MANAGER SKILL AND PORTFOLIO SIZE WITH RESPECT TO A BENCHMARK

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- Thank you for coming.

1. INTRODUCTION

- Active and Passive World of Investing
- Oftentimes we just match or try to beat our benchmark
- Is there any point in asking HOW MANY stocks should we buy to achieve the best chance of success?
- We propose a theoretical framework about this issue.

2. THE MODEL

Assumption 1: In any given index, 50% of the stocks will outperform and 50% will underperform.

Assumption 2: Stock either outperforms or underperforms (1 or 0), magnitude is unimportant.

Assumption 3: A portfolio manager's constant skill lies in the probability to pick a "winner" versus a "loser".

Assumption 4: The benchmark and portfolio are equally-weighted.

2. THE MODEL

We introduce the notion of omega (ω), where ω >1 if a portfolio manager is more likely to pick a good stock versus a bad stock.

To get a rough idea of how ω is related to probabilities, if ω = 1.1 and a manager is picking the 1st stock, the probability of picking a good one is about 0.5238.

2. THE MODEL

Two Possible Selection Methods for a group of n stocks out of a universe of N stocks.

Method 1: Bulk Selection

Method 2: Sequential Selection

2. THE MODEL

Bulk Selection: This means that the portfolio manager selects the stocks into the portfolio ALL AT ONCE using his/her skill. Another way to think of it: the winners and losers are selected independently of each other and the exact number of stocks is not known a prior.

Mathematically, this is governed by the Fisher Noncentral Hypergeometric Distribution.

Sequential Selection: This means that the portfolio manager decides a prior how many of the stocks in the benchmark to chose. Then he/she selects them ONE AT A TIME using his/her skill.

Mathematically, this is governed by the Wallenius Noncentral Hypergeometric Distribution

2. THE MODEL

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- Bulk Selection – no path dependency
- No skill (ω=1), then probability of getting 3 good: 39.68%
- Skill (ω=1.1), then probability of getting 3 good: 41.49%
- For 3: Numerator: $\binom{5}{3}\binom{5}{2}\omega^3$
- For 3: Denominator: $\binom{5}{0}\binom{5}{5}\omega^0+\binom{5}{1}\binom{5}{4}\omega^1+\binom{5}{2}\binom{5}{3}\omega^2+\binom{5}{3}\binom{5}{2}\omega^3+\binom{5}{4}\binom{5}{1}\omega^4+\binom{5}{5}\binom{5}{0}\omega^5$

2. THE MODEL

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking more good stocks than bad stocks in a portfolio of 5 stocks?

- We need to sum up the probabilities of selecting 3, 4 and 5 stocks. The result is 53.4%

TABLE 1 A simple example of picking 5 stocks from the noncentral fisher distribution

This table shows a simple example of the noncentral Fisher distribution in the context of portfolio selection. The table shows the probability of selecting 0, 1, … , 5 good stocks in a portfolio of 5 stocks chosen from a 10-stock benchmark with an equal amount of good and bad stocks.

Number of Good Stocks	Numerator	Denominator	Probability of Event (%)
0	1.00	320.81	0.31
1	27.50	320.81	8.57
2	121.00	320.81	37.72
3	133.10	320.81	41.49
4	36.60	320.81	11.41
5	1.61	320.81	0.50

2. THE MODEL

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- Sequential Selection – path dependency, thus slightly more difficult calculation
- So once all combinations have been computed, you add them – in this case probability of 3 good stocks = 41.98%
- Similar steps for 4, 5 stocks to derive the probability of picking more good than bad stocks (54.39%).

TABLE 2 The different possible paths to picking 5 stocks

This table shows the different paths that can occur when selecting five stocks from a universe of 10 stocks. A ''1'' indicates that a good stock has been picked, while a ''0'' indicates a bad stock was picked. There are 10 possible combinations of picking five stocks consisting of three good stocks from a universe of 10 stocks containing an equal number of good and bad stocks. The probability of picking any given stock in the sequence is shown in the second section of the table, below the paths, and the probability of any individual sequence is shown at the bottom of the table.

Path 1	Path 2	Path 3	Path 4	Path 5	Path 6	Path 7	Path 8	Path 9	Path 10
1	1	1	0	0	0	1	0	1	1
1	0	1	1	1	0	0	1	1	0
1	1	0	1	0	1	1	1	0	0
0	1	1	1	1	1	0	0	0	1
0	0	0	0	1	1	1	1	1	1
Probabilities of Individual Picks									
52.38	52.38	52.38	47.62	47.62	47.62	52.38	47.62	52.38	52.38
46.81	53.19	46.81	57.89	57.89	42.11	53.19	57.89	46.81	53.19
39.76	52.38	60.24	52.38	47.62	64.71	52.38	52.38	60.24	47.62
69.44	45.21	45.21	45.21	59.46	59.46	54.79	54.79	54.79	59.46
64.52	64.52	64.52	64.52	52.38	52.38	52.38	52.38	52.38	52.38
Probabilities of Entire Sequence									
4.37	4.26	4.31	4.21	4.09	4.04	4.19	4.14	4.24	4.13

2. THE MODEL

Portfolio Manager selects n stocks from a benchmark of N stocks. There are 50% "good" stocks and 50% "bad" stocks. Good stocks provide a 10% return and bad stocks a -10% return.

We will then compare a portfolio manager's performance against the benchmark via the Information Ratio.

When the portfolio manager draws from Fisher or Wallenius, we will know the expected number of good stocks. Thus, expected return and standard deviation of the portfolio are given by:

$$E(r_P) = p^* r_g + (1 - p^*) r_b,$$

$$S(r_P) = (r_g - r_b) \frac{\sqrt{\sigma_x^2}}{n},$$

2. THE MODEL

We can show that the Information Ratio of the portfolio will be:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P)}{S(r_P)}.$$

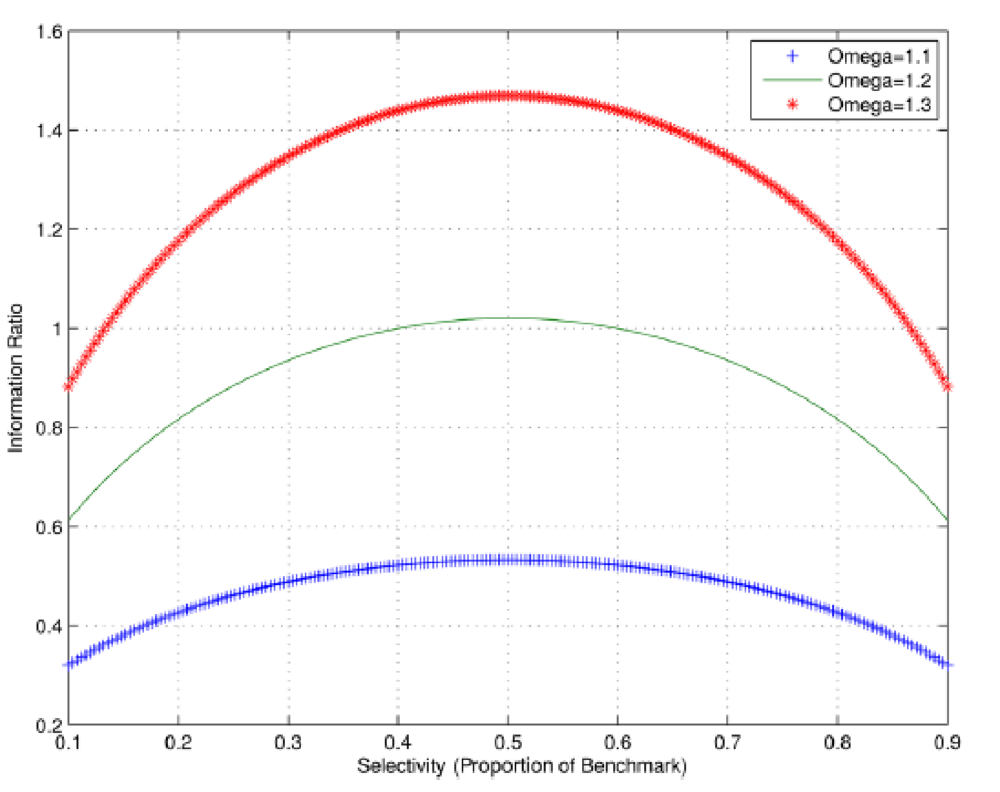
We also look at the Downside Information Ratio:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P) - E(r_{BM})}{SS(r_P - r_{BM})}$$

$$SS(r_P - r_{BM}) = \sqrt{\psi \sum_{i=1}^{n_l} \left[\min\left(0, r_{P,i} - r_{BM}\right)\right]^2 \cdot f(r_{P,i} - r_{BM}),}$$

3. BEHAVIOR OF MODEL

Example: N=500, n(g) = 250 n(b) = 250, ω=1.1 What is optimal selectivity ratio?

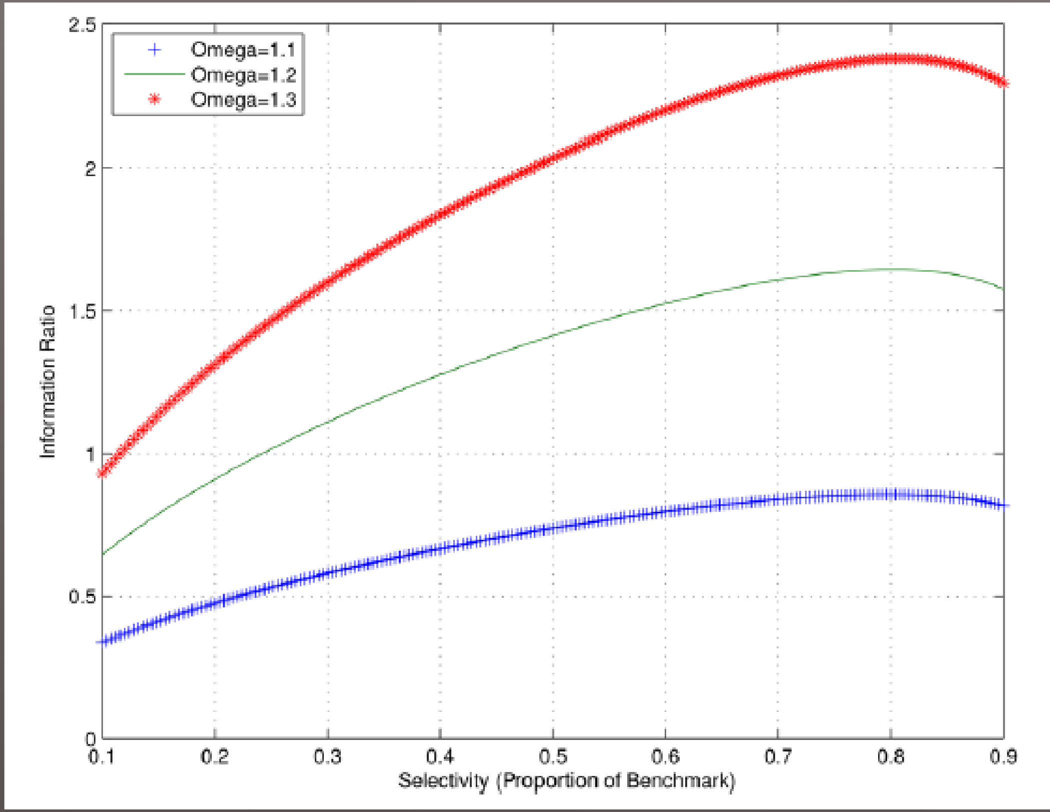


Bulk Selection = 50%

Note: For all ω, it's 50%!

3. BEHAVIOR OF MODEL

Example: N=500, n(g) = 250 n(b) = 250, ω=1.1 What is optimal selectivity ratio?

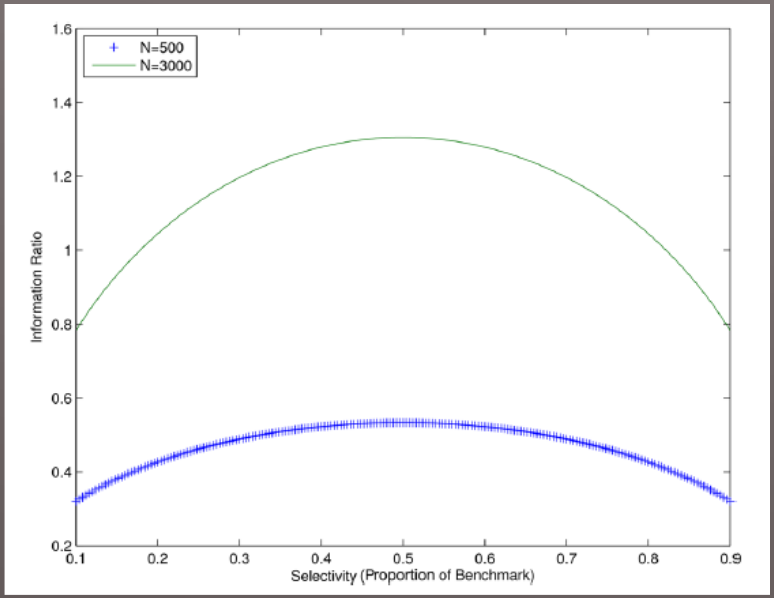


Sequential ~ 80%

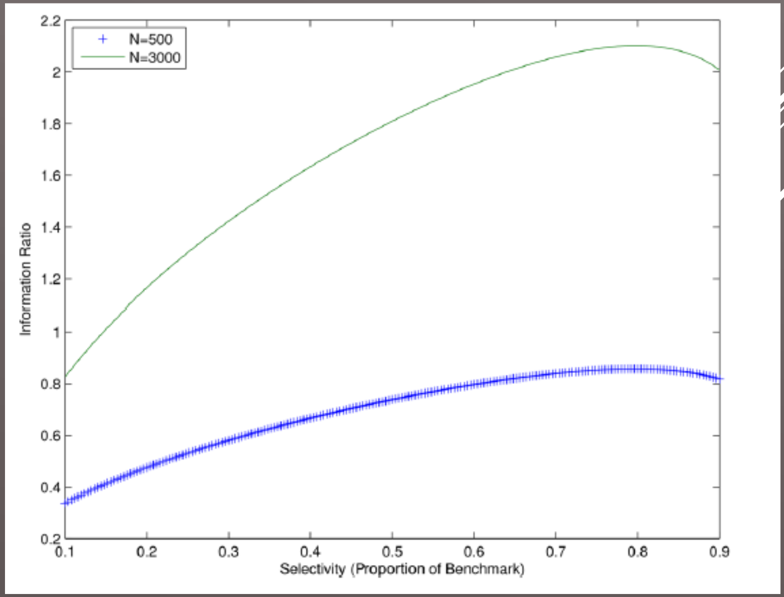
Note: For all ω, it's 80% (for reasonable values of ω)!

3. BEHAVIOR OF MODEL

Question: How do more stocks in benchmark affect the result?



Same selectivity ratio, but higher IR.



4. CHARACTERISTICS OF MODEL

There are some general characteristics about the model's predictions.

Characteristic 1. Given a benchmark universe of stocks, N, the highest Information Ratio for a manager with skill level ω is obtained at a selectivity ratio (n/N) between 50% and 80%. For the bulk selection method, it is always at 50%. For the sequential selection method, it is near 80% for reasonable values of ω.

Characteristic 2. Given a manager with skill level ω that stays constant as the universe increases, a larger universe, M, will result in a larger Information Ratio, which is approximately $\sqrt{M/N}$ larger.

Characteristic 3. Given a certain selectivity ratio, the Information Ratio for the sequential selection method will always be higher than the Information Ratio for the bulk selection method given a constant level of skill level, ω.

5. THE IMPERFECTION OF IR

For most applications, the Information Ratio (IR) is thought to be a reliable measure of performance versus a benchmark.

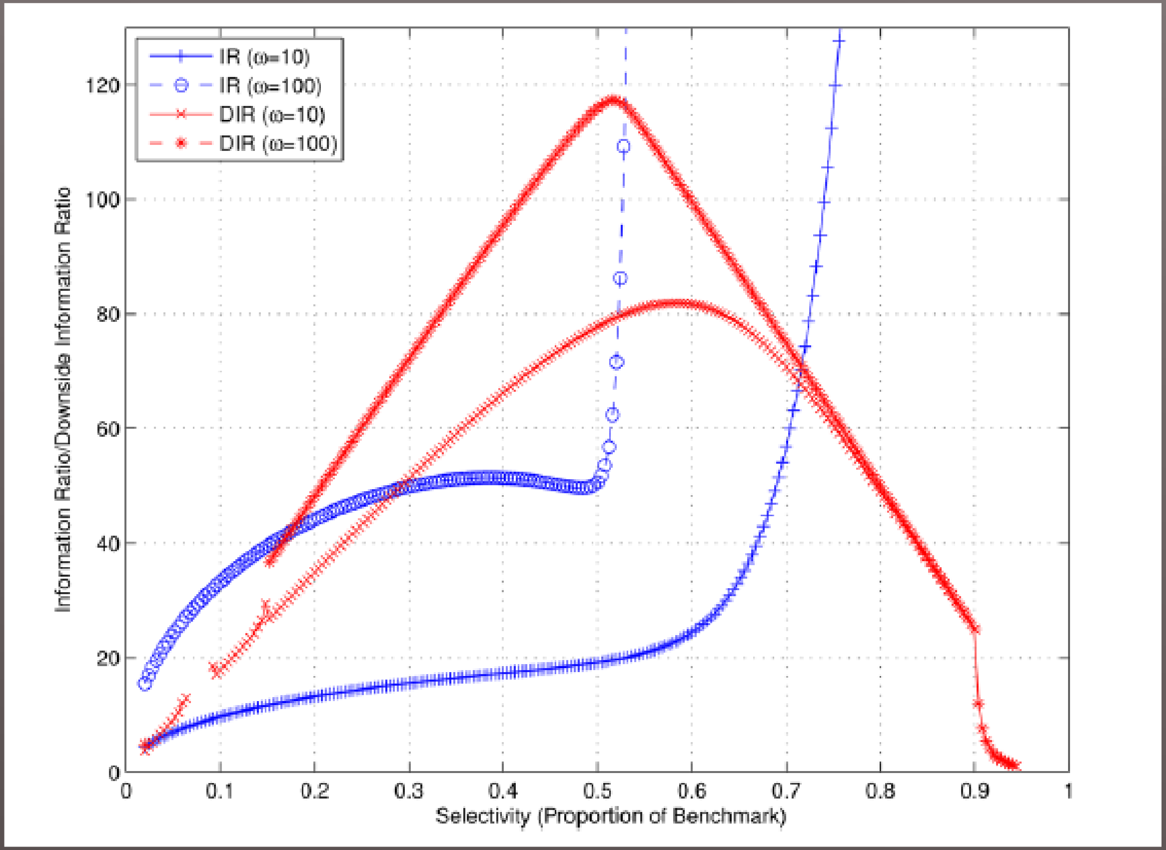
In our theoretical framework, when skill is very large, this measure performs very poorly.

For sequential picking, at very high levels of skill, the optimal IR is at 100% or complete indexing (TE declines faster than E(r)).

The problem is that at high levels of skill, although the probability of underperforming the benchmark is tiny, however because the distribution of returns isn't centered around zero – IR is much less relevant, but DIR becomes appropriate criterion.

5. THE IMPERFECTION OF IR

- However, the Downside Information Ratio (DIR) resolves this problem as can be seen in graph.



Bottom Line: With the more appropriate DIR, as skill goes to infinity, sequential chooses 50% of portfolio.

6. RELAXING MODEL ASSUMPTIONS

The model has certain simplifying assumptions about the investment universe.

Assumption 1: In any given index, 50% of the stocks will outperform and 50% will underperform.

Assumption 2: Stock either outperforms or underperforms (1 or 0), magnitude is unimportant.

Assumption 3: A portfolio manager’s skill lies in the probability to pick a “winner” versus a “loser.”

Assumption 4: The benchmark and portfolio are equally-weighted.

6. RELAXING MODEL ASSUMPTIONS

Relax Assumption 1: In practice benchmarks are not required to have an equal number of winners and losers; the benchmark return will be calculated for whatever proportions exist.

Example: r(1) = +10%, r(2) = -10%, r(3) = -10% r(bm)= -3.33%. Only 1/3 of stocks are "winners."

In order to investigate the robustness of this assumption, we performed 100,000 Wallenius (sequential selection) simulations at each selectivity ratio, under different winner/loser proportions. The conclusions in this presentation have been also affirmed for the Fisher (bulk selection) simulations.

Each simulation randomly picks one stock at a time and recalculates the probabilities of the next pick based on how many winners/losers have been picked before. The simulation stops once the desired selectivity ratio has been reached.

6. RELAXING MODEL ASSUMPTIONS

As the % of winners in the benchmark shrinks, the manager has fewer chances for their skill to shine. Their IR declines as a result.

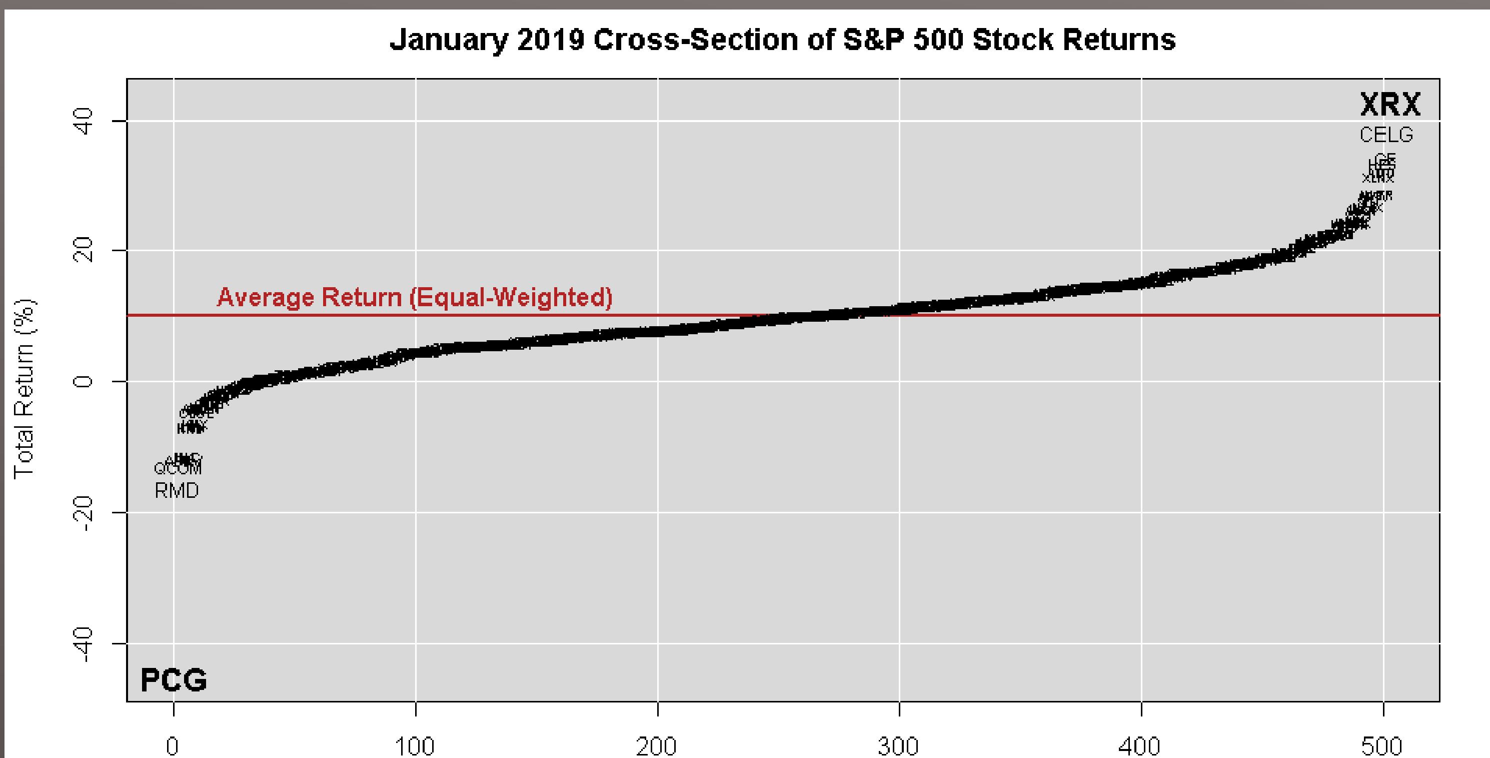
Monte Carlo Simulations Varying % of Winners vs. Losers

	Proportion of Winners in the Benchmark				
	50%	**48%**	**40%**	**30%**	**20%**
Benchmark Return (%)	0.000	-0.400	-2.000	-4.000	-6.000
Optimal Selectivity Ratio (%)	79.0	79.0	79.4	79.4	79.6
Expected Return at Optimal Selectivity	0.197	-0.203	-1.813	-3.839	-5.878
Information Ratio at Optimal Selectivity	0.855	0.854	0.851	0.791	0.697

However their optimal behavior is relatively unchanged: IR is still maximized by holding ~80% of the benchmark.

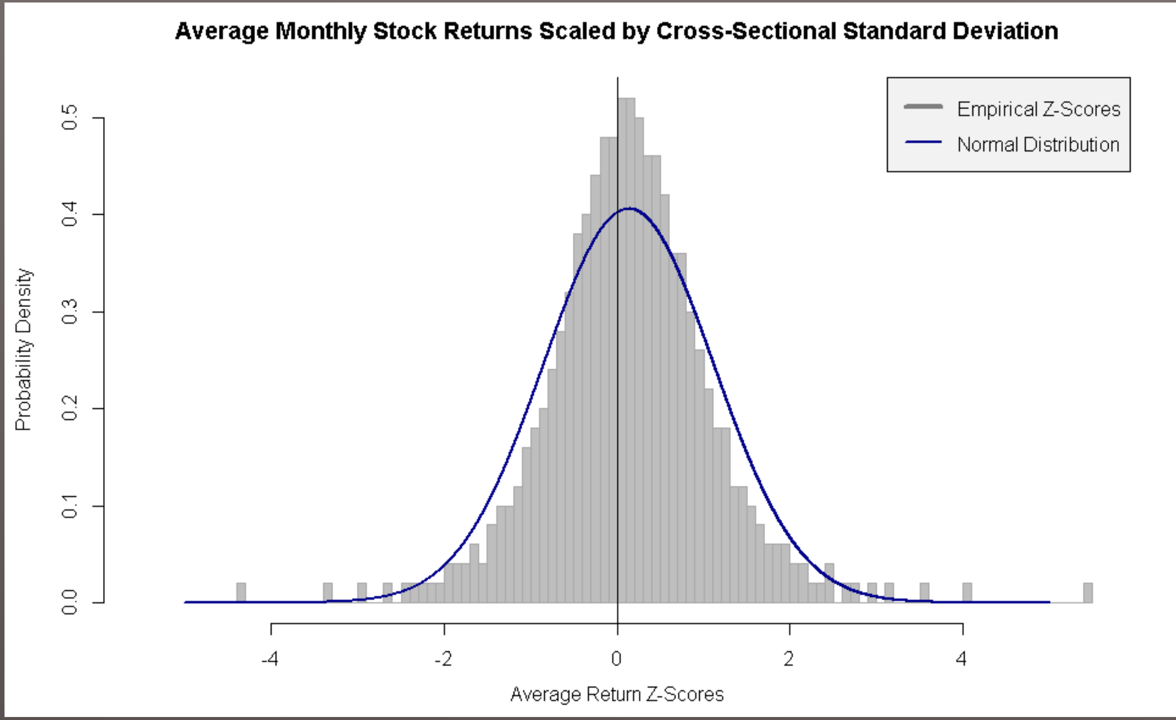
6. RELAXING MODEL ASSUMPTIONS

Relax Assumption 2: The empirical distribution of stock returns is much more continuous than the assumed binary, ±10% outcomes. It also typically has excess kurtosis and skewness.



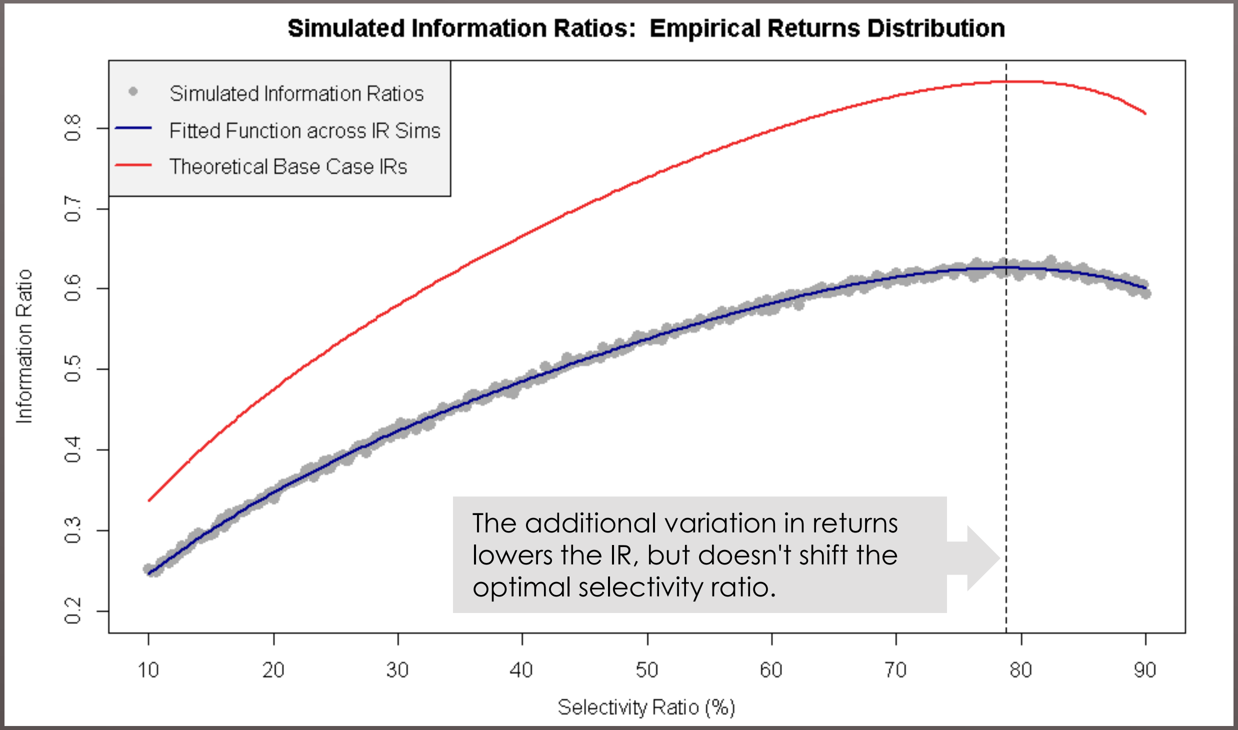
6. RELAXING MODEL ASSUMPTIONS

We could relax this assumption in one given period (i.e. month) or across time and cross-section. Either way, it would not matter. We chose to compute Z-scores of each stock return (see Chincarini & Kim (2006)) for S&P 500 stocks each period and across time to get an "average" distribution of Z-scores (similar to relative returns).



6. RELAXING MODEL ASSUMPTIONS

To simulate this effect, each benchmark stock has a Z-score "return" assigned to it based on the frequency distribution from the prior slide. As stocks are sampled from the benchmark their unique "returns" follow them into the manager's portfolio.



6. RELAXING MODEL ASSUMPTIONS

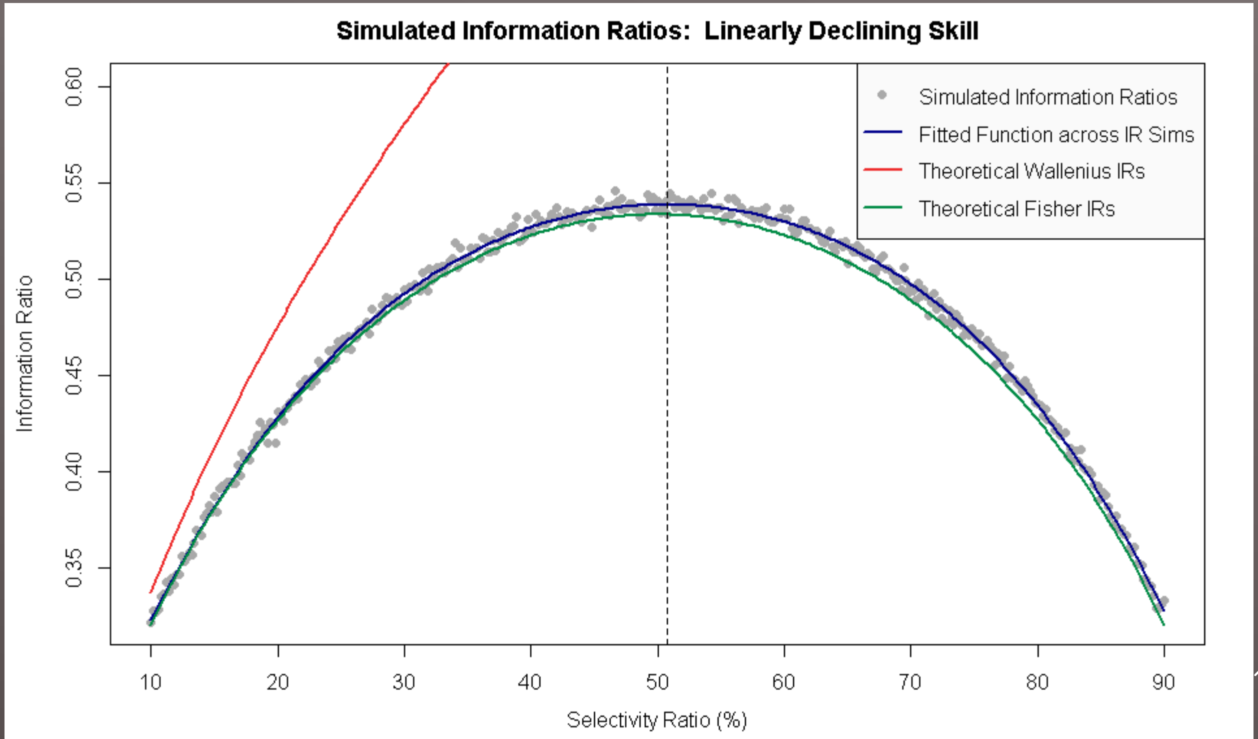
Relax Assumption 3: One might posit that a portfolio manager's skill in picking good stocks either (1) starts high and slowly declines; (2) only applies to a subset of the benchmark that they know well; (3) reaches some saturation point; or (4) follows a stochastic process that might reward or punish an expected level of skill.

Four scenarios have been studied through Monte Carlo simulation:

1. Sequentially declining skill.
2. Skill for only a subset of the winner population.
3. Skill saturation point.
4. Skill as a random variable.

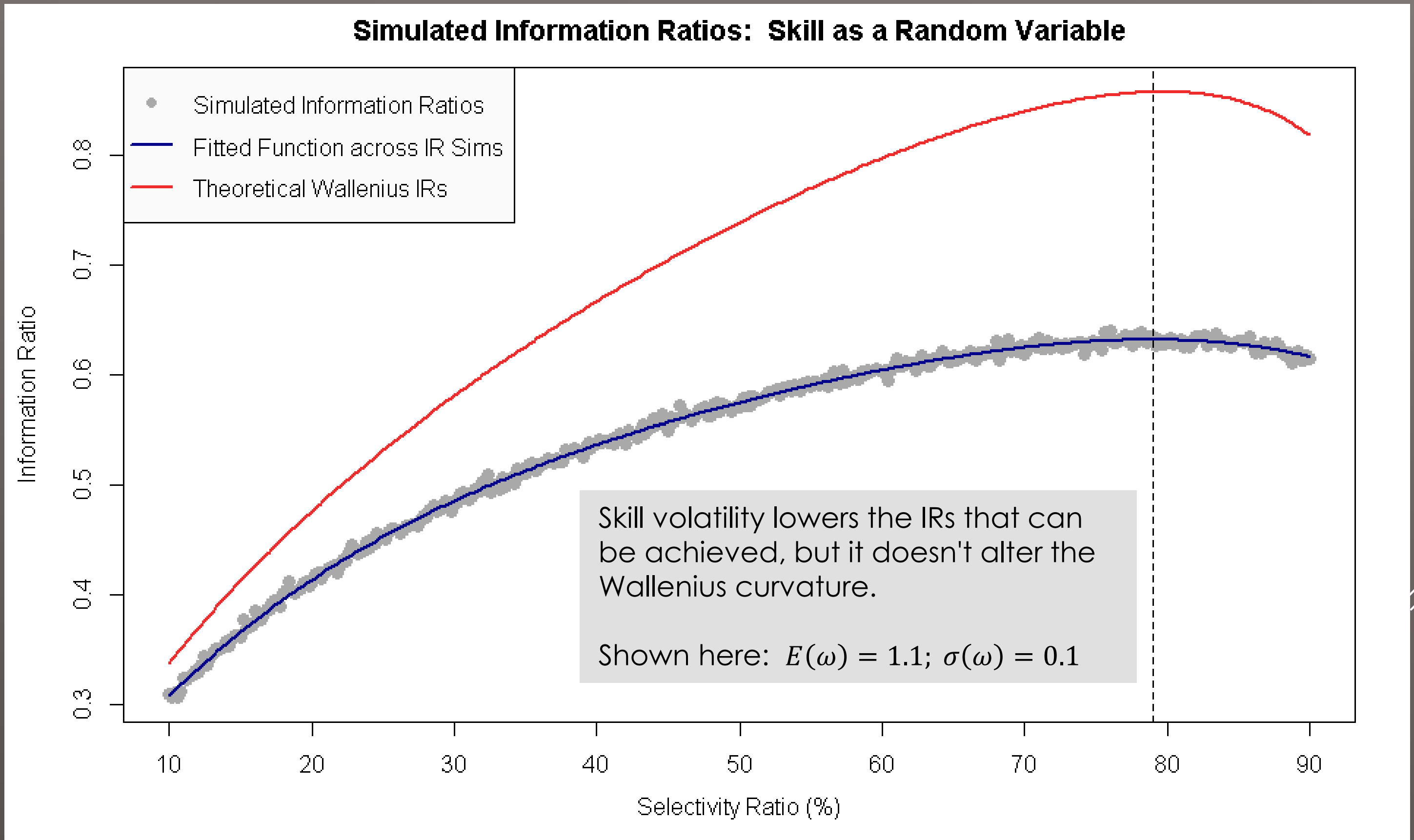
6. RELAXING MODEL ASSUMPTIONS

If the manager's skill declines linearly after each stock pick, the probability boost that typically comes from a sequential process (Wallenius) is overwhelmed by that deterioration. As a result the Information Ratios converge towards the bulk selection method (Fisher).



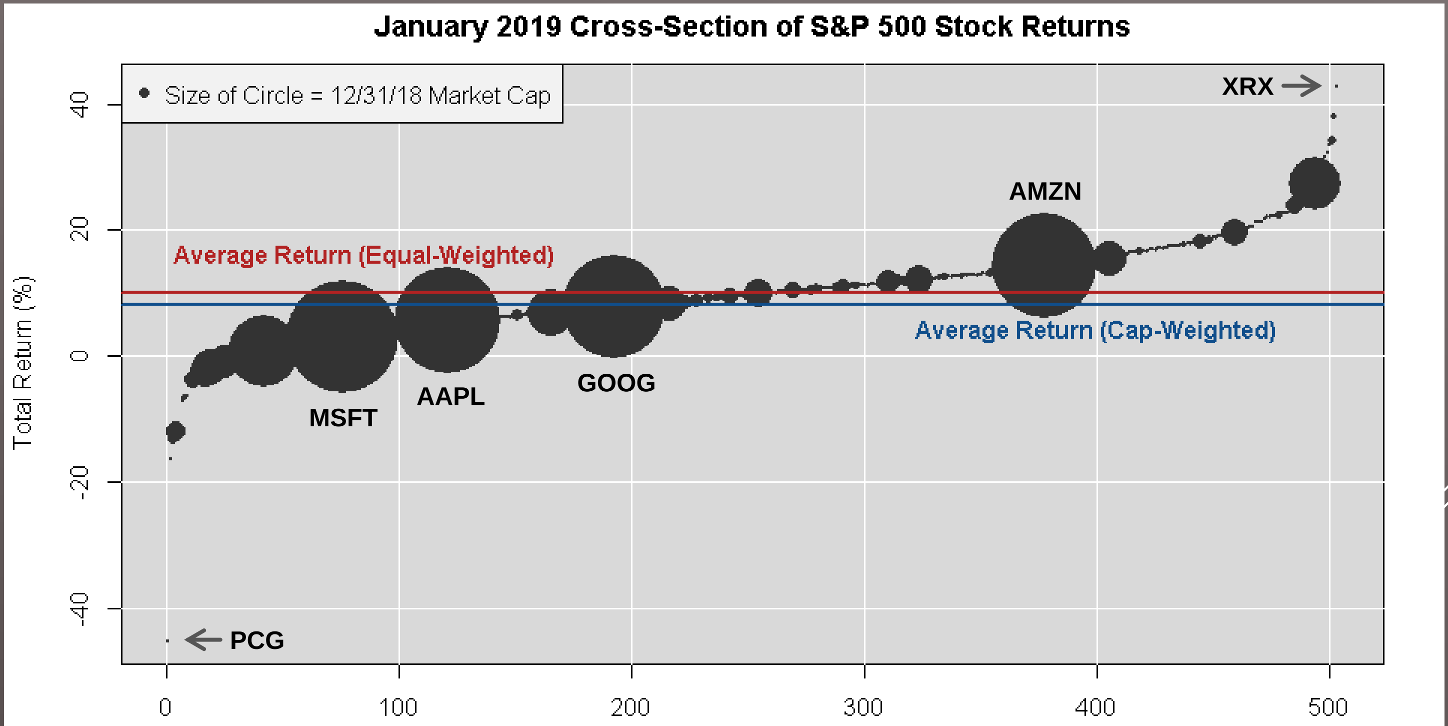
6. RELAXING MODEL ASSUMPTIONS

The other three scenarios produce intuitive results, for the most part maintaining the Wallenius properties.



6. RELAXING MODEL ASSUMPTIONS

Relax Assumption 4: Many benchmarks (e.g. the S&P 500) are weighted according to market capitalizations. Such an approach can lead to a handful of stocks having an outsized gravitational pull on the benchmark's return.



6. RELAXING MODEL ASSUMPTIONS

Introducing cap-weighted effects into the simulation framework reduces the Information Ratios that the manager can achieve.

Monte Carlo Simulations for Cap-Weighted Benchmark Possibilities

	Correlation of Market Cap and Winner Designation			Compare to Equal Wtd Bench
	Positive	None	Negative	
Benchmark Return (%)	1.250	0.000	-1.250	0.000
Optimal Selectivity Ratio (%)	78.0	78.4	78.6	79.0
Expected Return at Optimal Selectivity	1.448	0.203	-1.050	0.197
Information Ratio at Optimal Selectivity	0.397	0.407	0.406	0.855

Once again though, the optimal selectivity ratio to attain the best IR varies only slightly from the theoretical ~80%.

6. RELAXING MODEL ASSUMPTIONS

SUMMARY: Relaxing the simplifying assumptions required for the theory does not significantly alter its theoretical conclusion of which selectivity ratio maximizes the manager's Information Ratio.

In fact, the consistency of results across different assumptions reinforces the validity of the theory.

7. SUMMARY

- Traditionally, enhanced portfolio management has considered the tracking error with a benchmark and mean-variance type optimization, as well as other ad hoc techniques to find OPTIMAL portfolios.
- Our approach is a new one. First, it asks what percentage of a benchmark should the manager choose to maximize Information Ratio. Second, it uses some concepts never applied to these portfolio problems (according to the best of our knowledge).
- The results are extremely interesting, in that, many enhanced managers do not hold such a large portion of their benchmark (50 to 80%).

8. FURTHER RESEARCH

- We are currently writing a second paper describing the results from relaxing the theoretical assumptions.
- This follow-up paper will also introduce an intertemporal simulation framework, whereby stocks are periodically added to a portfolio and held for varying lengths of time.
- We also hope to study actual managed investment vehicles. One of the limitations has been that it is very hard to find data of enhanced managers that hold more than 40% of the underlying benchmark.

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