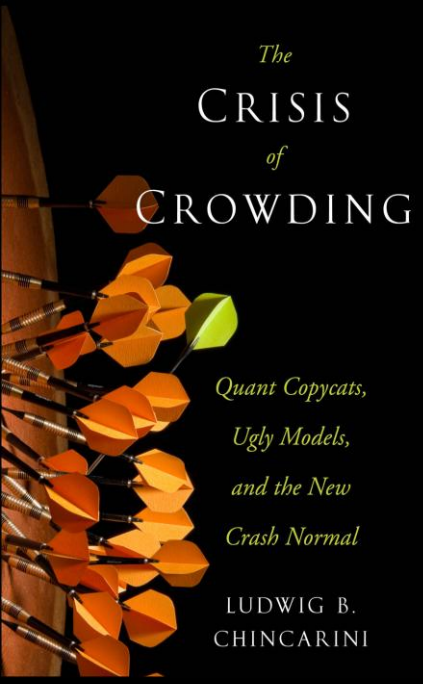
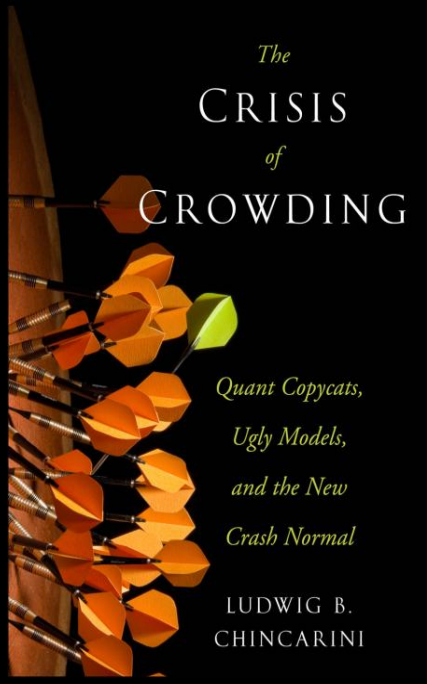


Selectivity, Skill and Benchmark Concentration: A Mapping Technique for the Information Ratio.

Bolshakov, Chincarini, and Jerison



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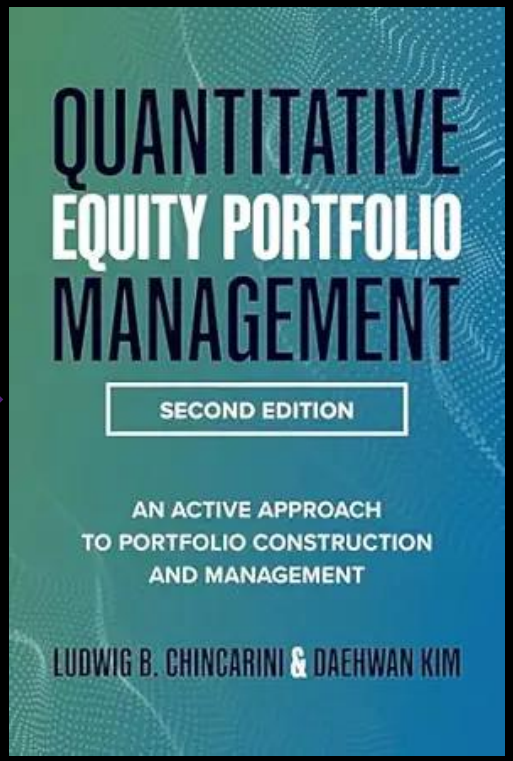


Remembering some important people related to the BIS and the Bank of Mexico

My late, great friend, Angus Butler. My late former boss and friend, Andrew Crockett.

They both made my life richer. Thank you



1. Prior Work on Selectivity Theory

- Chincarini, Ludwig (w/ Andrei Bolshakov). "Manager Skill and Portfolio Size with Respect to a Benchmark." European Financial Management, February, 2020.

- Chincarini, Ludwig (w/ Andrei Bolshakov and Christopher Lewis). "Enhanced Indexing and Selectivity Theory" European Financial Management, September 2022.

2. The model

Assumption 1: Winner stocks are stocks with a return better than the median stock return in the group.

Assumption 2: Stocks are distributed normally with mean mu and standard deviation sigma.

Assumption 3: A portfolio manager's constant skill lies in the probability to pick a "winner" versus a "loser". This skill doesn't diminish with the number of stocks he/she picks.

2. The model

We introduce the notion of omega (ω), where $\omega > 1$ if a portfolio manager is more likely to pick a good stock versus a bad stock.

To get a rough idea of how ω is related to probabilities, if $\omega = 1.1$ and a manager is picking the 1st stock, the probability of picking a good one is about 0.5238.

2. The model

Two Possible Selection Methods for a group of n stocks out of a universe of N stocks.

Method 1: Bulk Selection

Method 2: Sequential Selection

2. The model

Bulk Selection: This means that the portfolio manager selects the stocks into the portfolio ALL AT ONCE using his/her skill.

Mathematically, this is governed by the Fisher Noncentral Hypergeometric Distribution.

Sequential Selection: The portfolio manager decides ex-ante how many of the stocks in the benchmark to chose. Then he/she selects them ONE AT A TIME using his/her skill.

Mathematically, this is governed by the Wallenius Noncentral Hypergeometric Distribution

2. The Model

Model stock returns so that they represent the actual distribution of returns and try to find a closed-form expression for the information ratio.

Can we model the stock returns? If we do, has impact on empirical usage.

Do basic results still hold?

3. The Model

GOALS:

- Find a mapping from observable Information Ratio (IR) to latent skill (ω).

  Derive analytical formulas for:

  Expected Active Return

  Tracking Error

  Information Ratio

  Create a direct mapping: IR $\rightarrow$ ω (manager skill).

- Build a framework that works beyond Fisher and Wallenius selection.
- Make selectivity theory testable with real-world data.

4. The Model Framework

- Step 1: Draw stock returns from a realistic distribution of stock returns.
- Step 2: Divide stocks into 'good' and 'bad' relative performers (compared to median of the returns).
- Step 3: Manager selects more good than bad stocks according to skill ω. For example, use Wallenius to pick $N(p) < N$ stocks for the portfolio with omega>1, more likely to pick winners.
- Step 4: Derive E(Active Return), E(TE), and E(IR) analytically.
- Step 5: Connect parameters (omega, N, N(p), mu, sigma) to IR.

4. The Model Framework

- Step 6: Compare to simulations for accuracy.
- Derivation Complicated, but an approximate formula was derived.
- Key: Works with selectivity theory (Wallenius or Fischer) or any other stock picking mechanism that can be represented by mean and standard deviation of good stocks.
- MORE INFO: Chincarini, Ludwig B. and Bolshakov, Andrei and Jerison, Daniel, A Mapping Technique for Selectivity Theory. Available at SSRN: https://ssrn.com/abstract=5421420 or http://dx.doi.org/10.2139/ssrn.5421420

4. The Model Framework

$$\mathbf{E}(r_P) \approx \mu + \sigma\left(\frac{2\mu_x}{N_P} - 1\right)\left(\sqrt{\frac{2}{\pi}} + \frac{1}{N}\cdot\frac{4-\pi}{2\sqrt{2\pi}} - \frac{1}{N^2}\cdot\frac{-96+40\pi-3\pi^2}{16\sqrt{2\pi}}\right). \tag{3}$$

$$\begin{aligned}\mathrm{Var}(r_P) &\approx \sigma^2\left[\frac{1}{N_P}\cdot\frac{\pi-2}{\pi} + \frac{\sigma_x^2}{N_P^2}\cdot\frac{8}{\pi}\right] \\ &+\frac{\sigma^2}{N}\left[\frac{2}{\pi} + \frac{1}{N_P}\cdot\frac{2(\pi-3)}{\pi} + \frac{\sigma_x^2}{N_P^2}\cdot\frac{4(4-\pi)}{\pi}\right] \\ &+\frac{\sigma^2}{N^2}\left[-\frac{16(\pi-3)-(4-\pi)^2}{4\pi} - \frac{1}{N_P}\cdot\frac{120-60\pi+7\pi^2}{4\pi}\right. \\ &\quad -\frac{\mu_x}{N_P}\cdot\frac{(4-\pi)^2}{\pi} + \frac{\mu_x^2}{N_P^2}\cdot\frac{(4-\pi)^2}{\pi} \\ &\quad \left.+\frac{\sigma_x^2}{N_P^2}\cdot\frac{3(4-\pi)^2+(96-40\pi+3\pi^2)}{2\pi}\right]\end{aligned} \tag{4}$$

$$\mathbf{E}(r_P - r_B) \approx \sigma\left(\frac{2\mu_x}{N_P} - 1\right)\left(\sqrt{\frac{2}{\pi}} + \frac{1}{N}\cdot\frac{4-\pi}{2\sqrt{2\pi}} - \frac{1}{N^2}\cdot\frac{-96+40\pi-3\pi^2}{16\sqrt{2\pi}}\right). \tag{5}$$

$$\begin{aligned}\mathrm{Var}(r_P - r_B) &\approx \sigma^2\left[\frac{1}{N_P}\cdot\frac{\pi-2}{\pi} + \frac{\sigma_x^2}{N_P^2}\cdot\frac{8}{\pi}\right] \\ &+\frac{\sigma^2}{N}\left[-1 + \frac{2}{\pi} + \frac{1}{N_P}\cdot\frac{2(\pi-3)}{\pi} + \frac{\sigma_x^2}{N_P^2}\cdot\frac{4(4-\pi)}{\pi}\right] \\ &+\frac{\sigma^2}{N^2}\left[-\frac{2(\pi-3)}{\pi} - \frac{1}{N_P}\cdot\frac{120-60\pi+7\pi^2}{4\pi}\right. \\ &\quad +\frac{1}{4}\left(\frac{2\mu_x}{N_P} - 1\right)^2\frac{(4-\pi)^2}{\pi} \\ &\quad \left.+\frac{\sigma_x^2}{N_P^2}\cdot\frac{3(4-\pi)^2+(96-40\pi+3\pi^2)}{2\pi}\right].\end{aligned} \tag{6}$$

$$\mathrm{IR} \approx \frac{\left(\frac{2\mu_x}{N_P}-1\right)\left(\sqrt{\frac{2}{\pi}}+\frac{1}{N}\cdot\frac{4-\pi}{2\sqrt{2\pi}}-\frac{1}{N^2}\cdot\frac{-96+40\pi-3\pi^2}{16\sqrt{2\pi}}\right)}{\sqrt{\blacksquare}} \tag{7}$$

5. Extension to a Single-Factor World

- Embed selectivity theory inside a CAPM / single-index framework.
- Manager skill is assumed to identify positive idiosyncratic shocks (ε).
- Market risk enters through β differences.
- If $\beta P \approx \beta B$, mapping between IR and ω remains largely intact.
- This provides a much more realistic asset-pricing setting.

5. Extension to a Single-Factor World

$$\mathbf{E}(\tilde{r}_P - \tilde{r}_B) \approx (\overline{\beta}_P - \overline{\beta}_B) \cdot \mu_M + (2p^* - 1) \cdot \sigma_\epsilon \left(\sqrt{\frac{2}{\pi}} + \frac{1}{N} \cdot \frac{4-\pi}{2\sqrt{2\pi}} \right). \tag{11}$$

$$\begin{aligned} \mathrm{Var}(\tilde{r}_P - \tilde{r}_B) \quad &\approx (\overline{\beta}_P - \overline{\beta}_B)^2 \cdot \sigma_M^2 + \sigma_\epsilon^2 \left[\tfrac{1}{N_P} \cdot \tfrac{\pi-2}{\pi} + \tfrac{\sigma_x^2}{N_P^2} \cdot \tfrac{8}{\pi} \right] \\ &\quad + \tfrac{\sigma_\epsilon^2}{N} \left[-1 + \tfrac{2}{\pi} + \tfrac{1}{N_P} \cdot \tfrac{2(\pi-3)}{\pi} + \tfrac{\sigma_x^2}{N_P^2} \cdot \tfrac{4(4-\pi)}{\pi} \right]. \end{aligned} \tag{12}$$

$$\begin{aligned} \mathrm{IR} \quad &= \quad \frac{\mathbf{E}(\tilde{r}_P - \tilde{r}_B)}{\mathrm{SD}(\tilde{r}_P - \tilde{r}_B)} \\ &\approx \quad \frac{(\overline{\beta}_P - \overline{\beta}_B) \cdot \mu_M + (2p^* - 1) \cdot \sigma_\epsilon \left(\sqrt{\frac{2}{\pi}} + \frac{1}{N} \cdot \frac{4-\pi}{2\sqrt{2\pi}} \right)}{\sqrt{(\overline{\beta}_P - \overline{\beta}_B)^2 \sigma_M^2 + \sigma_\epsilon^2 \left[\frac{1}{N_P} \frac{\pi-2}{\pi} + \frac{\sigma_x^2}{N_P^2} \frac{8}{\pi} \right] + \frac{\sigma_\epsilon^2}{N} \left[-1 + \frac{2}{\pi} + \frac{1}{N_P} \frac{2(\pi-3)}{\pi} + \frac{\sigma_x^2}{N_P^2} \frac{4(4-\pi)}{\pi} \right]}}. \end{aligned} \tag{13}$$

6. Verifying the Model with Simulations

- 100,000 Monte Carlo simulations validate the formulas.
- Analytical IR closely matches simulated IR.
- Mapping tables can convert an observed IR into implied ω.

Table 1: Mapping from Error Returns and Omega to Information Ratio (Portfolio Drawn from Benchmark - Approximate Method)

		N= 100.00			N= 500.00			N= 1000.00		
	ω	10%	50%	80%	10%	50%	80%	10%	50%	80%
Theory	1.05	0.0616	0.1349	0.1558	0.1376	0.3017	0.3499	0.1946	0.4266	0.4951
Sims.	1.05	0.0612	0.1318	0.1630	0.1356	0.3063	0.3453	0.1970	0.4242	0.5002
Theory	1.10	0.1204	0.2635	0.3045	0.2688	0.5893	0.6837	0.3801	0.8334	0.9674
Sims.	1.10	0.1180	0.2606	0.3045	0.2669	0.5866	0.6808	0.3800	0.8302	0.9617
Theory	1.15	0.1766	0.3864	0.4467	0.3942	0.8641	1.0030	0.5573	1.2220	1.4193
Sims.	1.15	0.1717	0.3858	0.4443	0.4016	0.8638	1.0001	0.5564	1.2273	1.4170
Theory	1.20	0.2303	0.5040	0.5831	0.5142	1.1271	1.3092	0.7270	1.5940	1.8525
Sims.	1.20	0.2374	0.5051	0.5751	0.5138	1.1280	1.3077	0.7283	1.5903	1.8522
Theory	1.25	0.2818	0.6168	0.7141	0.6292	1.3793	1.6035	0.8897	1.9507	2.2689
Sims.	1.25	0.2838	0.6171	0.7116	0.6346	1.3854	1.6042	0.8873	1.9463	2.2717
Theory	1.30	0.3313	0.7252	0.8403	0.7397	1.6216	1.8868	1.0459	2.2933	2.6698
Sims.	1.30	0.3301	0.7169	0.8379	0.7368	1.6234	1.8928	1.0371	2.2870	2.6612
Theory	1.35	0.3789	0.8294	0.9620	0.8460	1.8546	2.1602	1.1962	2.6228	3.0566
Sims.	1.35	0.3744	0.8303	0.9577	0.8480	1.8525	2.1588	1.1935	2.6286	3.0529

6. Verifying the Model with Simulations

- Same with Single Index Model

For beta, betas chosen once for benchmark and portfolio manager is stuck with beta of stock. We don't manage beta in simulation.

Table 2: Mapping from Single Index Returns and Omega to Information Ratio for the Single Index Model (Portfolio Drawn Independently - Approximate Method)

		N= 100.00			N= 500.00			N= 1000.00		
	ω	10%	50%	80%	10%	50%	80%	10%	50%	80%
Theory	1.05	0.0550	0.1470	0.1275	0.0967	0.3027	0.3207	0.2346	0.3788	0.5058
Sims.	1.05	0.0546	0.1438	0.1344	0.0949	0.3073	0.3164	0.2367	0.3762	0.5109
Theory	1.10	0.1138	0.2756	0.2760	0.2276	0.5903	0.6541	0.4197	0.7843	0.9781
Sims.	1.10	0.1113	0.2727	0.2758	0.2257	0.5875	0.6514	0.4194	0.7808	0.9723
Theory	1.15	0.1699	0.3984	0.4180	0.3526	0.8651	0.9730	0.5965	1.1717	1.4298
Sims.	1.15	0.1650	0.3978	0.4155	0.3598	0.8647	0.9699	0.5950	1.1763	1.4274
Theory	1.20	0.2237	0.5161	0.5541	0.4723	1.1281	1.2788	0.7658	1.5425	1.8630
Sims.	1.20	0.2307	0.5171	0.5460	0.4716	1.1290	1.2771	0.7670	1.5391	1.8628
Theory	1.25	0.2752	0.6288	0.6850	0.5870	1.3803	1.5726	0.9282	1.8981	2.2793
Sims.	1.25	0.2771	0.6292	0.6823	0.5922	1.3864	1.5733	0.9257	1.8939	2.2821
Theory	1.30	0.3247	0.7372	0.8109	0.6972	1.6226	1.8555	1.0841	2.2397	2.6802
Sims.	1.30	0.3233	0.7288	0.8087	0.6944	1.6244	1.8611	1.0761	2.2328	2.6717
Theory	1.35	0.3723	0.8414	0.9324	0.8032	1.8556	2.1285	1.2341	2.5682	3.0670
Sims.	1.35	0.3677	0.8424	0.9281	0.8057	1.8535	2.1272	1.2313	2.5738	3.0632
Theory	1.40	0.4181	0.9417	1.0498	0.9053	2.0801	2.3923	1.3785	2.8846	3.4408
Sims.	1.40	0.4175	0.9374	1.0471	0.9021	2.0807	2.3803	1.3773	2.8647	3.4483
Theory	1.80	0.7341	1.6338	1.8740	1.6086	3.6279	4.2447	2.3744	5.0664	6.0671
Sims.	1.80	0.7297	1.6293	1.8623	1.6081	3.6090	4.2328	2.3784	5.0686	6.0555
Theory	2.00	0.8661	1.9230	2.2292	1.9021	4.2746	5.0439	2.7902	5.9778	7.2007
Sims.	2.00	0.8637	1.9120	2.2247	1.8917	4.2642	5.0393	2.7976	5.9680	7.2272
Theory	3.00	1.3689	3.0278	3.6845	3.0195	6.7454	8.3317	4.3744	9.4590	11.8717
Sims.	3.00	1.3619	3.0105	3.6532	3.0147	6.7413	8.3494	4.3910	9.4707	11.8523

Note: This table shows the simulated and theoretical results of information ratio based on a Wallenius picking procedure by a manager with a given ω for 100,000 simulations. The returns of the underlying errors are $N(0, 0.50)$ and for the market of $N(0.12, 0.20)$. The results are generated for the single index portfolio, where returns of all stocks are given by $\tilde{r}_i = \beta_i \tilde{r}_M + \epsilon_i$. The betas for the benchmark are chosen once for each benchmark and for each portfolio. That is, we assume the betas are fixed. The average beta for the portfolios are 0.9785 , 1.0106 , 0.9944 , 0.9753 , 0.9969 , 0.9961 , 1.0211 , 0.9851 , and 1.0004 and for the benchmark are 0.9868 , 1.0056 , 1.0003 , 0.9981 , 0.9968 , 0.9988 , 1.0050 , 0.9912 , and 0.9997. Selectivity levels are listed for 10, 50, and 80%. This took 35968.59 seconds to run on a supercomputer with 11 processors.

7. Limitations … Until Now …

- So far the paper results are for an equal-weighted benchmark and an equal-weighted portfolio.
- We are working on expanding the model for a fixed benchmark, but with unequal weights.
- This is the first time that we are presenting these new and exciting results.

8. Unequal Weights and Selectivity

- We allow the benchmark to have arbitrary weights that sum to 1.
- We assume that the portfolio manager adjusts his/her weights so as to minimize tracking error (other strategies would be dominated).

8. Unequal Weights and Selectivity

Proposition 1 (Minimum tracking-error weights)**.** *Under the maintained assumption that stock returns are i.i.d. with common variance* σ^2 *(so that* $\Sigma = \sigma^2 I_N$*), the weights minimizing* $\mathrm{Var}(r_P - r_B)$ *over* w^P *supported on* P *and subject to* $\sum_{i\in P} w_i^P = 1$ *are*

$$w_i^P \;=\; w_i^B + \frac{c}{N_P}, \qquad i \in P, \qquad c := \sum_{i\in Q} w_i^B \;=\; 1 - \sum_{i\in P} w_i^B.$$

Proof. Write $a = w^P - w^B$, so that $a_Q = -w_Q^B$ is fixed, since $w_i^P = 0$ off P. Because Σ is diagonal, $\mathrm{Var}(r_P - r_B) = \sigma^2\, a^\top a$. Minimizing $a_P^\top a_P$ subject to $\mathbf{1}^\top w_P^P = 1$, i.e. $\mathbf{1}^\top a_P = c$, is minimizing the Euclidean norm of a_P on an affine hyperplane; the minimizer is the point on the hyperplane closest to 0, namely $a_P = \frac{c}{N_P}\mathbf{1}$ (the Lagrange multiplier λ solves $2a_P = \lambda\mathbf{1}$, and $\mathbf{1}^\top a_P = c$ pins $\lambda/2 = c/N_P$). Hence $w_i^P - w_i^B = c/N_P$ for $i \in P$. □

8. Unequal Weights and Selectivity

Theorem 1 (Expected active return)**.**

$$\boxed{\mathbf{E}(r_P - r_B) \;\approx\; \sigma\,\mathrm{EU}\left(\frac{2\mu_x}{N_P} - 1\right)}$$

where $\mathrm{EU} := \mathbf{E}[\mathbf{E}(U \mid M)] \approx \sqrt{2/\pi} + \frac{1}{N}\cdot\frac{4-\pi}{2\sqrt{2\pi}} - \frac{1}{N^2}\cdot\frac{-96+40\pi-3\pi^2}{16\sqrt{2\pi}}$*. This expression is independent of* w^B *and equals the equal-weight expected active return in eq. (5).*

Note: All proofs are in the new appendix to the paper.

8. Unequal Weights and Selectivity

Theorem 2 (Tracking-error variance)**.**

$$\boxed{\begin{aligned}\mathrm{Var}(r_P - r_B) \;\approx\; \sigma^2\,\mathrm{VU}\left[\frac{\mathbf{E}(c^2)}{N_P} + (N - N_P)H\right] \\ + \sigma^2\left[\mathrm{Var}(F)\,\mathrm{EUL2} + \mathbf{E}(F)^2\,\mathrm{Var}(D)\right]\end{aligned}}$$

where $\mathrm{Var}(D) := \mathrm{EUL2} - 4\,\mathrm{EU}^2$,

$$\mathrm{VU} := \mathbf{E}[\mathrm{Var}(U \mid M)] \approx \frac{\pi - 2}{\pi} + \frac{2(\pi - 3)}{\pi N} - \frac{120 - 60\pi + 7\pi^2}{4\pi N^2},$$
$$\mathrm{EUL2} := \mathbf{E}\left[(\mathbf{E}(U \mid M) - \mathbf{E}(L \mid M))^2\right] \approx \frac{8}{\pi} + \frac{4(4 - \pi)}{\pi N} + \frac{3(4 - \pi)^2 + (96 - 40\pi + 3\pi^2)}{2\pi N^2},$$

and, with $\rho := (N - N_P)/N$, $a := N/2$ *and* $n_U := N/2 - \mu_x$, *the moments of* c *and* F *are*

$$\mathbf{E}(c^2) = \rho^2 + \mathrm{Var}(c), \qquad \mathrm{Var}(c) = \frac{N_P(N - N_P)}{N - 1}\left(H - \frac{1}{N^2}\right),$$
$$\mathbf{E}(F) = \frac{\mu_x}{N_P} - \frac{1}{2},$$
$$\mathrm{Var}(F) = \frac{\mathrm{Var}(xc)}{N_P^2} - \frac{2}{N_P}\mathrm{Cov}(xc, X) + \mathrm{Var}(X),$$

Note: All proofs are in the new appendix to the paper.

8. Unequal Weights and Selectivity

Corollary 1 (Information ratio)**.**

$$\mathrm{IR} \approx \frac{\sigma\,\mathrm{EU}\left(\dfrac{2\mu_x}{N_P} - 1\right)}{\sqrt{\sigma^2\,\mathrm{VU}\left[\dfrac{\mathbf{E}(c^2)}{N_P} + (N - N_P)H\right] + \sigma^2\left[\mathrm{Var}(F)\,\mathrm{EUL2} + \mathbf{E}(F)^2\,\mathrm{Var}(D)\right]}}$$

with all quantities as in Theorems 1 and 2.

Note: All proofs are in the new appendix to the paper.

8. Unequal Weights and Selectivity

- Single Index Version

$$\mathbf{E}(\tilde{r}_P - \tilde{r}_B) \;\approx\; (\beta_P - \beta_B)\,\mu_M + \sigma_\epsilon\,\mathrm{EU}\left(\frac{2\mu_x}{N_P} - 1\right) \tag{11}$$

$$\begin{aligned}\mathrm{Var}(\tilde{r}_P - \tilde{r}_B) \;&\approx\; (\beta_P - \beta_B)^2\sigma_M^2 + V_\epsilon,\\ V_\epsilon \;&:=\; \sigma_\epsilon^2\,\mathrm{VU}\left[\frac{\mathbf{E}(c^2)}{N_P} + (N - N_P)H\right]\\ &\quad + \sigma_\epsilon^2\left[\mathrm{Var}(F)\,\mathrm{EUL2} + \mathbf{E}(F)^2\,\mathrm{Var}(D)\right]\end{aligned} \tag{12}$$

$$\mathrm{IR} \;\approx\; \frac{(\beta_P - \beta_B)\,\mu_M + \sigma_\epsilon\,\mathrm{EU}\left(\frac{2\mu_x}{N_P} - 1\right)}{\sqrt{(\beta_P - \beta_B)^2\sigma_M^2 + V_\epsilon}} \tag{13}$$

where V_ϵ is the selection (idiosyncratic) part of the tracking-error variance, and EU, VU, EUL2, $\mathrm{Var}(D)$ and the moments of c and F are those of Theorems 1 and 2 with $\sigma = \sigma_\epsilon$.

$$\beta_B := \sum_{i=1}^{N} w_i^B \beta_i \qquad \text{and} \qquad \beta_P := \sum_{i \in P} w_i^P \beta_i$$

Note: All proofs are in the new appendix to the paper.

8. Unequal Weights and Selectivity

- The general formula works:

Table 3: Mapping from Error Returns and Omega to Information Ratio (Portfolio Drawn from Benchmark - Approximate Method) Unequal Weight Benchmark

		N= 100.00			N= 500.00			N= 1000.00		
	ω	10%	50%	80%	10%	50%	80%	10%	50%	80%
Theory	1.05	0.0531	0.0814	0.0799	0.1037	0.1376	0.1314	0.1097	0.1246	0.1162
Sims.	1.05	0.0532	0.0861	0.0828	0.1047	0.1381	0.1289	0.1097	0.1216	0.1199
Theory	1.10	0.1036	0.1589	0.1560	0.2026	0.2688	0.2565	0.2143	0.2433	0.2267
Sims.	1.10	0.1028	0.1586	0.1573	0.1990	0.2684	0.2562	0.2086	0.2322	0.2242
Theory	1.15	0.1519	0.2328	0.2286	0.2969	0.3939	0.3757	0.3141	0.3566	0.3320
Sims.	1.15	0.1504	0.2330	0.2254	0.2976	0.3916	0.3775	0.3140	0.3562	0.3222
Theory	1.20	0.1981	0.3035	0.2977	0.3871	0.5134	0.4893	0.4094	0.4647	0.4323
Sims.	1.20	0.1974	0.3078	0.3036	0.3860	0.5102	0.4897	0.4110	0.4687	0.4333
Theory	1.25	0.2423	0.3712	0.3638	0.4735	0.6277	0.5976	0.5006	0.5681	0.5279
Sims.	1.25	0.2394	0.3678	0.3654	0.4740	0.6270	0.6004	0.5015	0.5673	0.5283

Table 4: Mapping from Single Index Returns and Omega to Information Ratio for the Single Index Model (Portfolio Drawn Independently - Approximate Method) Unequal Weight Benchmark

		N= 100.00			N= 500.00			N= 1000.00		
	ω	10%	50%	80%	10%	50%	80%	10%	50%	80%
Theory	1.05	0.0445	0.1009	0.0705	0.0763	0.1791	0.1372	0.1135	0.1290	0.1153
Sims.	1.05	0.0446	0.1058	0.0734	0.0770	0.1799	0.1346	0.1134	0.1260	0.1190
Theory	1.10	0.0951	0.1784	0.1466	0.1750	0.3099	0.2623	0.2180	0.2477	0.2259
Sims.	1.10	0.0941	0.1784	0.1478	0.1715	0.3094	0.2619	0.2123	0.2365	0.2233
Theory	1.15	0.1433	0.2523	0.2192	0.2693	0.4347	0.3814	0.3178	0.3609	0.3311
Sims.	1.15	0.1417	0.2527	0.2159	0.2700	0.4325	0.3832	0.3177	0.3606	0.3213
Theory	1.20	0.1895	0.3230	0.2883	0.3594	0.5539	0.4950	0.4131	0.4691	0.4314
Sims.	1.20	0.1888	0.3273	0.2942	0.3580	0.5510	0.4955	0.4147	0.4731	0.4324
Theory	1.25	0.2338	0.3906	0.3544	0.4457	0.6679	0.6033	0.5043	0.5725	0.5270
Sims.	1.25	0.2308	0.3872	0.3560	0.4461	0.6674	0.6061	0.5052	0.5716	0.5274

8. Unequal Weights and Selectivity

- Does anything change? Yes, the optimal level DECLINES with benchmark concentration.

- Concentration is measured as:

$$H = \frac{1}{N}\sum_i (w_i^B)^2 \text{ and } \kappa = N^2 H$$

- Where kappa is relative to an equal-weighted benchmark.

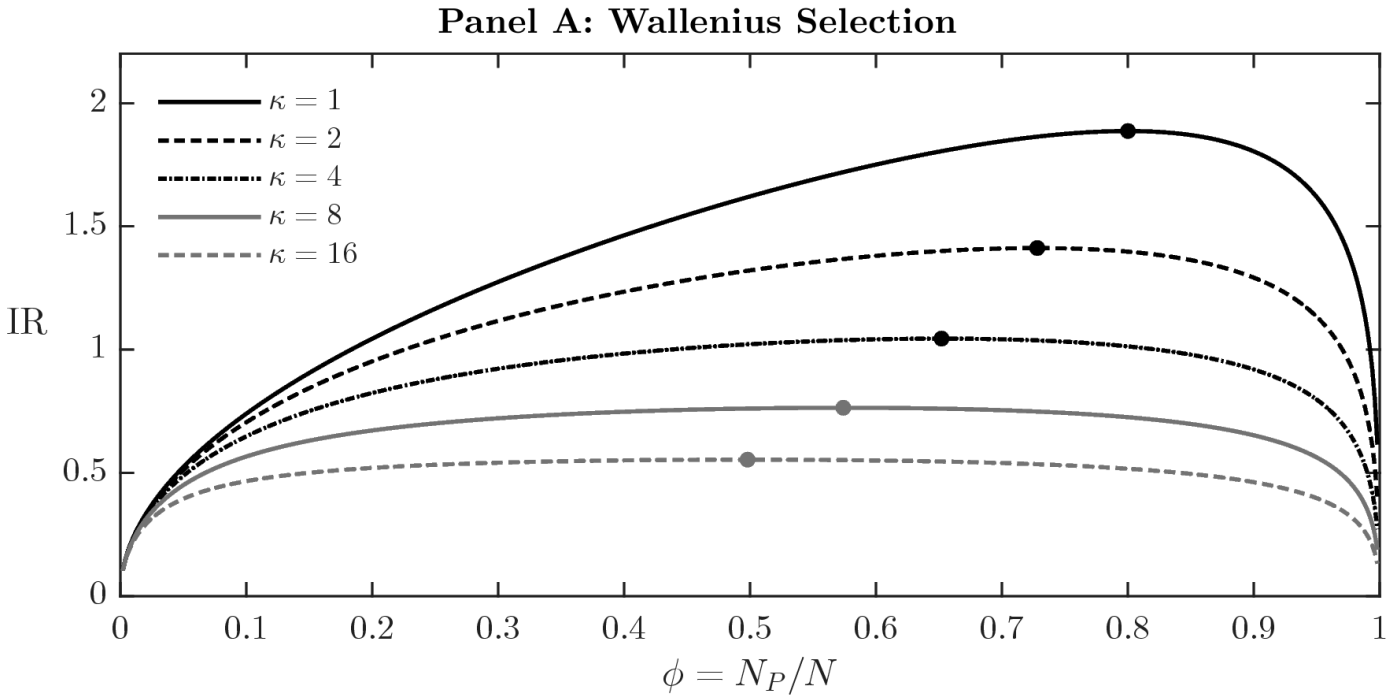
8. Unequal Weights and Selectivity

Table 5: Optimal Selectivity Ratio and Information Ratio at the Optimum by Benchmark Concentration

		Optimal selectivity ratio ϕ^*					Information ratio at the optimum			
ω	κ	Large N	N=100	N=500	N=1000	N=3000	N=100	N=500	N=1000	N=3000
Panel A: Wallenius selection										
1.1	1	0.797	0.790	0.796	0.797	0.797	0.305	0.684	0.967	1.676
1.1	2	0.731	0.720	0.730	0.730	0.730	0.229	0.514	0.727	1.260
1.1	4	0.657	0.650	0.656	0.656	0.657	0.170	0.381	0.539	0.933
1.1	8	0.580	0.570	0.578	0.579	0.579	0.124	0.279	0.394	0.683
1.1	16	0.503	0.490	0.500	0.502	0.502	0.090	0.202	0.286	0.495
1.3	1	0.801	0.790	0.800	0.801	0.801	0.840	1.887	2.670	4.626
1.3	2	0.729	0.720	0.728	0.729	0.729	0.629	1.412	1.998	3.462
1.3	4	0.654	0.650	0.652	0.653	0.653	0.465	1.045	1.478	2.561
1.3	8	0.576	0.570	0.574	0.575	0.576	0.340	0.764	1.081	1.873
1.3	16	0.500	0.490	0.498	0.499	0.499	0.247	0.553	0.783	1.357
2.0	1	0.831	0.820	0.830	0.830	0.831	2.266	5.095	7.210	12.495
2.0	2	0.720	0.710	0.718	0.720	0.720	1.638	3.678	5.204	9.016
2.0	4	0.633	0.630	0.632	0.633	0.633	1.202	2.697	3.816	6.612
2.0	8	0.555	0.550	0.554	0.554	0.555	0.877	1.968	2.784	4.823
2.0	16	0.483	0.480	0.482	0.482	0.483	0.635	1.424	2.015	3.490

8. Unequal Weights and Selectivity

Figure 4: **Information Ratio, Selectivity and Benchmark Concentration.** This figure shows the theoretical information ratio of Corollary 1 as a function of the selectivity ratio $\phi = N_P/N$ for a benchmark of $N = 500$ stocks and a manager with $\omega = 1.3$, for concentration ratios $\kappa = 1$, 2, 4, 8 and 16. Panel A uses Wallenius selection and Panel B Fisher selection, with the exact Wallenius and Fisher moments at each (N, N_P, ω). The dot on each curve marks the optimal selectivity ratio ϕ^*.



8. Unequal Weights and Selectivity

- Practical Implications:

1. The concentration of the benchmark will affect the relative information ratios achievable with skill.

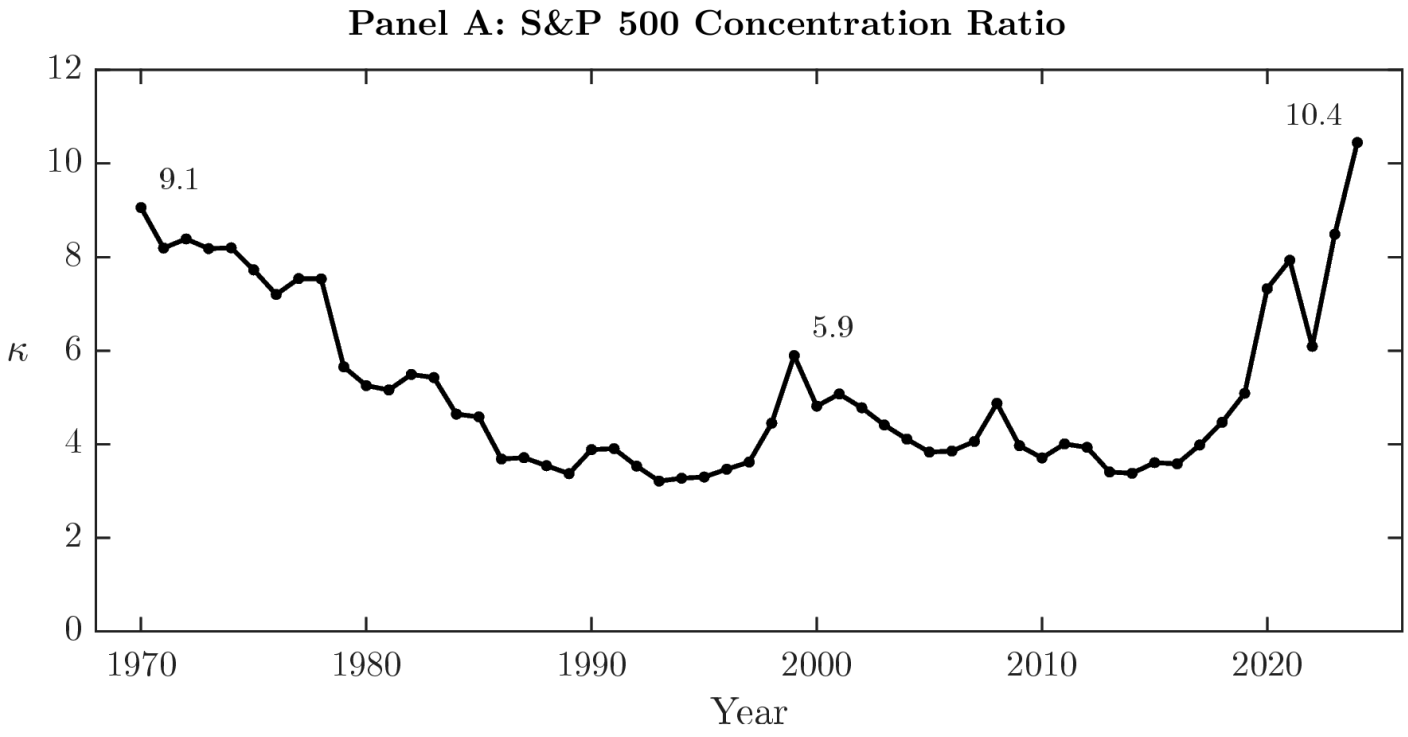
2. The concentration might dictate altering strategies with concentration.

3. Today, S&P 500 is very concentrated – hard to distinguish skill.

8. Unequal Weights and Selectivity

- Practical Implications:

Figure 5: **S&P 500 Concentration and the Implied Optimal Selectivity Ratio, 1970–2024.** Panel A shows the concentration ratio $\kappa = N/N_{\text{eff}}$ of the S&P 500 at the end of each year, computed from CRSP market capitalizations of the index constituents. Panel B shows the optimal selectivity ratio implied by Proposition 5 for a Wallenius (solid) and a Fisher (dashed) manager. The dotted lines mark the equal-weight optima, 0.797 and 0.5.



8. Unequal Weights and Selectivity

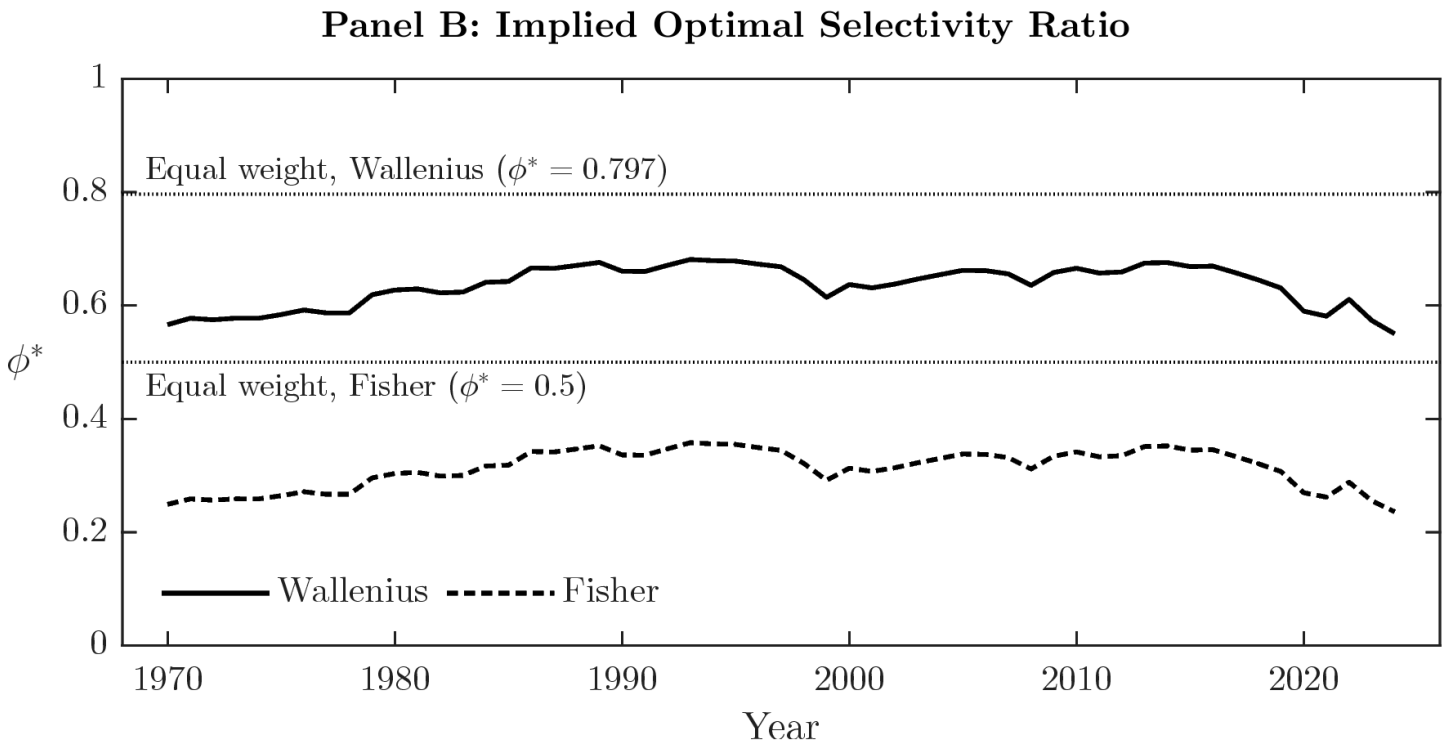
- Practical Implications:

Table 7: S&P 500 Concentration and Its Implications for Active Managers, 1970–2024

				Optimal ϕ^*		IR* relative to $\kappa = 1$		Track-record multiplier	
Year	N	N_{eff}	κ	Wallenius	Fisher	Wallenius	Fisher	Wallenius	Fisher
Equally weighted	500	500.0	1.00	0.797	0.500	1.000	1.000	1.00	1.00
1970	498	55.0	9.06	0.566	0.249	0.385	0.499	6.74	4.02
1975	498	64.5	7.73	0.584	0.265	0.414	0.529	5.82	3.57
1980	500	95.2	5.25	0.627	0.304	0.494	0.608	4.10	2.71
1985	500	109.1	4.58	0.642	0.318	0.525	0.637	3.63	2.47
1990	500	128.7	3.89	0.661	0.337	0.565	0.673	3.14	2.21
1995	500	151.2	3.30	0.678	0.355	0.607	0.710	2.72	1.98
1999	500	84.7	5.89	0.615	0.292	0.469	0.583	4.55	2.94
2000	500	103.7	4.81	0.637	0.313	0.513	0.626	3.80	2.55
2005	500	130.4	3.83	0.662	0.338	0.568	0.676	3.10	2.19
2010	500	134.9	3.71	0.666	0.342	0.576	0.684	3.01	2.14
2014	502	147.9	3.38	0.676	0.352	0.600	0.705	2.78	2.01
2015	504	138.6	3.61	0.669	0.345	0.583	0.690	2.94	2.10
2019	505	98.3	5.09	0.631	0.307	0.501	0.614	3.99	2.65
2020	505	68.3	7.33	0.590	0.270	0.425	0.540	5.55	3.43
2024	503	47.9	10.45	0.550	0.236	0.361	0.473	7.69	4.48

8. Unequal Weights and Selectivity

- Practical Implications:



9. An Application of Mapping Model

Disculpa, pero ¿y qué? ¿Por qué debería importarme?

One of the goals of portfolio managers is a high information ratio. Investors often times invest in managers with high realized information ratios.

If we have a more reliable estimate of future information ratio, we can invest better.

Using our method, we predict future better.

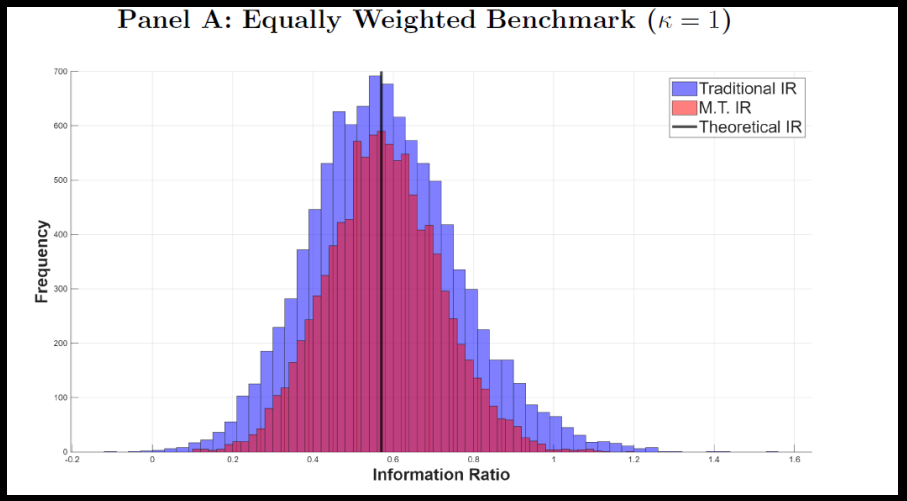
9. An Application of Mapping Model

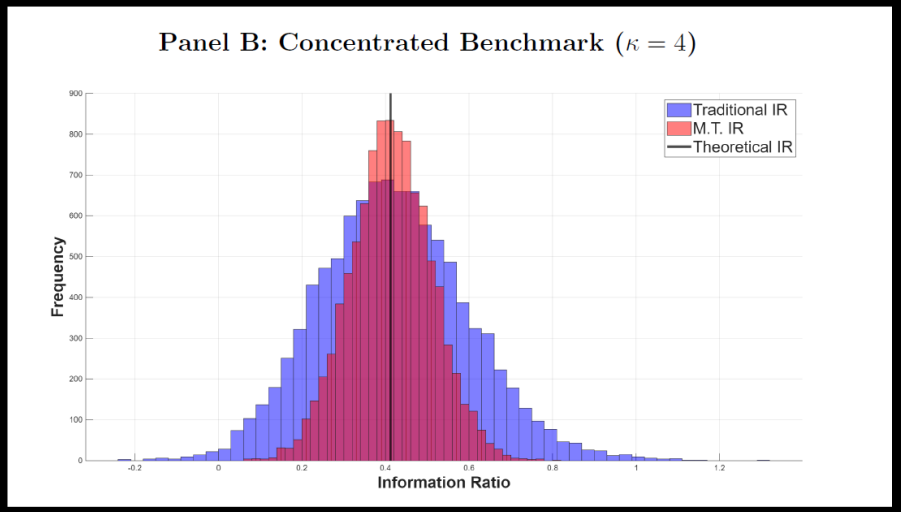
- The mapping theorem works for any stock picking process that can be summarized by a mean and standard deviation of picking.
- Estimating these parameters from the data, may reduce the noise from "lucky" stocks in a single period model.
- Thus, forward-looking information ratios might be more accurate using the mapping technique than traditional techniques.

9. An Application of Mapping Model

- We perform simulations using Wallenius, Fisher, and Binomial picking mechanisms.
- We find mapping technique is better and more concentrated around true IR of manager even with small sample of data.
- This becomes MORE IMPORTANT, the more concentrated an index is.

9. An Application of Mapping Model





9. An Application of Mapping Model

Table 8: Traditional and Mapping-Technique Estimates of the Information Ratio, Equal-Weight and Concentrated Benchmarks

		Traditional IR^H		Mapping IR^{MT}		Improvement	
Selection	True IR	MAE	MSE	MAE	MSE	MAE	MSE
Panel A: Equally weighted benchmark ($\kappa = 1$), $T = 36$ periods							
Wallenius	0.5705	0.1463	0.0346	0.1120	0.0200	23%	42%
Fisher	0.4831	0.1436	0.0331	0.1117	0.0198	22%	40%
Binomial	0.9397	0.1641	0.0445	0.1328	0.0279	19%	37%
Panel B: Concentrated benchmark ($\kappa = 4$), $T = 36$ periods							
Wallenius	0.4121	0.1418	0.0322	0.0774	0.0095	45%	71%
Fisher	0.3491	0.1401	0.0313	0.0781	0.0096	44%	69%
Binomial	0.7105	0.1531	0.0379	0.0905	0.0129	41%	66%
Panel C: Equally weighted benchmark ($\kappa = 1$), $T = 120$ periods							
Wallenius	0.5705	0.0788	0.0099	0.0619	0.0060	21%	40%
Fisher	0.4831	0.0765	0.0092	0.0600	0.0056	22%	39%
Binomial	0.9397	0.0882	0.0124	0.0710	0.0079	20%	36%
Panel D: Concentrated benchmark ($\kappa = 4$), $T = 120$ periods							
Wallenius	0.4121	0.0759	0.0091	0.0429	0.0029	43%	69%
Fisher	0.3491	0.0745	0.0087	0.0420	0.0028	44%	68%
Binomial	0.7105	0.0823	0.0107	0.0487	0.0037	41%	65%

9. An Application of Mapping Model

Warum? Perche? Por que?

• In essence, we separates skill (omega) from return luck (high return stocks).

- This occurs, even when we estimate sample statistics. That is, we measure the cross-sectional stock returns from the simulated data, rather than assume we know it.

- For now, we can only verify when we know the ex-ante picking process.

10. Summary

Selectivity theory can now be connected to real-world performance measures.

- Information Ratio becomes interpretable as implied stock-selection skill.
- Framework generalizes earlier selectivity theory.
- Provides foundation for future empirical tests using manager holdings data.
- Creates a bridge between theory and practice.

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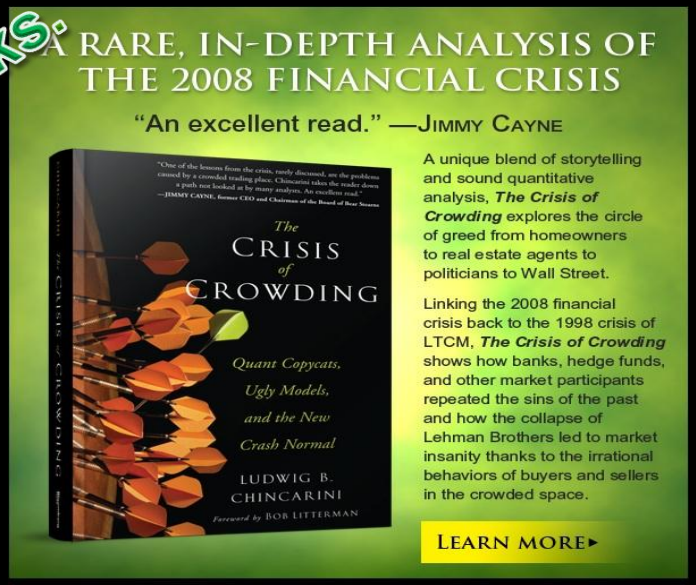
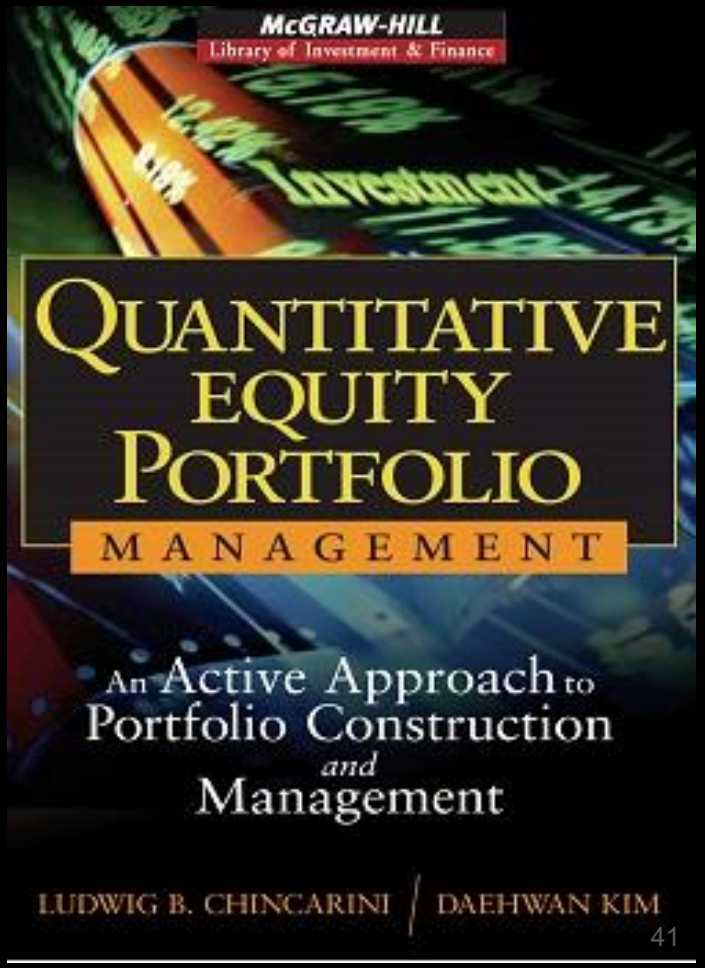




Open Discussion for all Participants

Further Notes

- The paper available on SSRN currently has all the derivations used in the paper.

- MORE INFO: Chincarini, Ludwig B. and Bolshakov, Andrei and Jerison, Daniel, A Mapping Technique for Selectivity Theory. Available at SSRN: https://ssrn.com/abstract=5421420 or http://dx.doi.org/10.2139/ssrn.5421420

Appendix A: Extra Details about Simple Model of Selectivity (Bolshakov and Chincarini (2020))

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- **Bulk Selection** – no path dependency
- No skill (ω=1), then probability of getting 3 good: 39.68%
- Skill (ω=1.1), then probability of getting 3 good: 41.49%
- For 3: Numerator: $\binom{5}{3}\binom{5}{2}\omega^3$
- For 3: Denominator:
$\binom{5}{0}\binom{5}{5}\omega^0+\binom{5}{1}\binom{5}{4}\omega^1+\binom{5}{2}\binom{5}{3}\omega^2+\binom{5}{3}\binom{5}{2}\omega^3+\binom{5}{4}\binom{5}{1}\omega^4+\binom{5}{5}\binom{5}{0}\omega^5$

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking more good stocks than bad stocks in a portfolio of 5 stocks?

- We need to sum up the probabilities of selecting 3, 4 and 5 stocks. The result is 53.4%

TABLE 1 A simple example of picking 5 stocks from the noncentral fisher distribution

This table shows a simple example of the noncentral Fisher distribution in the context of portfolio selection. The table shows the probability of selecting 0, 1, … , 5 good stocks in a portfolio of 5 stocks chosen from a 10-stock benchmark with an equal amount of good and bad stocks.

Number of Good Stocks	Numerator	Denominator	Probability of Event (%)
0	1.00	320.81	0.31
1	27.50	320.81	8.57
2	121.00	320.81	37.72
3	133.10	320.81	41.49
4	36.60	320.81	11.41
5	1.61	320.81	0.50

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- **Sequential Selection** – path dependency, thus slightly more difficult calculation
- So once all combinations have been computed, you add them – in this case probability of 3 good stocks = 41.98%
- Similar steps for 4, 5 stocks to derive the probability of picking more good than bad stocks (54.39%).

TABLE 2 The different possible paths to picking 5 stocks

This table shows the different paths that can occur when selecting five stocks from a universe of 10 stocks. A ''1'' indicates that a good stock has been picked, while a ''0'' indicates a bad stock was picked. There are 10 possible combinations of picking five stocks consisting of three good stocks from a universe of 10 stocks containing an equal number of good and bad stocks. The probability of picking any given stock in the sequence is shown in the second section of the table, below the paths, and the probability of any individual sequence is shown at the bottom of the table.

Path 1	Path 2	Path 3	Path 4	Path 5	Path 6	Path 7	Path 8	Path 9	Path 10
1	1	1	0	0	0	1	0	1	1
1	0	1	1	1	0	0	1	1	0
1	1	0	1	0	1	1	1	0	0
0	1	1	1	1	1	0	0	0	1
0	0	0	0	1	1	1	1	1	1
Probabilities of Individual Picks									
52.38	52.38	52.38	47.62	47.62	47.62	52.38	47.62	52.38	52.38
46.81	53.19	46.81	57.89	57.89	42.11	53.19	57.89	46.81	53.19
39.76	52.38	60.24	52.38	47.62	64.71	52.38	52.38	60.24	47.62
69.44	45.21	45.21	45.21	59.46	59.46	54.79	54.79	54.79	59.46
64.52	64.52	64.52	64.52	52.38	52.38	52.38	52.38	52.38	52.38
Probabilities of Entire Sequence									
4.37	4.26	4.31	4.21	4.09	4.04	4.19	4.14	4.24	4.13

2. The model

Portfolio Manager selects n stocks from a benchmark of N stocks. There are 50% "good" stocks and 50% "bad" stocks. Good stocks provide a 10% return and bad stocks a -10% return.

We will then compare a portfolio manager's performance against the benchmark via the Information Ratio.

When the portfolio manager draws from Fisher or Wallenius, we will know the expected number of good stocks. Thus, the expected return and standard deviation of the portfolio are given by:

$$E(r_P) = p^* r_g + (1 - p^*) r_b,$$

$$S(r_P) = (r_g - r_b) \frac{\sqrt{\sigma_x^2}}{n},$$

2. The model

We can show that the Information Ratio of the portfolio will be:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P)}{S(r_P)}.$$

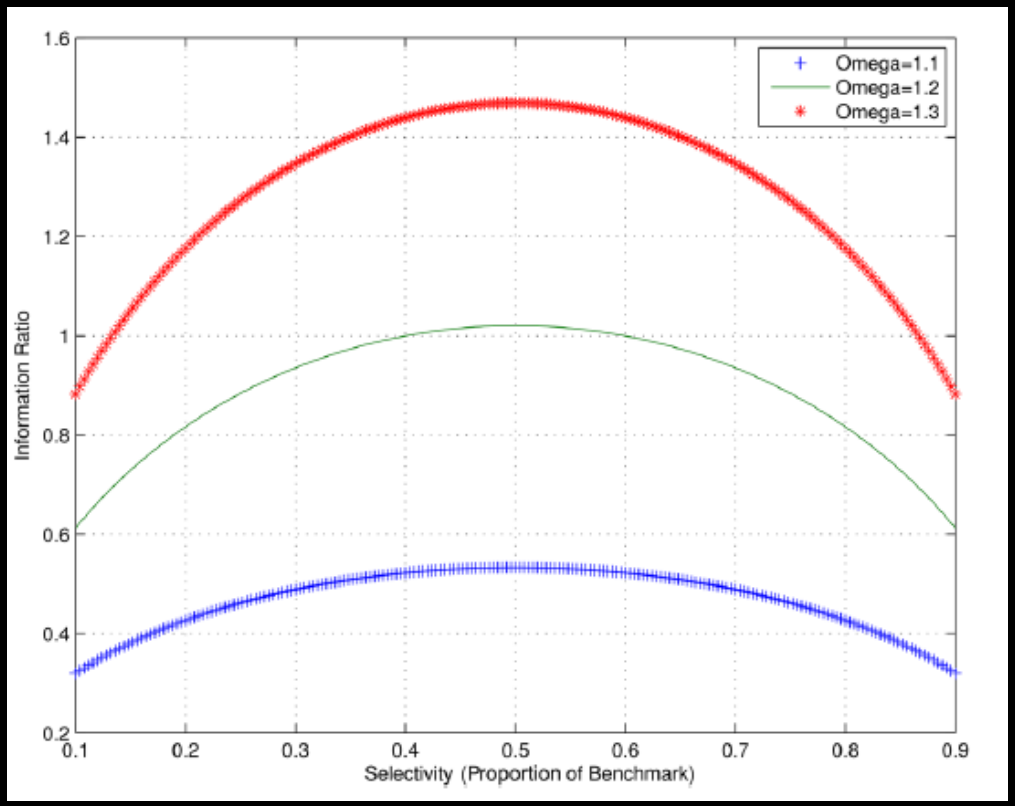
We also look at the Downside Information Ratio:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P) - E(r_{BM})}{SS(r_P - r_{BM})}$$

$$SS(r_P - r_{BM}) = \sqrt{\psi \sum_{i=1}^{n_l} \left[\min\left(0, r_{P,i} - r_{BM}\right)\right]^2 \cdot f(r_{P,i} - r_{BM}),}$$

3. Behavior of model

Example: N=500, n(g) = 250 n(b) = 250, ω=1.1 What is optimal selectivity ratio?

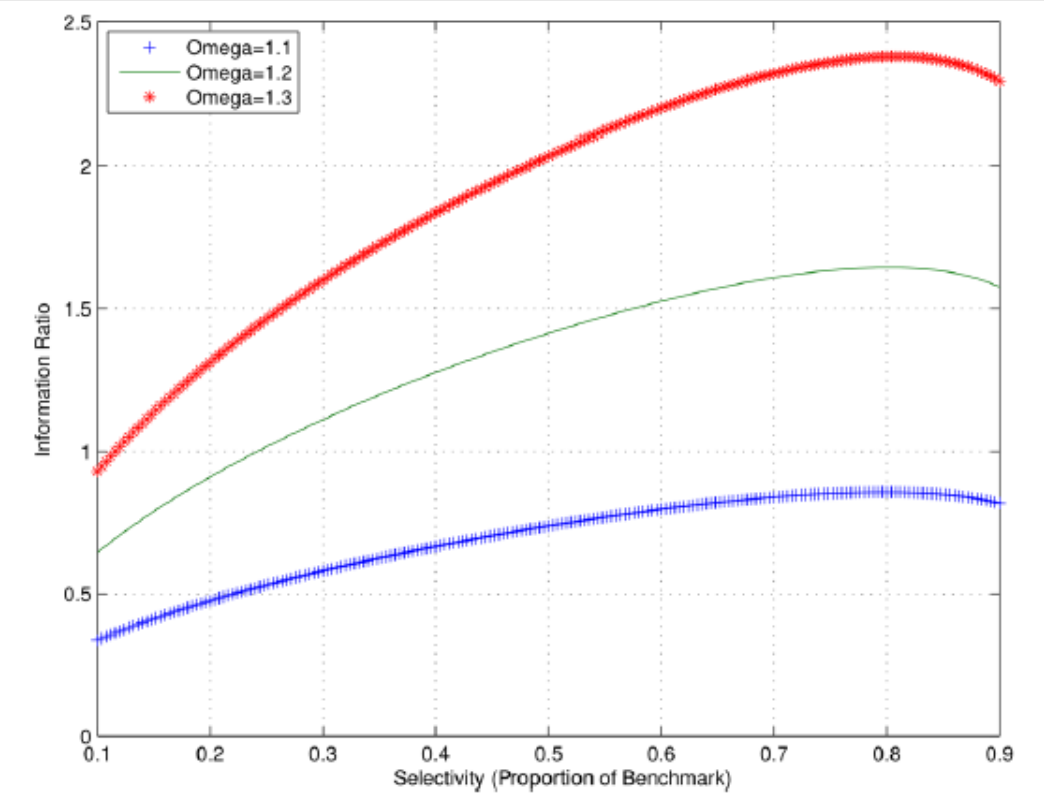


Bulk Selection = 50%

Note: For all ω, it's 50%!

3. Behavior of model

Example: N=500, n(g) = 250 n(b) = 250, ω=1.1 What is optimal selectivity ratio?

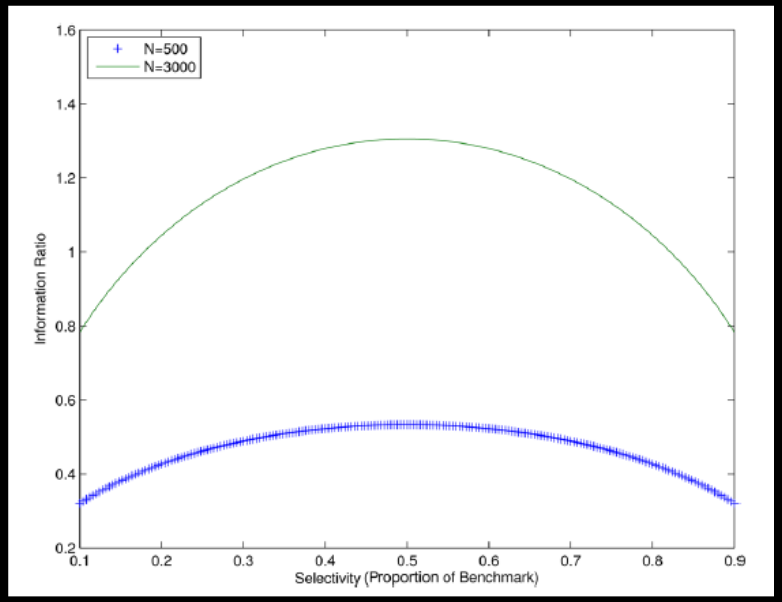


Sequential ~ 80%

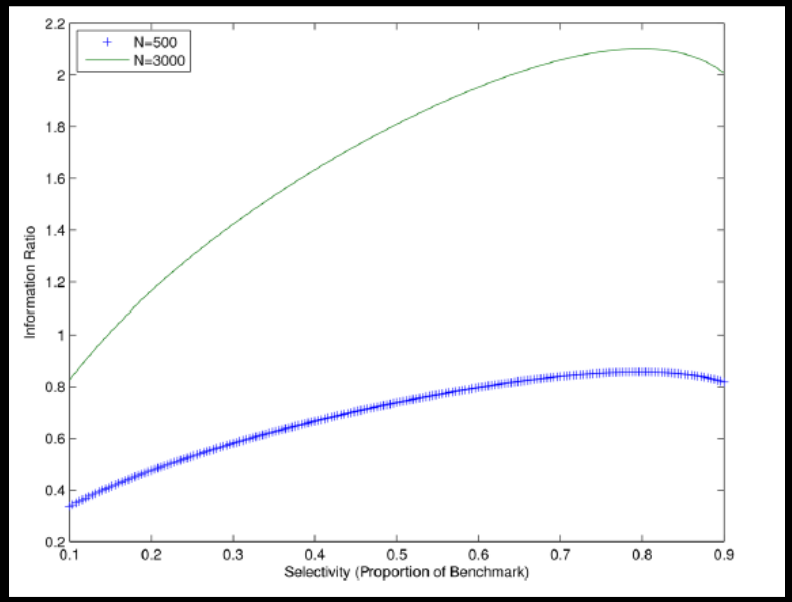
Note: For all ω, it's 80% (for reasonable values of ω)!

3. Behavior of model

Question: How do more stocks in benchmark affect the result?



Same selectivity ratio, but higher IR.



4. Characteristics of model

There are some general characteristics about the model's predictions.

Characteristic 1. Given a benchmark universe of stocks, N, the highest Information Ratio for a manager with skill level ω is obtained at a selectivity ratio (n/N) between 50% and 80%. For the bulk selection method, it is always at 50%. For the sequential selection method, it is near 80% for reasonable values of ω.

Characteristic 2. Given a manager with skill level ω that stays constant as the universe increases, a larger universe, M, will result in a larger Information Ratio, which is approximately $\sqrt{M/N}$ larger.

Characteristic 3. Given a certain selectivity ratio, the Information Ratio for the sequential selection method (Wallenius) will always be higher than the Information Ratio for the bulk selection (Fisher) method given a constant level of skill level, ω.

5. The imperfection of IR

For most applications, the Information Ratio (IR) is thought to be a reliable measure of performance versus a benchmark.

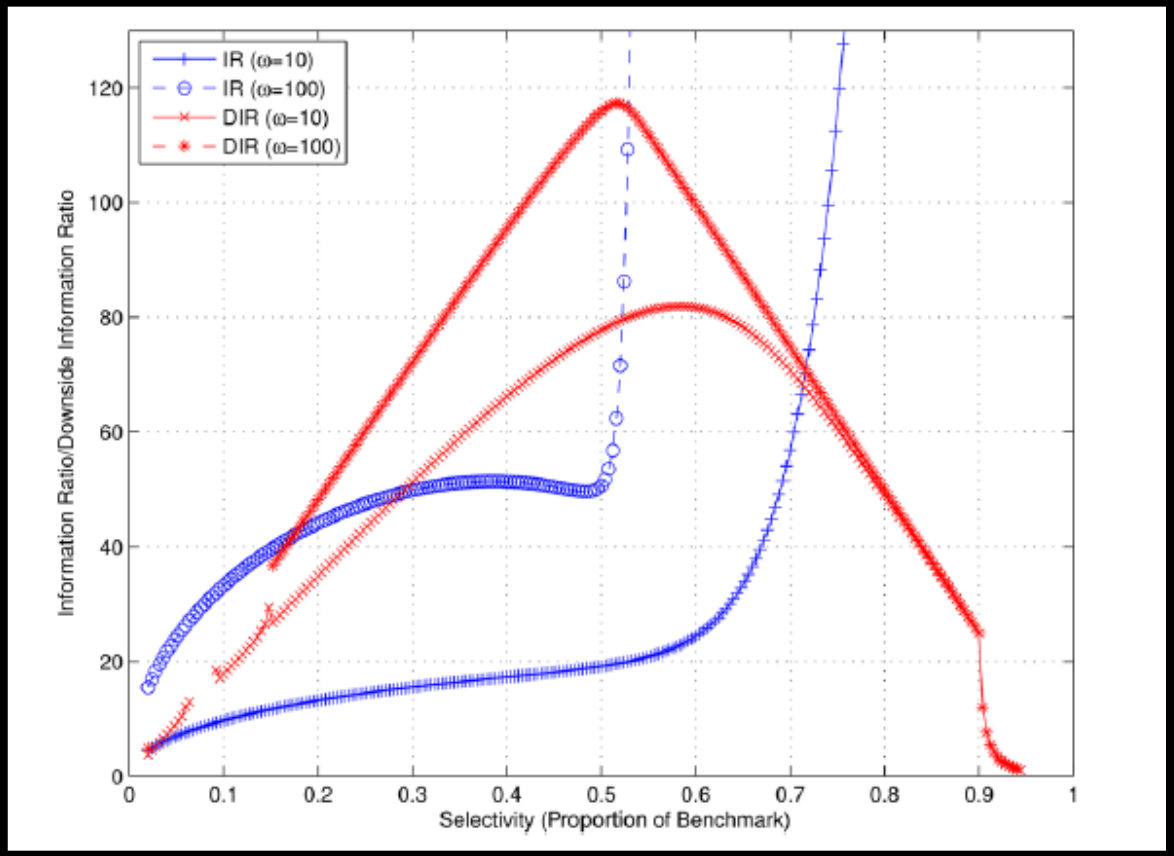
In our theoretical framework, when skill is very large, this measure performs very poorly.

For sequential picking, at very high levels of skill, the optimal IR is at 100% or complete indexing (TE declines faster than E(r)).

The problem is that at high levels of skill, although the probability of underperforming the benchmark is tiny, because the distribution of returns isn't centered around zero – IR is much less relevant, but DIR becomes appropriate criterion.

5. The imperfection of IR

- However, the Downside Information Ratio (DIR) resolves this problem as can be seen in graph.



Bottom Line: With the more appropriate DIR, as skill goes to infinity, sequential chooses 50% of portfolio.

