Flexible Insurance for Separate Accounts

Ludwig Chincarini"

With the advances in technology and lower costs to trading, separate account platforms and online brokers should be able to offer customized portfolio protection to their clients with the click of a button. This paper uses basic concepts from the option literature to show how this insurance could be offered in two convenient forms. In one form, it would represent a straight cash payment by customers, and in another form, investors would exchange a portion of the upside of their returns for the protection. The paper uses the Black-Scholes model to show the benchmark costs associated with this type of protection, as well as performs a sintulation with an actual portfolio over various sub-periods from 2000 to 2006.

Introduction

The concept of protecting oneself from unlikely, but possible events have existed at least since 225 A.D., when the Roman jurist, Ulpian, prepared the first recorded life expectancy tables. Insurance has entered the financial markets in the form of home insurance, default insurance, and most commonly in the form of options. The option market, which began trading on exchanges in **1973,** has blossomed with the aid of the theoretical advances of Black and Scholes **(1973).** Options are now a common method used to hedge the principal invested in some underlying asset, be it a stock or bond. Although this has provided a way for individuals to hedge or protect their stock investments, it is far from perfect. Firstly. it is usually done one stock at a time, while an individual may own a portfolio of stocks. Thus, it is difficult and inefficient to hedge since it ignores the correlation amongst the stocks in one's portfolio. In otherwords, individuals are paying too much for their insurance by hedging stocks individually. Secondly, many stocks owned by individuals do not have listed options available to trade. This means that the individual would have to resort to the institutional structured product department, which is much less liquid, and hence, much more expensive. Thirdly, it still remains very difficult for the individual to understand exactly what is being protected. In otherwords, protection of a few stocks in one's portfolio does not really give an investor a good idea of how his entire portfolio is being protected. The temporary aspect of most protection also turns all investors into active managers, needing to roll-over their option agreements as they expire. This is a very tedious task. There is no easy way to protect one's entire portfolio. Fourthly, although Long-Term Equity Anticipation Securities[1] (LEAPs) are

* **Assistant Professor, Department of Economics, Pomona College. Claremont, CA.**
E-mail: chincarinil@hotmail.com.

[1] **LEAPs are exchange traded options that usually expire in 2-5 years.**

becoming more common, there is no easy way for investors to protect their investments for periods longer than two years.

Thus, although portfolio insurance exists in some fragmented ways, there is no easy way for the average investor to protect his or her entire portfolio. There are many difficulties in creating a system that would enable the average investor to protect his or her entire portfolio. The most obvious is the difficulty in the synthetic creation of such protection, and secondly, a platform for such a system would be needed to easily bring this type of option to investors. This paper addresses the first difficulty by conceptualizing the possibility of offering portfolio insurance to investors in a couple of user-friendly formats.[2] The second difficulty will be analyzed in later sections of this document. The basic idea is that through the advances in technology, both online brokerage systems and separate account platforms will provide the flexibility for creating an efficient portfolio insurance product for average investors.

First, this paper discusses some preliminary concepts associated with portfolio insurance analysis. Then, it discusses the benefits of portfolio protection versus single stock protection and provides some illustrative tables on the costs of such an insurance program for individual portfolios. Subsequently, it discusses alternative ways to pay for the insurance and how one might logically price such insurance; how such an insurance system would allow for investors to sell, exchange, and adjust their already purchased insurance; the aggregation of all insurance positions so that the firm can perform the aggregate hedge or sell the hedges to an outside hedging group; investigates historical simulations of the pricing and hedging for a simple dynamic delta hedging technique.

Preliminary Concepts

There are many ways to price options, however, it may help to begin with the simple, yet, very common Black-Scholes formula (Black and Scholes (1973)).This method can be used to price European call or put options on an underlying asset, such as a stock. The equations are presented below for the price of a European call and put option:[3]

$$c = SN(d_1) - Xe^{-r(T-t)}N(d_2) \quad ...(1)$$

$$p = Xe^{-r(T-t)}N(-d_2) - SN(-d_1) \quad ...(2)$$

where, c is the price of the call option, p is the price of the put option, $T - t$ is the time until the option expires, r is the risk-free rate of interest, σ is the volatility of the underlying stock or instrument, N ( ) is the cumulative probability distribution for a standardized normal variable, S is the price of the stock or instrument, $d_1 = \frac{ln\left(\frac{S}{X}\right)(r+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$, and $d_2 = d_1 - \sigma\sqrt{T-t}$.

[2] The concept of portfolio insurance is an old one and was first popularized by Leland and Rubinstein (1981).

[3] The entire analysis works for American options as well, however the lack of a closed-form solution makes the analysis more tedious.

Given these formulas, one can easily price the value of protection on a single stock investment. For instance, assume that a person purchases a share of Anheuser-Busch (BUD) at $S = 51.16$ on February 16, 2007, with a risk-free rate of 5.36%, for one year (i.e., $T - t = 1$), with a volatility of 16.3% per year, and a strike price of $X = 51.16$ (i.e., protection of the entire investment). Given these parameters, the price of the protection for one year would be $2.07 or 4.05% of one's investment. However, this would guarantee that the investor had no less than $49.09 ($51.16 – $2.07) at the year end.

The Black-Scholes formula can be slightly altered to be of more general use in ideas related to portfolio insurance. In particular, we can assume that the strike price of the particular option can be represented as any particular growth rate of the underlying asset. That is, $X = Se^{\eta(T-t)}$, where, η is the growth rate of the stock price from its current value. Thus, an at-the-money strike (as in the example above) would be represented simply by having $\eta = 0$, that is the strike price would be the current price of the underlying. To guarantee at least the rate of risk-free interest, one would choose $\eta = r$, and to guarantee a minimum loss on one's principle, one could choose some $\eta < 0$. The generalized strike-price version of Black-Scholes for a call and a put is:[4]

$$c = S[N(d_1) - e^{(\eta - r)(T-t)}N(d_2)] \quad ...(3)$$

$$p = S[e^{(\eta - r)(T-t)}N(-d_2) - N(-d_1)] \quad ...(4)$$

One pleasant feature of this representation is that one can represent the put (or call) price as a percentage of the underlying asset. Thus, one can always compute the percentage cost of protecting one's investment. It also has the pleasant feature of not asking an investor what is his or her strike price in dollar terms, but rather asking him or her "What amount of your investment would you like to protect?" An investor can then answer all of it ($\eta = 0$), more than all of it ($\eta > 0$) or less than all of it ($\eta < 0$). This is more intuitive to an investor and will make much more sense when we consider entire portfolios.

Single Stock Protection to Portfolio Protection

In today's world, an investor usually must choose to hedge each individual stock in his or her portfolio. This is very inefficient and more costly than it should be for an investor. It would be unambiguously cheaper for the investor to hedge or protect the entire portfolio as a bundle. Although there are certain commercial banks who offer this sort of customization at a very high price, it is generally unavailable to the common investing public. Even when it is available, the price of hedging individual stocks and the complexity of understanding how to hedge it are so great that it is practically never done.

The hedging or protecting of the entire portfolio will be cheaper to the investor for the same reason that diversification reduces the non-systematic risk of a portfolio. By owning a portfolio of securities, one can reduce the total risk of the investment, and hence, reduce the price to

[4] Including continuous dividends, the put price becomes $p = S[e^{(\eta - r)(T-t)}N(-d_2) - e^{-q(T-t)}N(-d_1)]$, where q is the continuous dividend yield.

protect the portfolio. Thus, buying an option protection on individual security investments is completely inefficient for the investor who owns a portfolio of stocks.

In the above example, we showed that protecting the investment in BUD would cost the investor roughly 4.05% of the investment. However, if one considers a portfolio of 30 stocks in the retail industry with BUD as one of the stocks, the stock-specific volatility of the investment is reduced and so is the overall hedging cost in most cases.

The Black-Scholes equations for an individual stock can be used to price the options for the entire portfolio by using parameters relevant to the entire portfolio.[5] The Black-Scholes put equation for a basket of stocks or an entire portfolio is:

$$p_P = S_P[e^{(\eta - r)(T-t)}N(-d_2) - N(-d_1)] \quad ...(5)$$

where, S_P is the value of the portfolio, σ_P is the volatility of the portfolio, and all variables included in d_1 and d_2 are for the entire portfolio. Before considering the actual hedging or pricing issues, it might be illustrative to view the kind of results a Black-Scholes equation reveals to us about the price of a European put option on such a portfolio. Assume that the investor has $10,000 to invest and would like to protect his or her investment at a certain level η. Assume a value of $\sigma_P = 0.20$ and also assume an interest rate of $r = 5.36\%$, broadly in line with Treasury rates in February of 2007.

In this calculation, a self-financed portfolio is considered. Thus, the problem is to find the amount to be invested in the portfolio, such as to have an exact amount of additional cash to cover the purchase of the put option required to protect the portfolio at the desired level. The number of shares of stock that can be purchased with protection for this self-financed portfolio is

$$n = \frac{V}{p_P + S_P} \quad ...(6)$$

where, V is the value of investable funds and n is the number of units purchased of the portfolio. and S_P is the arbitrary value of one unit of the portfolio. Table 1 and Figure 1 illustrate the theoretical Black-Scholes prices for various levels of protection (–5% to 5%) of principal for various time horizons (one year to 10 years). One can see that the price of protection declines over time with protection of the 'European kind'. In fact, in the limit of $T \to \infty$, the price of protection approaches 0. This is a well-known theorem of perpetual European put options.[6]

[5] There are other ways of pricing these options, which include considering the differential equation of the entire basket's value by the variance-covariances of the Wiener processes of each individual stock and the weights of the portfolio. This, however, is more complicated and may be less precise, since there are a multitude of possible estimation errors, There is a vast literature on hedging basket options, including Moshe and Posner (1998). Atkinson and Alexandropoulos (2006). and several derivative textbooks.

[6] See Merton (1973). It is also true that perpetual European call options converge to the price of the underlying. Some people find this result puzzling. Surely, the price of an option that can never be exercised should be valueless or 0, but option pricing theory says it is equal to S. However, the underlying stock can never receive any distributions, hence, the value of the stock should also be 0. Thus, c = S = 0. One might even imagine strategies of selling out-of-the-money puts and purchasing shares of the company for zero cost. While this could be done, it is unlikely that there will be a demand for such products and the strategy will never be profitable. Thus, although it may seem unintuitive, there are no arbitrage opportunities left open.

Table 1: The Cost of Protecting Your $10,000 Investment European Style							
Years Protected	The Percent of Protection (η)						
	–5	*–2*	–1	*0*	1	*2*	5
1.	9601.00	9505.00	9470.00	9434.00	9395.00	9356.00	9226.00
	399.00	495.00	530.00	566.00	605.00	644.00	774.00
	4.16	5.20	5.59	6.00	6.43	6.89	8.39
2.	9612.00	9459.00	9400.00	9336.00	9268.00	9195.00	8952.00
	388.00	541.00	600.00	664.00	732.00	805.00	1048.00
	4.04	5.72	6.38	7.11	7.90	8.75	11.71
3.	9654.00	9464.00	9386.00	9301.00	9208.00	910.00	8757.00
	346.00	536.00	614.00	699.00	792.00	893.00	1243.00
	3.59	5.66	6.54	7.51	8.60	9.81	14.19
4.	9700.00	9488.00	9396.00	9294.00	9179.00	9053.00	8603.00
	300.00	512.00	604.00	706.00	821.00	947.00	1397.00
	3.09	5.40	6.42	7.60	8.94	10.46	16.23
5.	9744.00	9519.00	9418.00	9301.00	9169.00	9020.00	8475.00
	256.00	481.00	582.00	699.00	831.00	980.00	1525.00
	2.63	5.05	6.18	7.51	9.07	10.87	17.99
6.	9783.00	9554.00	9445.00	9317.00	9169.00	9000.00	8365.00
	217.00	446.00	555.00	683.00	831.00	1000.00	1635.00
	2.22	4.67	5.88	7.33	9.06	11.11	19.54
7.	9817.00	9589.00	9475.00	9338.00	9177.00	8989.00	8269.00
	183.00	411.00	525.00	662.00	823.00	1011.00	1731.00
	1.86	4.29	5.54	7.08	8.97	11.24	20.94
8.	9846.00	9623.00	9506.00	9363.00	9190.00	8986.00	8183.00
	154.00	377.00	494.00	637.00	810.00	1014.00	1817.00
	1.56	3.92	5.20	6.81	8.81	11.28	22.21
9.	9871.00	9655.00	9537.00	9389.00	9207.00	8988.00	8105.00
	129.00	345.00	463.00	611.00	793.00	1012.00	1895.00
	1.30	3.57	4.85	6.51	8.62	11.26	23.38
10.	9892.00	9686.00	9568.00	9416.00	9226.00	8994.00	8034.00
	108.00	314.00	432.00	584.00	774.00	1006.00	1966.00
	1.09	3.24	4.52	6.20	8.39	11.19	24.47

Note: This example was created using $r = 0.0536$ and $\sigma = 0.2155$. The cost of protection is calculated by using the theoretical price for non-dividend paying, European put options according to the Black Scholes formula. For every year protected and for every amount of protection, there are three rows. The first row represents the amount of the original $10,000 that is left for actual investment in the portfolio. The second row represents the amount of the original $10,000 that must be used to pay for the protection. The third row represents the percentage cost of the protection as a percentage of the amount actually invested.

Figure 1: The Price of portfolio Protection Over 10 Years on $10,000 Model: Black-Scholes on Non-Dividend, European Option

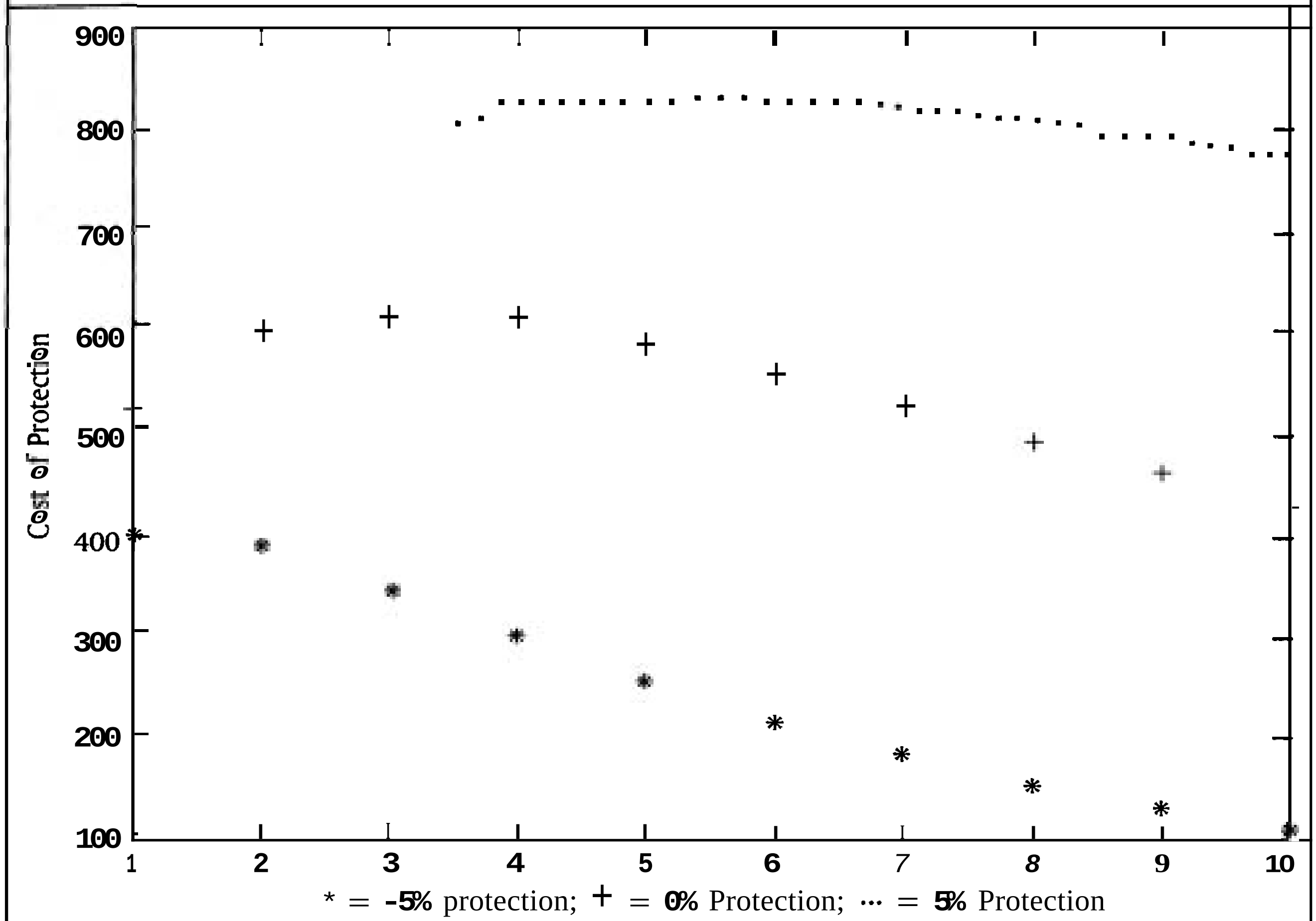


Table 1 contains the three elements for every η (The percent of protection) and for every time horizon protected (years protected). Thus, if an investor would like a protection level of 0%(meaning protect only the principal) for five years when the current interest rate is 5.36% and the volatility of the portfolio is estimated to be 20%, and with $10,000 to invest, one interprets the table as: The investor will have $9,301 (1st number) to invest in the portfolio and will use the remaining $699 (2nd number) to pay for the protection. This protection is effectively costing him 7.51% (3rd number) of the invested assets. One can read the table for any scenario one is interested in. Figure 1 shows how the price of this protection varies over 10 years for different levels of protection. It is interesting to note that the price of European protection on a non-dividend paying assets decreases as the time of protection increases.

Due to this unique feature of European options and the fact that the most common types of options on stocks are of the American type, it may be more interesting to examine Table 2 and Figure 2, which prices the same insurance for American options using the binomial option pricing model (Cox et al. (1979)).[7] It is assumed that there is no dividend payout. One can see

[7] One will notice the standard result that the price of the American put option is higher for every horizon of protection. This is partly due to the flexibility of American options and they also protect the buyer from large cash distributions.

Table 2: The Cost of Protecting Your $10,000 Investment American Style

Years Protected	The Percent of Protection (η)						
	–5	–2	–1	0	1	2	5
1.	9563.00	9463.00	9427.00	9388.00	9341.00	9293.00	9146.00
	437.00	537.00	573.00	612.00	659.00	707.00	854.00
	4.57	5.67	6.07	6.52	7.06	7.61	9.34
2.	9563.00	9377.00	9309.00	9235.00	9148.00	9059.00	8763.00
	437.00	623.00	691.00	765.00	852.00	941.00	1237.00
	4.57	6.64	7.42	8.29	9.31	10.39	14.12
3.	9588.00	9350.00	9253.00	9145.00	9020.00	8889.00	8418.00
	412.00	650.00	747.00	855.00	980.00	1111.00	1582.00
	4.30	6.95	8.08	9.35	10.86	12.50	18.79
4.	9631.00	9348.00	9223.00	9084.00	8922.00	8745.00	8087.00
	369.00	652.00	777.00	916.00	1078.00	1255.00	1913.00
	3.84	6.97	8.42	10.08	12.08	14.35	23.66
5.	9670.00	9359.00	9210.00	9040.00	8840.00	8613.00	7752.00
	330.00	641.00	790.00	960.00	1160.00	1387.00	2248.00
	3.41	6.85	8.58	10.62	13.13	16.10	29.00
6.	9712.00	9374.00	9206.00	9007.00	8767.00	8488.00	7408.00
	288.00	626.00	794.00	993.00	1233.00	1512.00	2592.00
	2.97	6.68	8.62	11.03	14.06	17.82	34.99
7.	9747.00	9394.00	9208.00	8980.00	8701.00	8368.00	7047.00
	253.00	606.00	792.00	1020.00	1299.00	1632.00	2953.00
	2.59	6.45	8.60	11.35	14.93	19.50	41.91
8.	9781.00	9417.00	9216.00	8959.00	8639.00	8252.00	6703.00
	219.00	583.00	784.00	1041.00	1361.00	1748.00	3297.00
	2.24	6.19	8.51	11.62	15.76	21.19	49.18
9.	9810.00	9442.00	9225.00	8942.00	8580.00	8133.00	6376.00
	190.00	558.00	775.00	1058.00	1420.00	1867.00	3624.00
	1.94	5.91	8.40	11.83	16.55	22.96	56.83
10.	983.00	9467.00	9237.00	8927.00	8523.00	8014.00	6065.00
	164.00	533.00	763.00	1073.00	1477.00	1986.00	3935.00
	1.67	5.63	8.26	12.01	17.32	24.79	64.87

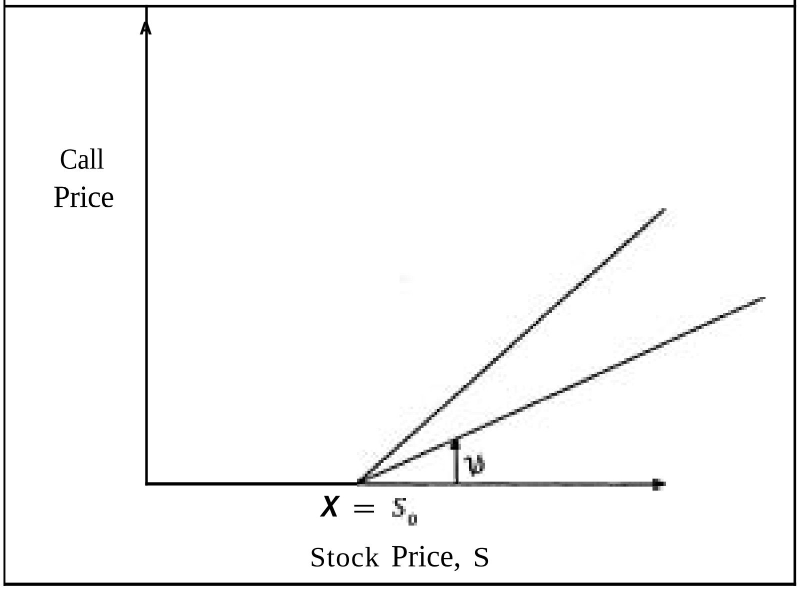
Note: This example was created using $r = 0.0536$ and $\sigma = 0.2155$. The cost of protection is calculated by using the theoretical price for non-dividend paying, European put options according to the Black-Scholes formula. For every year protected and for every amount of protection, there are three rows. The first row represents the amount of the original $10,000 that is left for actual investment in the portfolio. The second row represents the amount of the original $10,000 that must be used to pay for the protection. The third row represents the percentage cost of the protection as a percentage of the amount actually invested.

option on his portfolio. On the other hand, the investor is selling the investment firm a call on ψ % of the upside of his or her portfolio (see Figure **3).** The only fair price for this transaction is that both options are equal to each other. As before, the option offered by the 'firm to the customer is a standard put option on some fraction of the upside, thus, its price is:

$$p_P = S_P[e^{(\eta - r)(T-t)} N(-d_2) - N(-d_1)] \quad ...(7)$$

The implicit option offered by the investor to the firm can be thought of as an at-the-money strike, where the payoff is some fraction, ψ of the traditional payoff to a call option. A keen reader will observe that ψ, the upside sacrifice, and η the desired level of protection, are interdependent. Thus, by choosing one of them, an investor has implicitly already chosen the other. The price of this other option can be obtained from a slight modification of the price of a call option on the stock, as written below

Figure 3: Paying for Protection with the Upside, ψ



$$c_P^* = \psi S_P \left[N(d_1) - e^{(\eta^* - r)(T-t)} N(d_2)\right] \quad ...(8)$$

where, ψ is just the portion of the upside that is exchanged as payment to the investment firm from the investor and η^* can be a separate point from where the upside is exchanged. For instance, the investor may want protection for an amount, but may want to exchange excess returns from a point η^*. The equality of the two option prices makes the fair, non-arbitrage equilibrium. Thus,

$$p_P = c_P^* \quad S_P\left[e^{(\eta - r)(T-t)} N(-d_2) - N(-d_1)\right] = \psi S_P\left[N(d_1) - e^{(\eta^* - r)(T-t)} N(d_2)\right] \quad ...(9)$$

One can rearrange this to solve for ψ given the η and η^* parameters. The solution is

$$\psi = \frac{\left[e^{(\eta - r)(T-t)} N(-d_2) - N(-d_1)\right]}{\left[N(d_1) - e^{(\eta^* - r)(T-t)} N(d_2)\right]} \quad ...(10)$$

Table 3 illustrates the amount of upside an investor may be forced to give up for various protection horizons and for various levels of protection. One can see that for protecting the principal for one year (η = 0%), the investor must give up 53.5% of his or her upside. This decreases to **13%** for a protection level of **10** years. For comparison, Table 4 illustrates the upside cost modelling the option price for non-dividend paying, American put option.

Table 3: The Upside Cost of Protection							
Years Protected	The Percent of Protection (η = η*)						
	–5	–2	–1	0	1	2	5
1.	29.70	42.31	47.58	53.50	60.13	67.58	95.89
2.	17.74	29.46	34.83	41.16	48.61	57.38	94.23
3.	11.84	22.23	27.34	33.59	41.23	50.56	92.96
4.	8.36	17.47	22.24	28.25	35.84	45.41	91.91
5.	6.11	14.09	18.50	24.22	31.64	41.27	90.98
6.	4.58	11.57	15.63	21.04	28.25	37.84	90.14
7.	3.49	9.63	13.36	18.46	25.43	34.91	89.38
8.	2.70	8.09	11.53	16.33	23.03	32.37	88.67
9.	2.11	6.86	10.02	14.53	20.98	30.14	88.00
10.	1.66	5.86	8.76	13.00	19.18	28.16	87.37

Note: This example was created using $r = 0.0536$, and $\sigma = 0.2155$. The cost of protection is calculated by using the theoretical price for non-dividend paying, European put options according to the Black-Scholes formula.

Table 4: The Upside Cost of Protection							
Years Protected	The Percent of Protection (η = η*)						
	–5	–2	–1	0	1	2	5
1.	32.45	46.02	51.81	58.62	66.16	74.45	105.62
2.	20.14	34.15	40.47	48.17	57.25	67.86	114.27
3.	14.17	27.25	33.76	41.91	51.99	64.36	122.78
4.	10.36	22.56	29.12	37.56	48.39	62.34	134.31
5.	7.91	19.13	25.64	34.27	45.78	61.25	146.55
6.	6.12	16.54	22.92	31.69	43.82	60.67	161.69
7.	4.85	14.47	20.73	29.62	42.34	60.54	178.87
8.	3.87	12.78	18.88	27.90	41.20	60.78	196.60
9.	3.13	11.36	17.34	26.45	40.32	61.51	213.98
10.	2.54	10.17	16.02	25.21	39.64	62.45	231.94

Note: This example was created using $r = 0.0536$, and $\sigma = 0.20$. The cost of protection is calculated by using the theoretical price for non-dividend paying, American put option.

In this section, we present the cost of portfolio protection as a percentage of the upside of the portfolio. Offering the protection in this fashion may be of more interest to investors in separate accounts, rather than paying a fixed fee upfront. If priced correctly, the hedging firm or separate account firm should be indifferent to the payment mechanism.

Insurance and Exchange

It is natural to expect a portfolio's characteristics to change over time. This could be due to a variety of reasons. One such reason could be due to a corporate action on a particular stock in the portfolio. It could also be due to changes in exchange rates in a multi-currency portfolio. It may also be due to investor actions, such as the desire of investors to update their portfolios, either by purchasing more of the same portfolio or by selling one stock for another or by exchanging one portfolio for another portfolio. An investor that had purchased insurance on such a portfolio would most likely want the new portfolio to also have protection regardless of the reason for change. At a minimum, an investor would like to be offered the choice of protection or not.

One can think of this problem in two periods: t', the period before the change in the portfolio and t'', the period after the change in the portfolio. Before the change in the portfolio, we can value the option that the user has purchased on his or her portfolio. This option's value is $p_{t'}$.[8] The value of the option on the newly altered portfolio (be it a minor or major alteration) for the same time to expiration will be $p_{t''}$.

In the most simple manner, one can see that offering the user protection on the new investment is very simple requiring only that the investor pay the difference between the two prices. Again, this difference can be paid in cash or paid in terms of the amount of upside received by the investment firm. The hedging and unwinding of current positions will be more difficult, but again, this just translates into a higher price.

A very simple example may help illustrate the point. Suppose, an investor owns portfolio A at time t', shortly before liquidating this portfolio and purchasing portfolio B at time t''. Suppose that the investor originally purchased the portfolio A at time t^0, where, $t' - t^0 = 0.5$. For simplicity, assume that the underlying portfolio did not appreciate in value. Assume that the new portfolio B has a much higher volatility of $\sigma_B = 40\%$, but all else is equal. This investor would like to maintain the exact same duration and level of protection. Assume that the investor had originally bought protection for five years. From Table 2, we know that the investor originally invested $9,301 and paid $699 for the protection for five years. Now, with 4.5 years remaining until the expiration of the option, it is worth $704. The price of protection for 4.5 years on the more volatile portfolio B is $1,835.[9] Thus, at the time, the investor sells his or her current portfolio A for $9,301 and purchases $9,301 of portfolio B, he or she will have to pay a net of $1,131 ($1,835 – $704) to continue to maintain the same level of protection on the more volatile portfolio. One should also note that had the new portfolio had the same risk level, the investor would have had to pay net $0.00 (ignoring transaction costs).

[8] In the case where the portfolio has not changed at all, only the time remaining until expiration has changed making the option usually less valuable.

[9] Author's own calculations using the Black-Scholes formula. The numbers for this example are contained in Table 5.

Table 5: The Cost of Exchanging Portfolio Protection							
Years Protected	The Percent of Protection (η)						
	–5	–2	–1	0	1	2	5
1.	9601.00	9505.00	9470.00	9434.00	9395.00	9356.00	9226.00
	399.00	495.00	530.00	566.00	605.00	644.00	774.00
	358.00	412.00	430.00	449.00	469.00	489.00	552.00
	828.00	887.00	907.00	927.00	947.00	967.00	1028.00
	469.14	475.23	476.55	477.51	478.11	478.35	476.83
	63.07	72.08	75.36	78.79	82.38	86.12	98.41
2.	9612.00	9459.00	9400.00	9336.00	9268.00	9195.00	8952.00
	388.00	541.00	600.00	664.00	732.00	805.00	1048.00
	401.00	527.00	575.00	625.00	677.00	733.00	913.00
	1119.00	1283.00	1341.00	1399.00	1458.00	1519.00	1703.00
	717.29	755.88	765.85	774.22	780.93	785.91	790.12
	44.71	56.48	61.05	65.98	71.31	77.06	97.25
3.	9654.00	9464.00	9386.00	9301.00	9208.00	9107.00	8757.00
	346.00	536.00	614.00	699.00	792.00	893.00	1243.00
	369.00	542.00	611.00	685.00	765.00	851.00	1141.00
	1190.00	1443.00	1534.00	1627.00	1723.00	1821.00	2124.00
	820.29	901.00	922.93	941.88	957.56	969.77	983.97
	35.07	47.57	52.64	58.24	64.43	71.28	96.44
4.	9700.00	9488.00	9396.00	9294.00	9179.00	9053.00	8603.00
	300.00	512.00	604.00	706.00	821.00	947.00	1397.00
	324.00	526.00	611.00	705.00	808.00	921.00	1313.00
	1184.00	1513.00	1633.00	1758.00	1888.00	2022.00	2441.00
	860.44	986.34	1021.95	1053.29	1079.76	1100.84	1127.88
	28.65	41.25	46.55	52.51	59.24	66.81	95.77
5.	9744.00	9519.00	9418.00	9301.00	9169.00	9020.00	8475.00
	256.00	481.00	582.00	699.00	831.00	980.00	1525.00
	278.00	498.00	595.00	704.00	827.00	964.00	1453.00
	1143.00	1534.00	1680.00	1835.00	1996.00	2164.00	2695.00
	864.98	1035.65	1085.75	1130.59	1169.07	1200.22	1242.35
	23.95	36.36	41.75	47.93	55.00	63.10	95.19

Note: This example was created using $r = 0.0536$ and $\sigma = 0.2155$. For the second portfolio, the $\sigma = 0.40$. The cost of protection is calculated by using the theoretical price for non-dividend paying, European put options according to the Black-Scholes formula. For every year protected and for every amount of protection, there are three rows. The first row represents the amount of the original $10,000 that is left for actual investment in the portfolio. The second row represents the amount of the original $10,000 that must be used to pay for the protection. The third row represents the cost of the protection for the same portfolio until maturity if the protection was bought or sold after 0.5 years. The fourth row represents the cost of the protection on the new portfolio for protecting the same level as with the old portfolio. The fifth row is the cost difference between the two or in other words how much new money would have to be paid to maintain the same level of protection on the newly exchanged portfolio. The sixth row represents how much of the upside of the new portfolio would have to be given up to maintain the same level of protection rather than paying in cash.

all these option positions are added up to estimate the total risk inherent in these positions by the firm.

Consider many investors, m, and each of those investors holds n_i portfolios with protection, thus, the total number of portfolios with protection at any given moment is $N=\sum_{i=1}^{m} n_i$. Of course, the different portfolios can have different levels of protection and different characteristics.[12] Assuming, the investment house has properly calculated their hedge ratios or s, then each portfolio will have a A. Thus, these can be indicated by A, for portfolio j.

Thus, for each stock, we can aggregate over all of the portfolios to understand the aggregate amount of hedging needed for any particular stock, this amount is:

$$V_i^a = \sum_{j=1}^{N} \Delta_j w_{i,j} V_j \quad ...(11)$$

$$s_i^a = \frac{V_i^a}{P_i} \quad ...(12)$$

where, $w_{i,j}$ represents the weight of stock i in portfolio j, V_j is the dollar value of portfolio j, Δ_j is the appropriate delta-hedge parameters from the Black-Scholes formula[13] based on the volatility of the entire portfolio, and s_i^a represents the amount of shares to be sold in aggregate based on hedging n_i portfolios in aggregate.

The positions required to A-hedge can all be aggregated in this fashion and the corresponding aggregate dynamic hedging set in place. These hedge amounts can be computed at the close of any hedging interval (e.g., daily) and appropriate positions taken by the firm to hedge the risk. To the extent that people sell insurance,[14] there will be crossing which will result in bid-ask savings on the overall hedging costs.

Deaggregation and New Flows

Once the separate account firm has made all stock order transactions for hedging purposes in an omnibus account, the details of each order as they pertain to each customer should be stored in each account through a computer program and database system that would deaggregate the bulk orders and assign values to each individual's account. In addition, the relevant parameters for each individual client should be computed daily so as to have the new hedging parameters and aggregate and deaggregate accordingly.[15]

[12] It might be better to hedge the entire universe of portfolios for the investor, rather than each in isolation.

[13] This analysis does not depend on using the Black-Scholes model for valuing the hedge parameters or the option. It is used to simplify the discussion of the concepts. In fact, the firm should use their own proprietary hedging scheme based upon their own expertise.

[14] This possibility has not been explicitly considered thus far.

[15] To the extent that the overall positions of client accounts net out, the separate account firm may wish to hedge with broad market futures contracts and in some extreme cases to not hedge at all, since the overall firm risk might be very small.

Historical Simulations of Pricing and Hedging

In this section, an actual portfolio will be used as an example to illustrate the feasibility of such a system. A simple hedging technique is used to illustrate the concepts with real data. The hedging technique will hedge the portfolios by taking underlying positions in each stock based on the A of the underlying portfolio. This will be called "dynamic delta hedging". The actual hedging strategy is used for illustration purposes. Clearly, firms that offer these products will use more advanced proprietary techniques.

Empirical Methodology

The study period shall be divided into two sections: an in-sample estimation period, and an out-of-sample testing period. A rolling window shall be used to measure the volatility of the portfolio due to changing circumstances. For example, suppose at date t, the portfolio is purchased by investor one, and investor one also wishes to purchase protection equal to three years with no loss of principle ($\eta = 0$). Then, at date t, the portfolio will be priced using a Black-Scholes model with a volatility for the portfolio based on the preceding 90 business days. A standard A shall also be computed and the appropriate number of shares shall be sorted for the first hedging day. On the following day, t + 1, the volatility of the portfolio shall be reestimated for the previous 90 days and the A shall be computed again. This process continues until the maturity of the underlying protection.[16]

[16] These are very rudimentary hedging techniques to illustrate the concepts. Other simple methods, which at times may be more appropriate and less complicated include using futures indices to hedge. In the case using the S&P 500 or NASDAQ futures for hedging the portfolio, the corresponding β of the portfolio with each of the two indices shall be computed and the one with the highest historical $\bar{R}^2$ will be chosen. The hedging amount of the index or number of contracts to short of the index shall be chosen using the following formula $N_f = \frac{\Delta_i v_t}{N_f q S_t}$ where $\Delta_i = \Delta_p \beta$. The aggregation for any portfolio is completed by aggregating the adjusted hedging parameters. That is,

$$N_f = \frac{\sum_{j=1}^{N} \Delta_{i,j} V_j}{N_f q S_t} \qquad \text{...(13)}$$

For the multifactor hedging technique, the approach is similar. The return generating process for stock returns is given by some model:

$$rP = a + \beta_1 (r_M - r_f) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t \qquad \text{...(14)}$$

where, r_M, r_f, SMB_t, HML_t are the Fama-French market, size, and book-to-market premium. One the hedging parameters have been estimated, the hedging of the portfolio is completed by purchasing the relevant amount of each underlying factor portfolio. Thus, for each share of each portfolio on day t, $\Delta\beta_1$, $\Delta\beta_2$, and $\Delta\beta_3$ are shorted of each of three portfolios and this short is updated daily. The aggregation is completed similarly to the single factor case described above. Of course, for any separate account platform engaging in these types of insurance, a sophisticated derivatives hedging team should be hired and/or a sophisticated software should be purchased.

Table 6: The Investor's Portfolio			
Sl. No	Ticker	Company Name	**Industry**
1.	ADPT	Adaptec Inc.	Computer Communication Equip
2.	APH	Amphenol Corp.	Electronic Connectors
3.	AWR	American States Water Co.	Water Supply
4.	BEZ	Baldor Electric Co.	Motors and Generators
5.	BKH	Black Hills Corp.	Electric Services
6.	BSX	Boston Scientific Corp.	Surgical, Med Instr, Apparatus
7.	CBSS	Compass Bancshares Inc.	Commercial Banks
8.	CHE	Chemed Corp.	Home Health Care Services
9.	CRR	Carbo Ceramics Inc.	Mng, Quarry Nonmtl Minerals
10.	CTXS	Citrix Systems Inc.	Prepackaged Software
11.	CW	Curtiss-wright Corp.	Aircraft Parts, Aux Eq, Nec
12.	DELL	Dell Inc.	Electronic Computers
13.	DIS	Disney (Walt) Co.	Television Broadcast Station
14.	DLP	Delta & Pine Land Co.	Agriculture Production-crops
15.	DRS	DRS Technologies Inc.	Srch, Det, Nav, Guid, Aero Sys
16.	DY	Dycom Industries Inc.	Water, Sewer, Pipe Line Constr
17.	ECL	Ecolab Inc.	Special Clean, Polish Preps
18.	ETR	Entergy Corp.	Electric Services
19.	EXAR	Exar Corp.	Semiconductor, Related Device
20.	FCF	First Commonwlth Finl CP/PA	Commercial Banks
21.	FIF	Financial Federal Corp.	Misc. Business Credit Instn
22.	GPS	Gap Inc.	Family Clothing Stores
23.	GRB	Gerber Scientific Inc.	Special Industry Machy, Nec
24.	GT	Goodyear Tire & Rubber Co.	Tires and Inner Tubes
25.	IEX	Idex Corp.	Pumps and Pumping Equipment
26.	JCI	Johnson Controls Inc.	Public Bldg and Rel Furniture
27.	K	Kellogg Co.	Grain Mill Products
28.	KWR	Quaker Chemical Corp.	Misc. Pds of Petroleum and Coal
29.	MER	Merrill Lynch & Co Inc.	Security Brokers and Dealers
30.	MLNM	Millennium Pharmaceuticals	In Vitro, in Vivo Diagnostics
31.	MMS	Maximus Inc.	Management Services
32.	MOG.A	Moog Inc. -CL A	Aircraft Parts, Aux Eq, Nec
33.	MRD	Macdermid Inc.	Misc. Chemical Products

(Contd...)

Table 6: The Investor's Portfolio			(...contd)
SI. No	Ticker	Company Name	Industry
34.	MRK	Merck & Co.	Pharmaceutical Preparations
35.	MSM	Msc Industrial Direct -CL A	Industrial Mach and Eq-whsl
36.	MTSC	Mts Systems Corp.	Meas and Controlling Dev. Nec
37.	NSC	Norfolk Southern Corp.	Railroads, Line-haul Operatng
38.	ODFL	Old Dominion Freight	Trucking, Except Local
39.	PDE	Pride International Inc.	Drilling Oil And Gas Wells
40.	PDLI	Pdl Biopharma lnc.	Biological Pds, Ex Diagnstics
41.	PLFE	Presidential Life Corp.	Life Insurance
42.	PSD	Puget Energy Inc.	Electric and Other Serv Comb
43.	SNPS	Synopsys Inc.	Prepackaged Software
44.	SXT	Sensient Technologies Corp.	Industrial Organic Chemicals
45.	TMK	Torchmark Corp.	Life Insurance
46.	TNL	Technitrol Inc.	Electronic Components, Nec
47.	TRB	Tribune Co.	Newspaper: Pubg, Pubg and Print
48.	UGI	Ugi Corp.	Gas and Other Serv Combined
49.	WLM	Wellman Inc.	Plastic Matl, Synthetic Resin
50.	WON	Westwood One Inc.	Amusement and Recreation Svcs

Note: **This hedging portfolio was equally-weighted at the start of every hedging period.**

Since the performance of such hedging strategies will be determined partly by the pricing of such insurance contracts and the time period examined, focus will be on illustrating the concepts rather than on the specific results. The following portfolio was chosen by randomly selecting 50 stocks from the S&P 1500 at the beginning of 2000. An equal-weight portfolio was created on January 2, 2000 and various other dates. Table 6 shows the stocks in this portfolio. The portfolio was then hedged and the customer was charged in two ways: straight cash for the protection and a percentage of the upside of the portfolio. The inputs to the pricing and hedging of the options was a straight Black-Scholes formula, using a rolling 90-day volatility as the volatility input, a treasury risk-free rate of interest as the respective interest rate, and the other parameters were, $S = X$ (that is, a protection of the principle for the corresponding horizon), and the time to maturity varied from as little as one year to as many as six years. Various combinations of hedging periods were considered for the time period from January 2000 to January 2006. The options were of an European variety.

Empirical Results

The performance of these portfolios for an investor that purchased protection and the cost of hedging to the company is presented in Table 7. The first three columns represent the returns to the investor from this portfolio with and without hedging. For example, for the customer

Table 7: Hedging Costs and Investor Returns

Portfolio	Investor Returns (%)			Hedging Costs ($ per share)	Revenue ($ per share)		Net P/L ($ per share)	
	None	Cash	Upside	A	Cash	Upside	Cash	Upside
Portfolio (6-Year Protection)January 2, 2000 – January 2, 2006	11.51	11.30	10.99	17.07	3.61	6.98	–13.46	–10.09
Portfolio (5-Year Protection)January 2, 2000 –January 2, 2005	12.84	12.53	12.05	17.51	3.94	8.16	–13.57	–9.35
Portfolio (5-Year Protection)January 2, 2001 –January 2, 2006	12.48	10.75	9.72	21.26	24.12	32.36	2.86	11.10
Portfolio (4-Year Protection)January 2, 2000 –January 2, 2004	11.64	11.41	11.07	17.19	3.52	7.14	–13.67	–10.05
Portfolio (4-Year Protection)January 2, 2001 – January 2, 2005	15.17	12.84	11.18	21.10	23.88	35.74	2.78	14.64
Portfolio (4-Year Protection)January 2, 2002 – January 2, 2006	12.01	9.28	5.26	16.74	23.07	49.03	6.33	32.29
Portfolio (3-Year Protection)January 2, 2000 – January 2, 2003	3.69	2.70	3.19	19.45	4.84	2.07	–14.61	–17.38
Portfolio (3-Year Protection)January 2, 2001 –January 2, 2004	13.66	9.93	9.12	20.59	23.66	26.03	3.07	5.44
Portfolio (3-Year Protection)January 2, 2002 – January 2, 2005	15.50	12.02	6.00	15.82	20.20	49.51	4.38	33.69
Portfolio (3-Year Protection)January 2, 2003 – January 2, 2006	18.44	14.73	5.44	12.44	20.07	60.21	7.63	47.77
Portfolio (2-Year Protection)January 2, 2000 – Jan 2, 2002	7.56	5.97	6.09	14.86	5.11	4.11	–9.75	–10.75
Portfolio (2-Year Protection)January 2, 2001 –January 2, 2003	3.63	-3.09	2.05	20.53	22.46	4.98	1.93	–15.55

(Contd...)

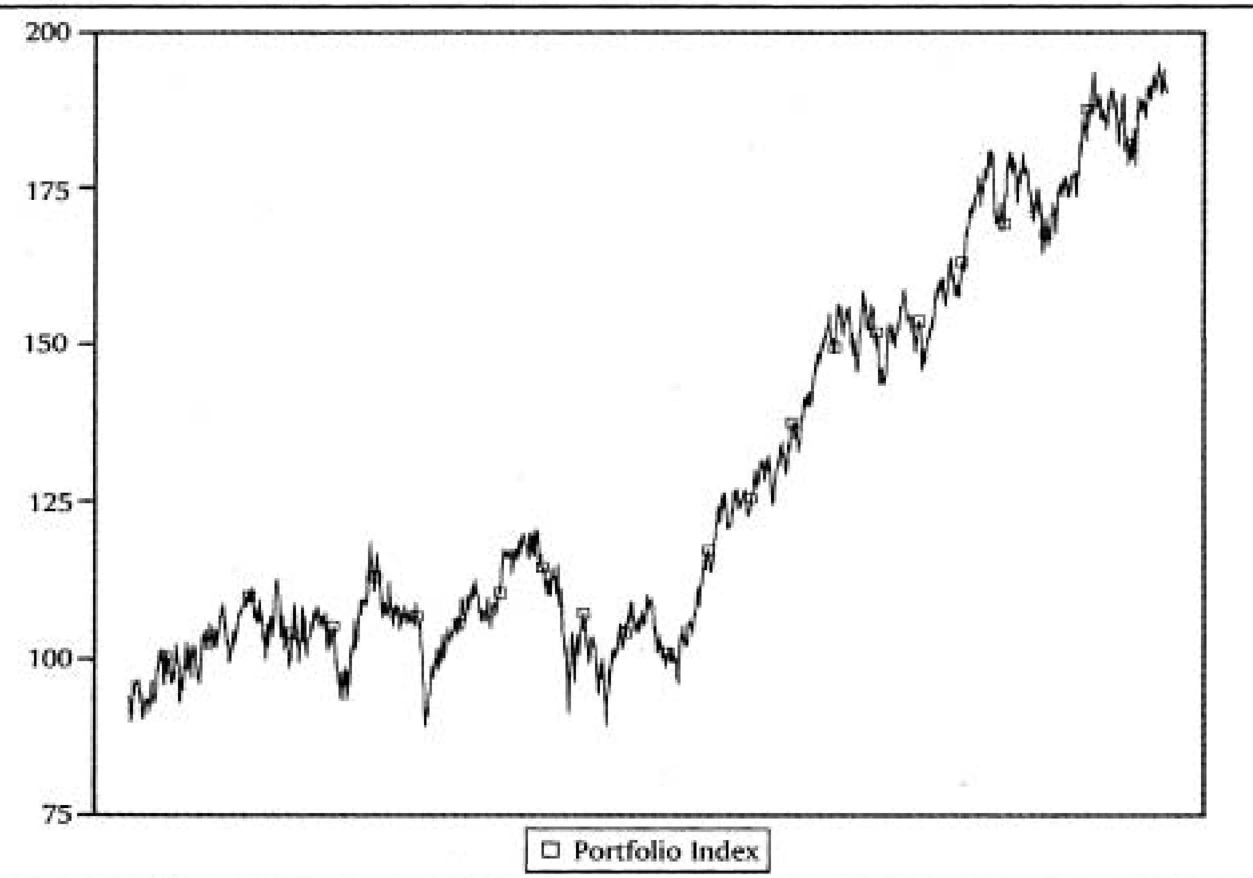
Table 7: Hedging Costs and Investor Returns *(...contd)*								
Portfolio	Investor Returns (%)			Hedging Costs ($ per share)	Revenue ($ per share)		Net P/L ($ per share)	
	None	Cash	Upside	A	Cash	Upside	Cash	Upside
Portfolio (2-Year Protection) January 2, 2002 – January 2, 2004	14.44	9.21	4.52	14.55	17.18	30.71	2.63	16.16
Portfolio (2-Year Protection) January 2, 2003 – January 2, 2005	26.48	21.29	6.33	11.10	16.37	57.81	5.27	46.71
Portfolio (2-Year Protection) January 2, 2004 – January 2, 2006	9.06	5.85	2.75	12.75	12.39	22.59	-0.36	9.84
Portfolio (1-Year Protection) 2000	4.02	2.97	2.76	11.18	4.77	5.28	-6.41	–5.90
Portfolio (1-Year Protection) 2001	3.14	-0.63	1.40	13.17	18.82	8.37	5.65	-4.80
Portfolio (1-Year Protection) 2002	-4.54	–8.90	0	12.47	12.85	0	0.38	–12.47
Portfolio (1-Year Protection) 2003	39.46	29.89	6.27	8.73	11.95	40.83	3.22	32.10
Portfolio (1-Year Protection) 2004	15.10	9.92	3.30	10.50	8.95	20.02	–1.55	9.52
Portfolio (1-Year Protection) 2005	1.44	–2.47	0.61	8.25	7.95	1.62	-0.30	-6.63
Average	11.75	8.45	5.72	15.11	12.74	22.55	–2.36	7.44

Note: The first three columns represent the returns to the investor from this portfolio with and without hedging. The first column is the annualized return of the portfolio without hedging, the second column is the return of the portfolio after subtracting the put premium, and the third column is the return to the investor after subtracting the upside taken by the firm. The fourth column represents the hedging cost from a daily A hedging program. The costs are in $-per-share. The next two columns represent the revenues to the hedger for offering the protection. The fifth column represents the revenue from the put premium plus any associated interest and the sixth column represents the revenue from taking a percentage of the upside as agreed in the protection contract. The last two columns represent the profit and loss from the hedging program for the two types of payment.

that chose to buy the equal-weighted portfolio on January 2, 2000 and buy protection until January 2, 2006 (six-year protection), the annualized return of this portfolio over the period was 11.51%. Had the investor paid cash for protection, the return of this portfolio would have been 11.3%, and had the investor paid upside, rather than upfront cash, the annualized return would have been 10.99%.

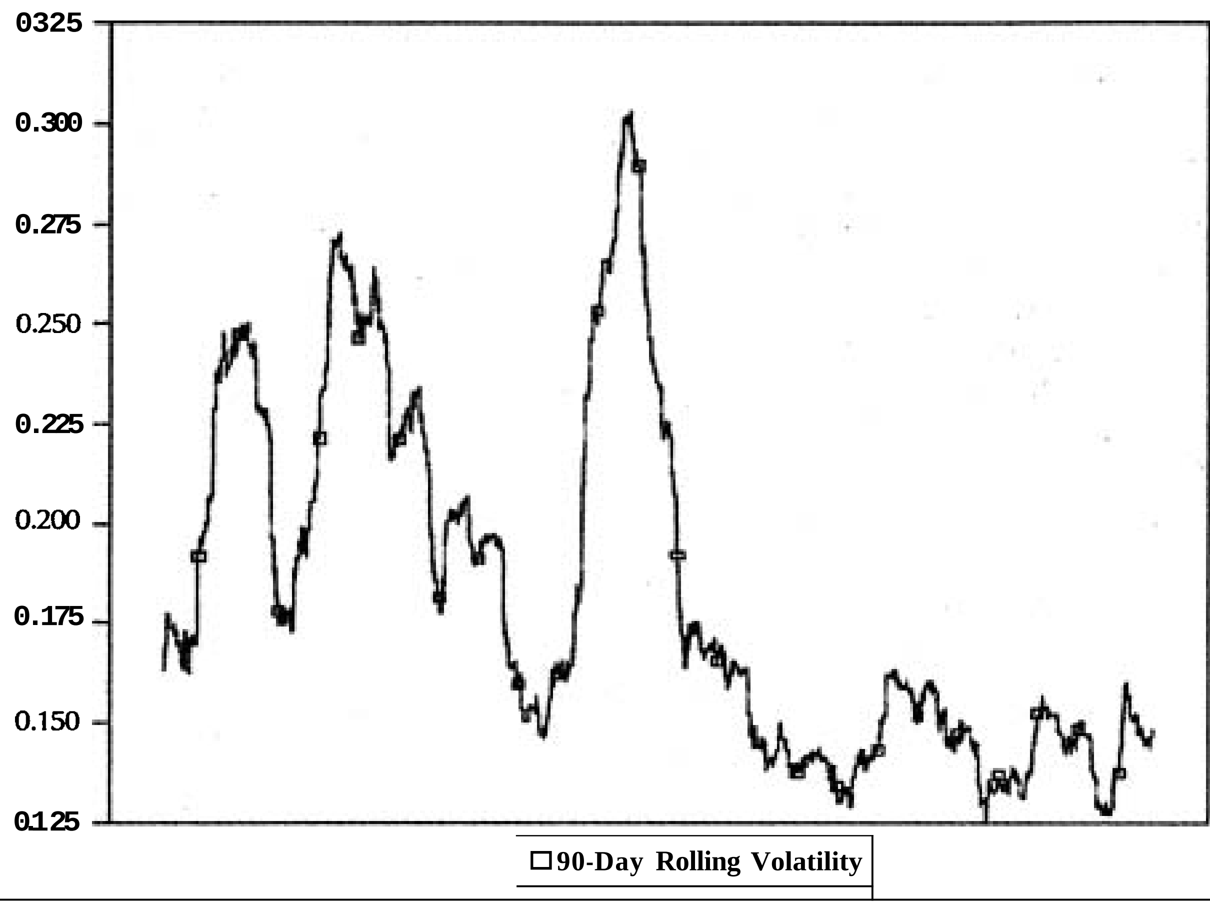
The fourth column represents the hedging cost from a daily A hedging program using the parameters already described. The costs are in $-per-share. Thus, these costs represent the dollar cost of dynamic hedging. Thus, to hedge, this portfolio from January 2, 2000 to January 2, 2006 would have cost the hedger $17.07 per portfolio unit. The next two columns represent the revenues to the hedger for offering the protection. In the case of a cash payment, they would receive the put option premium as described by Black-Scholes ($2.81) plus the interest on the premium which makes a total revenue of $3.61. If the investor had chosen instead to pay upside, at the time of signing the contract, the upside parameter, was equal to 0.057. Thus, over the period, the hedging firm would have collected a total of $6.98. The growth of the index over the entire period is depicted in Figure 4.

Figure 4: The Value of the Portfolio Index (January 2000–January 2006)



The last two columns represent the profit and loss from the hedging program for the two types of payment. In this case, both hedging programs produced a large loss to the hedging company of $13.46 and $10.09 per unit of the portfolio hedged. There are many reasons for this hedging loss. Some of the reasons have to do with the implementation of a very simple

Figure 5: The 90-Day Rolling Volatility of the Portfolio (January2000–January 2006)



dynamic hedging and pricing methodology based upon Black-Scholes. In this particular case, the main reason for the losses on hedging are due to an incorrect input of volatility. Figure 5 depicts the 90-day rolling volatility of the portfolio over the entire period. Over the investment horizon, the actual volatility turned out to be much higher than estimated on the first day of pricing the protection.

The values for other hedging periods are also presented in this table. The final column represents the average of all the respective hedging periods. On average, the upside premium made money for the hedging firm, while the cash option lost money. This is quite logical since this was a period in which the portfolio increased dramatically. Although from the investor point of view, hedging never seems worthwhile, this is a natural case when one buys protection on a rising portfolio.

Other Practical Applications of Such Insurance

The Real-Estate Market

Most people have most of their wealth tied up in their house. In some senses, this is unfortunate, because it means that a large part of their wealth is invested in an illiquid, undiversified holding. With the advent of many futures, contracts based on an index of home prices in various cities across the United States, possibilities will emerge to aid home owners in transporting this investment into other more liquid vehicles. The concepts in this paper apply well here too. Suppose a homeowner in San Francisco has a house worth $600,000 and has $400,000 in equity. One way he or she could tap into this money is to ask for a home equity loan. Some home owners use home equity loans for current consumption purposes, such as buying a new car. Although that is frequently done, it's really not related to diversification. Instead, there are other home owners who might choose to use the home equity loan to invest in the broad stock market. In our specific case, the home owner would take a loan of let's say 80% of the $400,000, thus $320,000 at an interest rate of let's say 6% and invest in the S&P 500. If all goes well and the S&P 500 outperforms the home equity loan rate, the home owner has effectively diversified his holdings and come out well. If, on the other hand, the investments do poorly, the homeowner will find himself in a troublesome situation of having to pay back interest and a principle of which he currently has depleted somewhat.

An alternative way to provide this diversification would be for a financial intermediary to offer the homeowner the portfolio of home equity desired in exchange for a percentage ownership of the house. Using the example above, the financial intermediary would give the homeowner $320K in exchange for say 53.33% of the house. The financial intermediary would be repaid for this, when the home is eventually sold.[17]

In addition to providing cash for the equity for diversification purposes, the financial intermediary could also offer protection on the home value. Since homeowners might be cash constrained, the most suitable form of protection might be a percentage of the upside of the house appreciation in exchange for this protection. Thus, a homeowner would forefeit x% of the upside appreciation of the house over the next n years. This would allow the homeowner to remove the downside risk of a collapse in home prices. In fact, the financial intermediary might offer a combination of these two products.

As for the financial intermediary, this program is very similar to selling put protection on the value of the house in exchange for a call option on the upside of the housing value. The financial intermediary would most likely charge a slight premium than fair value in order to maintain a profitable business. The financial intermediary could hedge this option in a variety of ways, including shorting Real-Estate Investment Trusts (REITS) of the appropriate type, shorting the new CME real-estate fhtures contracts, and through other means, including options on REITS and futures.

ETF Equity Market

Exchange Traded Fund (ETF) market is exploding in recent years. Today, it is very similar to the mutual fund market 30 years ago. Enormous advantages of ETFs over mutual funds will slowly eat into the mutual fund business assets as well as grow new assets of their own. In the last year or so, some fancy twists on basic equity index, ETFs have crept into the market place, including ETFs whose stocks are weighted by fundamental factors, such as a company's dividend to price ratio or ETFs that represent leveraged exposure to various equity indices in the negative or positive direction. For example, some new ETFs give the owner exposure to 2× or –2× the returns of the S&P 500.

[17] The financial intermediary might also charge a large fee as a percentage of the final selling price of the house for this service, since they would be losing the opportunity cost of capital or the risk-free rate over the period. They might also specify a maximum time period over which the house must be sold.

An obvious next step is to launch ETFs using a variety of concepts discussed in this paper. For example, a series of ETFs could be offered based upon major stock indices, which promise x% exposure to the upside of the equity index, but with only a worst case downside exposure of –y%. For example, a whole host of ETFs could be launched various protection levels and percentage of upside as listed in Tables 3 and 4. Of course, rather than being customized to an individual's specific portfolio, these ETF products would be based on one particular index, like the S&P 500 or Nasdaq-100. Rather than any dynamic hedging, however, these ETF-insurance products could be constructed by purchasing futures contracts on the underlying equity index product as well as call and put options to create the desired exposure. For example, suppose an S&P 500 50-5-3 was created, which guaranteed the buyer 50% of the upside of the returns to the S&P 500, with a worse case loss of 5%, regardless of the S&P 500 return, and for three years. The investor would simply purchase this product with the amount of funds he or she desires (e.g., $100M). The index of the ETF and/or the required hedging would be accomplished by purchasing an equivalent amount of S&P 500 futures contracts to obtained the desired exposure to $100M. The number of S&P 500 puts can easily be calculated so as to hedge –5% of the portfolio, and the number of calls to purchase would be equivalent to the number to equate the costs of the put protection. This would implicitly determine the percent of exposure to the S&P 500. This could be rebalanced daily to maintain roughly the same desired exposure each day for customers.

These products could be expanded in a number of directions. First, a suite of products with various levels of protection and upside could be produced. Second. a variety of expiration modes could be examined. For example, rather than make the ETF a daily exposure to this protection, it could be expanded to one month, one year, or many years into the future.

Portfolio Insurance and the Crash of 1987

The proposal in this paper for insurance is cosmetic in nature, rather than being fundamentally different from ideas already present in the option literature, including dynamic portfolio insurance strategies. Many people blamed the crash of 1987 on program-driven portfolio insurance schemes. Since 1987, there have been other crashes in 1991 and 1997 that were not related directly to portfolio insurance but may have been related to broker-dealers hedging their written over-the-counter (OTC) options through dynamic hedging.[18] Rubinstein (2000) has argued that portfolio insurance was not the primary reason of the market crash of 1987. Also, even the proponents of portfolio insurance causing the crash admit that it was the large number of assets pursuing these strategies that may have led to the amplification of a downside market move. To the extent that these mechanisms at brokerage firm or separate account platforms are small with respect to the vast majority of investors, it is even more unlikely that they will have destabilizing effects.

Since 1987, the stock exchanges and futures exchanges have added circuit breakers and specific rules to reduce the impact of programmed trades on markets. One must remember that any program to offer customized portfolio protection to individuals in separate accounts will

[18] A great reference for the entire period of portfolio insurance is Jacobs (1999).

have to have hedging schemes that adjust prices for the actual behavior of markets. To the extent that future volatility is estimated incorrectly and to the extent that stock prices have extreme jumps that are not priced correctly, the offering of portfolio protection products will ultimately result in losses for the hedging firm.

Conclusion

With the advances in technology and trade processing, a broad range of interesting portfolio products can be made available to smaller investors either through separate account platforms or brokerage firms. One of these products is the protection of a customized portfolio over long horizons. One convenient form of this protection is to offer investors protection of their portfolio in exchange for a proportion of the upside returns. Theoretically, it is quite straightforward to create such a product, although it will require appropriate hedging algorithms by the firm offering the protection.

Practically, the advances of technology make offering such mass customized portfolio insurance very practical. This paper has gone through some of the basic elements of such a portfolio protection system and illustrates the kinds of offerings that might apply to investors. The paper also illustrates an example of the returns to investors and returns to the hedging firm from offering such a product. It also describes other areas in which such an insurance system might be offered, such as the real-estate market and the ETF market..

References

1. Atkinson C and Alexandropoulos C A (2006), "Pricing a European Basket Option in the Presence of Proportional Transactions Costs", Applied Mathematical Finance, Vol. 13, No. 3, pp. 191-214.
2. Black Fischer and Myron Scholes (1973), "The Pricing of Options and Corporate Liabilities", Journal of Political Economy, Vol. 81, No. 3, pp. 637-654.
3. Cox John, Ross Stephen and Mark Rubinstein (1979), "Option Pricing: A Simplified Approach", Journal of Financial Economics, Vol. 7, No. 3, pp. 229-263.
4. Jacobs Bruce I (1999), Capital Ideas and Market Realities, Blackwell Publishers.
5. Leland Hayne E and Mark Rubinstein (1981), "Replicating Options with Positions in Stock and Cash", Financial Analysts Journal, Vol. 37, No. 4, pp. 63-72.
6. Merton Robert C (1973), "Theory of Rational Option Pricing", Bell Journal of Economics and Management Science, Vol. 4, No. 1, pp. 141-183, Spring.
7. Moshe Arye Milevsky and Steven E Posoner (1998), "A Closed-Form Approximation for Valuing Basket Options", Journal of Derivatives, Vol. 5, No. 4, pp. 54-61.
8. Rubinstein Mark (2000), "Comments on the 1987 Stock Market Crash: Eleven Years Later", Risks in *Accumulation* Products, Society of Actuaries, pp. 1-6.

Reference # *45J-2008-07-01-01*