{V}_{t+j}}{V_t} - 1 \tag{13}$$

The gross tax amount ($V^0_{t+j} - \widetilde{V}_{t+j}$) is determined as

$$V^0_{t+j} - \widetilde{V}_{t+j} = \sum_{s=1}^{j} d_{t+s}(1 + r_f)^{j-s} \tag{14}$$

where d_t is the amount of tax the investor paid in time t, and r_f is the borrowing rate.

To account for dividend income as well as price appreciation, we also calculate the total return. First, any cash flow out of the portfolio is assumed to be invested in a risk-free asset with a fixed interest rate. Then the income return is the ratio of the value of all the risk-free assets at the time of liquidation to the initial value of the portfolio. The before-tax total return ($r_{t,b}$) is the sum of the before-tax price return and the income return, and the after-tax total return ($r_{t,a}$) is the sum of the after-tax price return and the income return. That is:

$$r_{p,b} = \frac{V^0_{t+j} + C_{t+j}}{V_t} - 1 \tag{15}$$

and

$$r_{p,a} = \frac{\widetilde{V}_{t+j} + C_{t+j}}{V_t} - 1 \tag{16}$$

where C_{t+j} is the total value of the risk-free assets that the investor has (due to cash outflows) at the time of liquidation. The total value of the risk-free assets is determined by

$$C_{t+j} = \sum_{s=1}^{j} c_{t+s}(1+r_f)^{j-s} \tag{17}$$

where c_t is the amount of income the investor receives from the portfolio in time t, and r_f is the risk-free rate.

Once we calculate the before-tax return and the after-tax return, we can calculate the effective tax rate, defined as the ratio of the total tax payment to the total capital gains:

$$\tau_e = \frac{r_b - r_a}{r_b - 1} = \frac{V^0_{t+j} - \widetilde{V}_{t+j}}{V^0_{t+j} - V_t} \tag{18}$$

TAX-EFFICIENCY FOR INDEX INVESTORS

To compare the t-smart investors and the naive investors, who want to mimic indexes, we use sector indexes. We do not use broad market indexes such as the S&P 500 or the Wilshire 5000, because using these indexes makes it difficult to replace a stock inside the index universe with a stock outside the index universe. We do not use "style" indexes such as the Russell indexes for similar reasons.[18]

We perform the simulation as follows. The naive investor does not care to manage taxes efficiently, and simply invests in every stock in the index. She rebalances the portfolio to adjust for entry and exit every rebalancing period. The t-smart investor chooses the same portfolio as the naive investor at the initial period. At every rebalancing period, the t-smart investor sells "loser" stocks (i.e., stocks that have capital losses) and replaces them with *characteristically matched stocks* from outside the index universe, to minimize realized capital gains.

Both the naive investor and the t-smart investor invest \$10,000 at the beginning of July 1990 in a sector index. Each liquidates the entire portfolio at the end of June 2000. We repeat the simulation for 18 different sector indexes.

To simplify the analysis, we assume that the investors make transactions only once a month, at the beginning of each month. Thus, at the beginning of each month, each investor makes a rebalancing decision. When the naive investor rebalances the portfolio, she uses the first in, first out (FIFO) method; i.e., she sells the oldest tax lot first. When the t-smart investor rebalances the portfolio, he realizes capital losses by selling loser stocks and replacing them with characteristically matched stocks. For some periods, the t-smart investor may not realize capital loss if there are no loser stocks at all. (This is what is usually called the "locked-in" situation.)

We create 18 sector indexes based on the North American Industrial Classification System (NAICS). A sector return is calculated by weighting all the stocks in the sector according to their market capitalization. The entry and the exit of firms are reflected as best as possible, given the available information. If a stock is delisted or exits from the sample, we assume that investors have sold the stock at the last available end-of-month price.[19]

Exhibit 5 reports the before-tax return, after-tax return, and the effective tax rate for each hypothetical

EXHIBIT 5
PERFORMANCE OF INDEX INVESTORS

Investor	Pre-tax Return	After-tax Return	Effective Tax Rate
Naive Index Investor	2.1030	1.5777	0.2640
Tsmart Index Investor	3.6779	2.8414	0.2158

Note: Returns represent the cumulative returns over the ten-year period. Thus, 1.57 indicates that the strategy had a return of 157% over ten years. The returns are generated over the period July 1990–June 2000.

EXHIBIT 6
EFFECTIVE TAX RATE BY SECTOR AND INCOME INVESTORS

Sector	Investors: Naive Income Investor	Investors: Tsmart Income Investor
Mining	0.2975	0.2246
Utilities	0.3356	0.2414
Construction	NA	0.1951
Consumer Staples	0.2637	0.2113
Basic Materials	0.2443	0.2259
Machinery and Electronics	0.2192	0.2079
Wholesale	0.2552	0.1817
Retail	0.2243	0.2265
Merchandise Stores	0.2335	0.2044
Transportation	0.3194	0.2406
Information	0.2385	0.2152
Finance	0.2496	0.2158
Real Estate	0.2670	0.1990
Professional Service	0.2211	0.2141
Administrative Service	0.2488	0.2905
Healthcare	0.3106	0.1849
Entertainment	0.2878	0.1880
Accommodations	0.2718	0.2173
Average	0.2640	0.2158

investor. Each entry in Exhibit 5 is an average across 18 different simulations.

The entries show that the pre-tax return is higher for the t-smart investor than for the naive investor. This is due to the momentum in price movement. When there is momentum in price movement, selling loser stocks and buying something else produces superior returns. Since the pre-tax return is much higher for the t-smart investor than for the naive investor, the gross taxes are higher for the t-smart investor than for the naive investor. In terms of the effective tax rate, however, the tax burden is lower for the t-smart investor than for the naive investor.

The pre-tax annualized return of the t-smart investor who attempts to generate all the capital losses is 16.68%. The naive investor achieves an annualized return of 11.99%. The after-tax returns of both investors are 14.41% and 9.93%, respectively. The effective tax rate for both investors differs by 4.82 percentage points. This is quite high. In effect, the t-smart investor gains an extra 4.82% per year through intelligent tax management that saves him on forgone earnings and on decreased income taxes.

The entire schedule of effective tax rates for each individual sector is shown in Exhibit 6. Note that the effective tax rate for the t-smart investor is sometimes lower than the long-term tax rate of 20%. This is because when the investor realizes losses, the loss reduces other taxable income, for which the short-term tax rate of 31% is applicable. Thus, the effective tax rate for the t-smart investor is not bounded below by the long-term tax rate. This shows the power of tax management.

To see the robustness of the results, we repeat the analysis for different subperiods. Exhibit 7 reports the effective tax rate for all the subperiods that are longer than five years. It is clear from the table that the results are very robust across periods, and not specific to any particular period.

Tax-Efficiency for Dividend-Income Investors

In the second case, both the naive investor and the t-smart investor regularly withdraw cash from their portfolios by cashing out dividends. They have little or no labor income, and need to generate cash from their portfolios. Analyzing the income investors is interesting because withdrawing cash from portfolios increases turnover, which creates more room for tax management.

The naive investor is a buy-and-hold investor. He buys a portfolio at the beginning of the simulation period, and holds it until the time of liquidation. In each period, he cashes out dividends and pays taxes on those dividends. The t-smart investor buys the same portfolio as the naive investor at the beginning of the simulation period. In each period, the t-smart investor withdraws the same

EXHIBIT 7
EFFECTIVE TAX RATES FOR SUBPERIODS

Starting	Investor	Ending: Jun-95	Jun-96	Jun-97	Jun-98	Jun-99	Jun-00
Jul-90	Naive	0.2498	0.2370	0.2351	0.2417	0.2477	0.2640
	Tsmart	0.1718	0.1900	0.1973	0.2024	0.2132	0.2158
Jul-91	Naive		0.2386	0.2356	0.2397	0.2470	0.2581
	Tsmart		0.1938	0.1980	0.2017	0.2073	0.2066
Jul-92	Naive			0.2339	0.2376	0.2450	0.2559
	Tsmart			0.1995	0.2036	0.2123	0.2106
Jul-93	Naive				0.2347	0.2396	0.2590
	Tsmart				0.2038	0.2063	0.2030
Jul-94	Naive					0.2340	0.2510
	Tsmart					0.2132	0.2034
Jul-95	Naive						0.2415
	Tsmart						0.2034

amount of cash from her portfolio as the naive investor, but tries to offset dividends with realized capital losses. She does so by selling loser stocks and replacing them with characteristically matched stocks. This way, the t-smart investor attempts to minimize the tax burden.

Both the naive investor and the t-smart investor invest $10,000 at the beginning of July 1990 in a portfolio of 50 stocks. Stocks are selected so that their dividend yields for the 12-month period, July 1989-June 1990, are closest to a given income rate (i.e., withdrawal rate). Income rates vary from 0.5% per year to 8.0% per year.

As before, we assume that the investors make transactions at the beginning of each month. At the beginning of each month, the naive investor withdraws cash from the portfolio at the given income rate. If there are dividend payouts, the naive investor uses dividends to generate cash outflows. If there are not enough dividends or no dividends, the investor sells stocks to generate cash outflow. The amount sold is proportional to the value of each stock holding.

At the beginning of each month, the t-smart investor also withdraws cash from the portfolio at the given income rate in a similar manner to the naive investor. When she has to sell stocks to generate cash outflow, however, she sells the biggest loser stocks instead of a portion of every stock.

While the t-smart investor's initial portfolio is identical to the naive investor's initial portfolio, the two portfolios will not be identical after the first period. Thus, the t-smart investor's dividend income may not always match the naive investor's dividend income, although total income including the proceeds from the sale of stocks is always same. Besides selling stocks to generate cash outflow, the t-smart investor also sells loser stocks and replaces them with characteristically matched stocks to minimize the tax burden.[20]

Both investors liquidate the portfolios at the end of June 2000. We repeat the simulation using different annual income rates for the initial portfolios, ranging from 0.5% to 8.0%.

Exhibit 8 presents the various returns and the effective tax rate of each investor. What we find for index investors is still true for the income investors. First, the pre-tax returns are higher for the t-smart investors than for the naive investors, although the difference is less now than before. Again, this is due to price momentum. Second, the effective tax rate is lower for the t-smart investor than for the naive investor, which shows the benefit of tax management.

In particular, the t-smart investor earns an annualized return of 13.79% and an after-tax return of 12.04%, while the naive investor earns a pre-tax total return of 13.44% versus an after-tax return of 11.33%. The effective tax rates are substantially different (4.69 percentage points). On an annualized basis, the naive investor loses 2.11% per year due to taxes, while the t-smart investor loses 1.75% per year.

Exhibit 9 summarizes the results for different levels of annual income targets.

Tax-Efficiency for Synthetic Dividend-Income Investors

One disadvantage of dividends is that they are taxed at income tax rates of individuals. Thus, a portfolio that pays a significant amount of dividends in any one year will necessarily generate tax payments for individuals at a much higher tax rate, thus costing the investor two sources of return loss: higher taxes, and forgone earnings on those taxes paid. As we have seen, in the case of the Vanguard 500 Index fund, dividend taxes account for a startling 26% of the total final value.

EXHIBIT 8
PERFORMANCE OF DIVIDEND-INCOME INVESTORS

Investor	Pre-tax Price Return	After-tax Price Return	Pre-tax Total Return	After-tax Total Return	Effective Tax Rate
Naive Div-Income Investor	1.6756	1.0729	2.5295	1.9268	0.2406
Tsmart Div-Income Investor	1.7461	1.2245	2.6408	2.1192	0.1937

Note: Returns represent the cumulative returns over the ten-year period. Thus, 1.92 indicates that the strategy had a return of 192% over ten years. The returns are generated over the period July 1990–June 2000.

EXHIBIT 9
EFFECTIVE TAX RATE BY DIVIDEND RATE AND DIVIDEND-INCOME INVESTORS

	Investors	
Annual Dividend Target (%)	Naive Div-Income Investor	Tsmart Div-Income Investor
0.5	0.2055	0.2108
1	0.2172	0.1573
1.5	0.2344	0.1775
2	0.2206	0.2051
2.5	0.2207	0.1916
3	0.2204	0.2023
3.5	0.257	0.1957
4.0	0.2386	0.2099
4.5	0.2225	0.1896
5	0.2582	0.1974
5.5	0.2395	0.2157
6	0.2757	0.205
6.5	0.2566	0.2061
7	0.2681	0.1754
7.5	0.2604	0.1834
8	0.254	0.1762
Average	0.2406	0.1937

We want to address the investor who is interested in a continuous dividend stream, but who wishes to minimize the tax burden that dividends cause. One way to do this is to construct a synthetic dividend portfolio, from which investors generate income by selling stocks rather than cashing out dividends. We call income investors who buy a portfolio of stocks that typically do not pay dividends synthetic dividend-income investors.[21]

Thus, the initial portfolio is created from a universe of stocks that did not pay any dividends for the 12 months prior to June 1990. The initial portfolio is created so that the characteristics of the initial portfolio are the same as those of the initial portfolios of the dividend-income investors who have the same income rate.

Exhibit 10 presents the returns and the effective tax rates for each investor. There are two interesting things to notice. First, the pre-tax returns are higher for the naive investor than for the t-smart investor. In fact, on an after-tax basis, the naive investor has an annualized return of 20.56%, while the t-smart investor has an after-tax return of 14.60%. Thus, the naive investor substantially outperforms the t-smart investor. This is because some of the stocks included in the initial portfolios had explosive returns in our sample period. It is unlikely that this pattern will be repeated in the future.

Despite this outperformance, which we would not have expected ex ante, the t-smart investor still has a much lower effective tax rate than the naive investor. Once again, this illustrates the power of tax management.

The performance of the synthetic dividend-income investors can also be compared to the performance of the dividend-income investors since the only difference between these two groups is whether the universe of stocks is restricted to non-dividend-paying stocks (see Exhibits 9 and 11). Comparing the pre-tax returns, the synthetic dividend-income investors fare much better than the dividend-income investors. This is not surprising, given the superior performance of growth stocks in the last decade.

Comparing the effective tax rate, we can observe that one can reduce the effective tax rate by as much as 1.16% by simply restricting the universe of stocks to non-dividend-paying stocks. The reduction in the effective tax rate is not as great for the t-smart investors, since the effective tax rate is already very low for the t-smart dividend-income investors.

COMPARISON OF EX POST RISK OF THE STRATEGIES

The t-smart investors replace loser stocks with stocks with similar characteristics so that the risk properties of portfolios do not change. To the extent that the risk properties of portfolios are determined by the characteristics of member stocks, tax management based on characteristic matching does not distort the risk properties of portfolios.

EXHIBIT 10
PERFORMANCE OF SYNTHETIC DIVIDEND-INCOME INVESTORS

Investor	Pre-tax Price Return	After-tax Price Return	Pre-tax Total Return	After-tax Total Return	Effective Tax Rate
Naive Syn Div-Income Investor	6.1929	4.7266	6.9536	5.4873	0.2290
Tsmart Syn Div-Income Investor	3.0739	2.3761	3.6062	2.9084	0.1919

Note: Returns represent the cumulative returns over the ten-year period. Thus, 5.48 indicates that the strategy had a return of 548% over ten years. The returns are generated over the period July 1990–June 2000.

There could be more to the risk of portfolios than the characteristics of stocks, however. Also, we may be missing certain important characteristics of stocks relevant to the risk of portfolios. Thus, it is useful to examine the ex post (realized) risk properties of portfolios, and to compare the risk properties of the t-smart investors to the risk properties of the naive investors.

To examine ex post risk, we calculate standard measures of risk: volatility (standard deviation), Sharpe ratio, and betas from the three-factor model. Exhibit 12 reports the median value of these measures for each group of portfolios, index investors, dividend-income investors, and synthetic dividend-income investors.

To determine whether tax management distorts the risk of portfolios significantly, we calculate the median difference of matched pairs of portfolios. For example, there are 18 portfolios of naive index investors and 18 corresponding portfolios of t-smart index investors. We calculate the difference in each risk measure of the 18 pairs of portfolios, and obtain the median difference. The median difference in volatility between the portfolios of naive index investors and of t-smart index investors is about 0.6 percentage points. This is about 10% of the median volatility.

These numbers can be also compared to the median difference of random pairs, i.e., the median difference of all possible pairs out of all the portfolios of index investors. The median difference in volatility of random pairs of the portfolios of index investors is about 1%. By comparing 0.6% to 1.0%, we can infer that matched pairs of portfolios are more similar than random pairs of portfolios.

Tax management with characteristic matching does distort the ex post risk properties of portfolios. In terms

EXHIBIT 11
EFFECTIVE TAX RATE BY DIVIDEND RATE AND SYNTHETIC DIVIDEND-INCOME INVESTORS

	Investors	
Annual Dividend Target (%)	Naive Syn Div-Income Investor	Tsmart Syn Div-Income Investor
0.5	0.2054	0.2165
1	0.2065	0.1785
1.5	0.2039	0.1771
2	0.2036	0.1952
2.5	0.2037	0.1982
3	NA	0.2131
3.5	0.2447	0.2035
4	0.2028	0.1695
4.5	0.2068	NA
5	0.2122	0.2094
5.5	0.2096	0.2
6	0.2874	0.1967
6.5	0.2713	0.2157
7	0.2339	0.1699
7.5	0.3088	0.1511
8	0.2342	0.1843
Average	0.229	0.1919

EXHIBIT 12
COMPARISON OF EX POST RISK

Investor	Bucket	Volatility	Sharpe Ratio	Market Beta	Size Beta	BM Beta
Index Investors	All	0.0459	0.209	0.9553	0.1012	0.1609
	Diff of Random Pairs	0.0097	0.0832	0.1491	0.1495	0.2836
	Diff of Matched Pairs	0.0059	0.0743	0.0746	0.06	0.1474
Div-Income Investors	All	0.1003	0.1801	1.0589	0.3038	-0.4706
	Diff of Random Pairs	0.0089	0.0265	0.1384	0.1038	0.3495
	Diff of Matched Pairs	0.0062	0.0136	0.1179	0.0931	0.1533
Syn Div-Income Investors	All	0.1358	0.2153	1.5657	0.8189	-1.2188
	Diff of Random Pairs	0.0136	0.0436	0.1916	0.1701	0.5421
	Diff of Matched Pairs	0.0075	0.0409	0.1906	0.1149	0.3783

of monthly volatility, tax management changes the risk measures up to 0.8 percentage points. The size of distortion is small, however, relative to the level of risk and also relative to the potential size of the distortion. Distortion in monthly volatility created by tax management is about 10% of the level of monthly volatility and about 60% of the possible distortion.

CONCLUSION

Investors who invest in mutual funds or tend to buy and sell securities frequently generate tax bills that reduce their total returns and make them less well-off. The extent of these losses is greater, the more short-term gains are realized, the longer the investment horizon due to forgone earnings, and the higher one's tax bracket.

We have highlighted certain investing strategies that directly focus on reducing the tax bill of investors. We show that index investors can reduce their effective tax rate by 4.8 percentage points and increase their after-tax returns by cleverly managing their investments. Although this is not possible with traditional mutual funds, even traditional index funds, new on-line brokers with portfolio-based systems and lower fees are making this much more feasible for retail investors.[22]

Fischer Black once asked why corporations pay dividends. Since dividends are taxed at ordinary income tax rates, which are generally higher than capital gains rates, one would also wonder why investors would hold dividend-paying stocks. In fact, there are a myriad of reasons why investors might hold dividend-paying stocks, including the offering of a stable source of income. This may be especially true for older investors.

Our research also considers two types of portfolios for "income-preferring" individuals. We show that a portfolio strategy that manages taxes efficiently will reduce the investor's tax burden by as much as 4.69 percentage points and increase after-tax return. We also go a step farther and consider a second portfolio that avoids dividend-paying stocks altogether and generates a synthetic dividend stream by selling stocks in every period. We show that this again reduces an investor's effective tax rate by 3.71 percentage points per year.

It is becoming easier to improve portfolio tax management for individuals, with declining trading costs and improved technology for investment management. These savings are too important for investors or portfolio managers to neglect.

APPENDIX A
THEORETICAL DERIVATIONS

Derivation of Equation (1)

The fund with investment V_t grows to $V_t(1 + r)$ after one period, and both short- and long-term taxes are paid of level $\tau_s l_s$ and $\tau_l l_l$. The value of the investment after one period is: $V_{t+1} = V_t(1 + r) - rV_t\,[\tau_s l_s + \tau_l l_l]$. This equals $V_t\,(1 + r\,[1 - \tau_s l_s - \tau_l l_l])$. Given the previous-period value, we know the next period's value is multiplied by $(1 + r\,[1 - \tau_s l_s - \tau_l l_l])$, so the value after j periods is $V_{t+j} = V_t\,(1 + r\,[1 - \tau_s l_s - \tau_l l_l])^j$.

Derivation of Maximum Loss from Forgone Earnings

To compute the maximum forgone earnings benefit for a period of j years at the reinvestment rate of the investment, r, we can use Equation (4) with $l_l = 1$. The return difference due to forgone earnings is: $r_{fe} = (1 + r)^j(1 - \tau_l) + \tau_l - [1 + r(1 - \tau_l)]^j$. This is the return difference between someone who totally postpones all capital gains at a uniform tax rate of τ_l and someone who realizes the full tax on the return each year at rate τ_l.

For the deferring investor, $\tilde{V}^0_{t+j} = V_t\,[(1+r)^j(1-\tau_l) + \tau_l]$. For the "churner" or non-deferring person, $\tilde{V}^1_{t+j} = V_t\,[(1 + r\,[1-\tau_l])^j\,(1-\tau_l) + [1 + (1-\tau_l)r]^j\,\tau_l] = V_t\,[1 + r(1-\tau_l)]^j$. Thus, $r_{fe} = (\tilde{V}^0_{t+j} - \tilde{V}^1_{t+j})/\,V_t = (1+r)^j(1-\tau_l) + \tau_l - [1 + r(1-\tau_l)]^j$. The tax rates are the same because we assume a yearly rebalancing period, so even the short-term investor pays only long-term gains.

We want to isolate just the forgone earnings. Using our parameters, the "churner" has $l_l = 1$, while the long-term investor has $l_l = 0$. The other parameters for both investors are $\tau_s = l_s = 0$ and τ_l. The difference is the maximum difference due to deferring capital gains. For example, for a tax rate of 20%, an annual return of 12%, and an investment of ten years (j = 10), r_{fe} = 18.37%, which works out to about 1.7 percentage point savings per year from deferring taxes.

Derivation of Equation (5)

Suppose an investor starts with V_t and invests it for one period. The investment would grow to $V_t(1+r)$. Taxes would have to be paid of $rV_t\tau_l l_l$, and the remaining $V_t\,[1 + r(1-\tau_l l_l)]$ could be reinvested. At this point, there are no forgone earnings. One period later, the forgone earnings will have started with $rV_t\tau_l l_l$ having grown at the risk-free rate of r_f, less the taxes on the risk-free investment less the original amount. Thus, forgone earnings at $t+2$ will be $rV_t\tau_l l_l\,[(1+r_f)(1-\tau_l^*) + \tau_l^* - 1]$.

This amount will continue compounding until, after j periods, the forgone earnings from the first tax payment are: $rV_t\tau_l l_l\,[(1+r_f)^{j-1}(1-\tau_l^*) + \tau_l^* - 1]$. In period $t+2$, the investor would pay taxes on the newly grown $V_t\,[1 + r(1-\tau_l l_l)]\,(1+r)$. These taxes would amount to

$$rV_t\tau_l l_l\,\underbrace{[1 + r(1-\tau_l l_l)]}_{\delta}\,[(1+r_f)^{j-2}(1-\tau_l^*) + \tau_l^* - 1]$$

forgone earnings at the end of j periods.

The total forgone earnings after j periods become the series:

$$rV_t\tau_l l_l[((1+r_f)^{j-1}(1-\tau_l^*) + \tau_l^* - 1) +$$
$$\delta\,((1+r_f)^{j-3}(1-\tau_l^*) + \tau_l^* - 1) + \ldots +$$
$$\delta^{j-2}\,((1+r_f)^1(1-\tau_l^*) + \tau_l^* - 1)]$$

$$= rV_t\tau_l l_l\left[(1-\tau_l^*)(1+r_f)\delta^{j-2}\times\right.$$
$$\left[1 + \left(\tfrac{1+r_f}{\delta}\right) + \left(\tfrac{1+r_f}{\delta}\right)^2 + \ldots + \left(\tfrac{1+r_f}{\delta}\right)^{j-2}\right] +$$
$$\left.\tau^*\left[\tfrac{1-\delta^{j-1}}{1-\delta}\right] - \left[\tfrac{1-\delta^{j-1}}{1-\delta}\right]\right]$$

$$= rV_t\tau_l l_l(1-\tau_l^*)\times$$
$$\left((1+r_f)\delta^{j-2}\left[\frac{1-\left(\frac{1+r_f}{\delta}\right)^{j-1}}{\left(1-\frac{1+r_f}{\delta}\right)}\right] - \left[\tfrac{1-\delta^{j-1}}{1-\delta}\right]\right)$$

Equation (5) is the total forgone earnings divided by V_t.

APPENDIX B
DATA

The data set we use is the Standard & Poor's Compustat Prices, Dividends, and Earnings (PDE) database. It includes monthly data on price, outstanding shares, and dividends, and annual data on the book value per share of stocks traded on major U.S. exchanges from the early 1970s to the present. The PDE database also provides the North American Industrial Classification System (NAICS) codes for each stock.

Some of the variables we construct deserve explanation.

- The size or market equity (ME) of a stock is the market capitalization of the stock, calculated as the price multiplied by the number of shares outstanding. Occasionally, the number of shares outstanding is not available. In this case, we go back up to nine months to find the data. For example, if the number of shares outstanding in June 1995 is not available, we may use the May 1995 figure instead. If the May 1995 figure is not available either, we use the April 1995 figure. In the worst case, we may use the September 1994 figure for June 1995.
- The market-to-book ratio (MB) of a stock is the ratio of the market value of the stock to the book value of the stock at the end of the fiscal year ending in the previous calendar year. The book value of a stock is calculated as the book value per share multiplied by the number of shares outstanding. The fiscal year ends in December for many companies, and the financial statements are not immediately available at the fiscal year-end. Therefore, we match MB for the fiscal year ending in calendar year t with the ME of June of year $t+1$.

EXHIBIT B-1
DISTRIBUTION OF ME AND MB BY SECTOR AT BEGINNING OF SAMPLE PERIOD

NAICS Code	Nobs	Mean(ME)	Std(ME)	Mean(MB)	Std(MB)
Mining	333	485.5239	1643.9044	3.2521	8.4588
Utilities	188	1292.3729	2106.1021	1.6318	0.9180
Construction	86	154.1245	432.0266	1.8278	1.9790
Consumer Staples	224	1416.4457	4434.4019	2.7521	3.5207
Basic Materials	554	1694.5057	5637.8398	5.9659	31.9987
Machinery and Electronics	1400	570.2094	2952.5085	3.5817	13.8541
Wholesale	247	284.7654	810.3242	5.6553	20.5612
Retail	152	635.5640	1470.4004	2.4395	1.9783
Merchandise Stores	88	1194.4250	4253.2144	3.5949	4.3693
Transportation	110	694.1501	1411.8362	2.2041	2.0219
Information	328	1595.5663	4535.6309	6.0224	10.2414
Finance	679	673.4146	2293.1216	2.0338	9.9811
Real Estate	120	150.2630	406.4402	4.6431	8.5134
Professional Service	223	449.3313	4540.1089	4.4761	9.3107
Administrative Service	112	527.8622	2133.4592	6.0860	23.6121
Healthcare	97	194.5341	604.0476	4.5274	10.0774
Entertainment	28	70.0756	110.3681	4.0359	6.4014
Accommodations	107	279.0764	1261.5140	3.2756	6.2621

EXHIBIT B-2
DISTRIBUTION OF ME AND MB BY SECTOR AT END OF SAMPLE PERIOD

NAICS Code		Mean(ME)	Std(ME)	Mean(MB)	Std(MB)
Mining	296	1077.8291	3568.1257	2.2779	4.0332
Utilities	146	3223.9802	4693.6694	1.8981	0.7018
Construction	101	308.6693	524.6833	2.5813	10.3925
Consumer Staples	295	3318.1924	12782.0060	4.5766	11.3450
Basic Materials	755	4357.1289	18625.0020	6.2607	27.0743
Machinery and Electronics	1697	2445.2283	14920.3490	4.4319	30.6952
Wholesale	290	726.9853	2589.8782	2.2954	3.6753
Retail	189	2483.3999	8649.6885	3.6646	7.1346
Merchandise Stores	137	3312.7334	18721.5610	7.5548	25.3614
Transportation	143	1463.4943	3615.3853	2.3466	3.6852
Information	656	5875.7158	26499.9650	10.4553	46.2876
Finance	1276	2414.3865	10390.7310	3.4433	47.9173
Real Estate	173	439.8446	1466.5139	5.9592	18.6713
Professional Service	351	1370.0181	12697.8130	7.9699	24.7927
Administrative Service	174	627.0928	2735.0869	4.0252	9.2931
Healthcare	138	438.2042	1407.2557	3.2173	5.9329
Entertainment	47	358.7691	654.6319	2.2930	2.3202
Accommodations	151	810.7056	4629.8433	2.4201	4.7881

- The dividend yield of a stock is the ratio of dividends paid for the previous 12 months to the latest end-of-month price. Dividends include cash-equivalent distribution as well as cash distribution.

We use two-digit NAICS codes to define a sector. Some of the two-digit NAICS codes are excluded for the analysis since there were too few stocks within those two-digit NAICS codes. The sectors included in the analysis are (two-digit NAICS codes in parentheses): mining (21), utilities (22), construction (23), consumer staples manufacturing (31), basic materials manufacturing (32), machinery and electronics manufacturing (33), wholesale trade (42), retail trade (44), general merchandise stores and retailers (45), transportation (48), information (51), finance and insurance (52), real estate and rental and leasing (53), professional services (54), administrative and support services (56), health care and social assistance (62), arts, entertainment, and recreation (71), and accommodations and food services (72). The sectors not included for the analysis are: agriculture, forestry, fishing and hunting (11), postal service and warehousing (49), management of companies and enterprises (55), educational services (61), other services (81), and public administration (92).

Exhibits B-1 and B-2 summarize the distribution of ME and MB by sectors.

Initial portfolios are created from stocks of certain dividend yields. Exhibit B-3 summarizes the distribution of ME and MB by dividend yields for the beginning of the sample period.

Since our analysis is based on monthly data, we make certain simplifying assumptions regarding exit. If a stock exits the sample, we assume investors have a chance to sell the stock at the last available end-of-month price. While some stocks exit due to bankruptcy, other stocks exit because they are bought by other companies. Therefore, our assumption does not bias our analysis in any one direction. Also, our analysis is comparative; i.e., we compare different investors under the same assumption, which provides another layer of protection against a bias.

To be included in the sample, a stock should satisfy certain restrictions including:

- A stock should have valid return data for the current month and two previous months. This requires four-month price data and three-month dividend data.
- A stock should have valid ME data for June and valid MB data for December of the previous year.
- A stock should have valid dividend yield data, which requires data on dividends for the previous 12 months.

ENDNOTES

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EXHIBIT B-3
DISTRIBUTION OF ME AND MB BY ANNUALIZED INCOME DIVIDEND LEVEL

Div	Nobs	Mean(ME)	Std(ME)	Mean(MB)	Std(MB)
5-10	168	1937.72	5853.71	4.47	19.16
11-15	212	887.24	1847.35	2.67	3.00
16-20	226	1142.99	2934.98	2.20	1.39
21-25	188	1561.62	4065.01	2.69	5.28
26-30	193	1883.31	5403.37	2.01	1.42
31-35	167	2106.81	5296.63	2.09	1.61
36-40	134	2038.19	4441.16	1.84	1.16
41-45	141	1404.90	3404.46	1.46	0.97
46-50	98	3271.80	8674.24	1.64	1.07
51-55	73	4146.97	10232.35	1.41	0.91
56-60	59	2081.79	5008.57	1.46	1.10
61-65	49	2242.26	4418.51	1.54	1.14
66-70	43	2586.59	6313.72	1.68	0.80
71-75	47	900.04	1234.58	1.44	0.43
76-80	57	1079.70	1873.45	1.48	0.80
81-85	29	986.06	1779.31	12.68	46.86
86-90	18	1624.37	2553.07	3.78	10.77
91-95	13	778.40	1194.52	1.33	1.42
96-100	13	412.35	645.50	1.36	1.04

[1]6% is the turnover for the Vanguard 500 in 1999. The historic turnover rates of the S&P 500, which it tracks, are 6.16% in 1999, 9.46% in 1998, 4.93% in 1997, 4.58% in 1996, 5% in 1995, and 3.78% in 1994. Turnover is defined as the minimum of the total buys and total sells in any given year divided by the total assets under management.

[2]See, for example, Dickson and Shoven [1993], Jeffrey and Arnott [1993], or Garland [1997].

[3]These are asset-weighted averages. The equal-weighted averages are much higher at 112% and 115%, respectively (computed from the Morningstar database as of August 31, 2000).

[4]Tax rates changed significantly over the 1976-1999 period. For simplicity, we use the most recent rates, which are at the lower end of the range of actual tax rates in effect for the period. This makes our results more clear. We also ignore the impact of state income tax rates, which vary significantly across states.

[5]See Appendix A.

[6]Even Morningstar's after-tax return is biased in favor of tax-efficient funds, because it has postponed taxes that will eventually be realized upon liquidation.

[7]After j periods, the investor liquidates the portfolio and pays long-term gains, τ_l on gains not yet realized. Thus, $\tilde{V}_{t+j} = V_{t+j} - (V_{t+j} - B_{t+j})\tau_l$. And: $\tilde{V}_{t+j} = [(1 + r(1 - \psi)]^j - \tau_l [(1 + r[1 - \psi]^j - (1 + (l_s + l_l - \psi)r)^j] = [1 + r(1 - \psi)]^j (1 - \tau_l) + [1 + (l_s + l_l - \psi)r]^j \tau_l$.

[8]The tax rate schedule at the time of this writing was as follows: For income between 0 and \$25,750, τ_s = 15% and τ_l = 10%; between \$25,750 and \$62,450, τ_s = 28% and τ_l = 20%; between \$62,450 and \$130,250, τ_s = 31% and τ_l = 20%; between \$130,250 and \$283,150, τ_s = 36% and τ_l = 20%; and \$283,150 or more, τ_s = 39.60% and τ_l = 20%.

[9]See Appendix A.

[10]See Appendix A.

[11]Borrowing and lending rates are assumed to be equal. In addition, Equation (5) assumes that the interest for lending is taxed at rate τ_l^* at the end of the investment horizon. An alternative method is to tax the interest for lending in every annual period at the rate τ_l^*. The appropriate formula in that case is:

$$\tilde{r}_{fe} = r\tau_l l_l \left([(1+r_f) - r_f\tau_l^*]\,\delta^{j-2} \left[\frac{1-\left(\frac{[(1+r_f)-r_f\tau_l^*]}{\delta}\right)^{j-1}}{\left(1-\frac{[(1+r_f)-r_f\tau_l^*]}{\delta}\right)} \right] - \left[\frac{1-\delta^{j-1}}{1-\delta}\right] \right)$$

[12]It is necessary to calculate a V^*_{t+j} since we do not know the hypothetical growth of the fund from forgone earnings of an interest-bearing security at the risk-free rate. We do know that our $\tilde{V}_{t+j}$, or the after-liquidation value of our investment, will be equal to $V^*_{t+j} - [\tilde{r}_{fe} + r\tau_l l_l ([1-\delta^j]/[1-\delta])]V_t$, where $\tilde{r}_{fe} V_t$ is our hypothetical amount of the investor's forgone earnings cost.

[13]This is lower than the \$12,100 that would be obtained using the investment's return to compute forgone earnings, which is exactly the difference between \$200(1 + r) and \$200(1 + r_f).

[14]When the investor lends and pays taxes on the forgone earnings, $\tau_1^* \neq 0$, the dollars from forgone earnings are slightly less than this.

[15]An interesting observation is that the effective tax rate converges to 100% as j tends to infinity in the r case. Consider:

$$\tau_e = \frac{\tilde{V}^0_{t+j} - \tilde{V}_{t+j}}{\tilde{V}^0_{t+j} - V_t} = \frac{(1+r)^j - (1+r(1-\tau_l))^j}{(1+r)^j} = 1 - \left[\frac{1+r(1-\tau_l)}{1+r}\right]^j$$

With $r > 0$ and $0 \leq \tau_l \leq 1$, the final term in brackets converges to 0 as $j \rightarrow 0$. Thus, effective tax rates converge paradoxically to 100%.

[16]If we take the 25-year investor and compute forgone earnings at the rate of r, we find that 28.32% of the pie is lost due to forgone earnings. Adding this to the 25-year losses from the short/long effect, we get 41%, which is close to the empirical Vanguard pie payment in taxes.

[17]Under the current tax code, realized capital losses can be deducted from other income up to \$3,000 per year, and any amount in excess of \$3,000 can be carried over to the next year. Thus, ignoring the \$3,000 limit does not distort the analysis much—the \$3,000 limit is rarely exceeded. Note also that while there are distinctions between long-term and short-term, long-term gains, if any, should be subtracted from short-term losses before short-term loss is determined, and short-term gains, if any, should be subtracted from long-term losses before long-term loss is determined.

[18]The ability to pick from outside the index universe is important. Otherwise, the number of shares in the portfolio would drop rapidly as we sell loser stocks.

[19]See Appendix B for a more detailed description of the data.

[20]The portfolios of the naive investor and the t-smart investor will have the same characteristics all the time, but the ex ante (or historical) dividend rates of two portfolios may not always be the same, since the dividend rate is not one of the controlled characteristics.

[21]While dividend policy does not change frequently, it is not possible to predict for sure which stocks will pay dividends and which stocks will not. We consider stocks that had not paid any dividend for the previous 12 months to be non-dividend-paying stocks. Occasionally, some of these stocks pay dividends in later periods.

[22]Examples are, Buy-and-Hold, Charles Schwab, ETRADE, Fidelity, FOLIO*fn*, and UNX.

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