

The
CRISIS
of
CROWDING

*Quant Copycats,
Ugly Models,
and the New
Crash Normal*

LUDWIG B.
CHINCARINI

***A Mapping Technique for
Selectivity Theory
Bolshakov, Chincarini, and Jerison***

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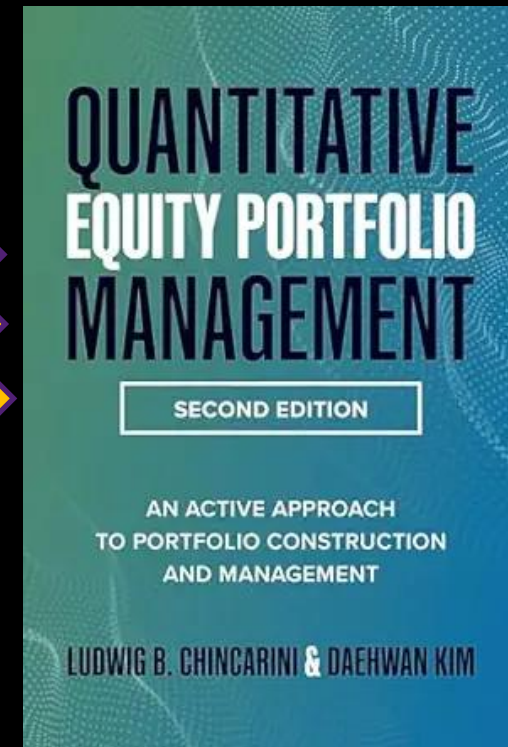
EFMA Annual Conference, Norway





- Thank you Prof Grillini and Chair Garcia and John Doukas and others.

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1. Brief Review of Selectivity Theory

- Chincarini, Ludwig (w/ Andrei Bolshakov). "Manager Skill and Portfolio Size with Respect to a Benchmark." European Financial Management, February, 2020.
- Chincarini, Ludwig (w/ Andrei Bolshakov and Christopher Lewis). "Enhanced Indexing and Selectivity Theory" European Financial Management, September 2022.

2. The model

Assumption 1: In any given index, 50% of the stocks will outperform and 50% will underperform.

Assumption 2: Stock either outperforms or underperforms (1 or 0), magnitude is unimportant.

Assumption 3: A portfolio manager's constant skill lies in the probability to pick a "winner" versus a "loser".

Assumption 4: The benchmark and portfolio are equally-weighted.

2. The model

We introduce the notion of omega (ω), where $\omega > 1$ if a portfolio manager is more likely to pick a good stock versus a bad stock.

To get a rough idea of how ω is related to probabilities, if $\omega = 1.1$ and a manager is picking the 1st stock, the probability of picking a good one is about 0.5238.

2. The model

Two Possible Selection Methods for a group of n stocks out of a universe of N stocks.

Method 1: Bulk Selection

Method 2: Sequential Selection

2. The model

Bulk Selection: This means that the portfolio manager selects the stocks into the portfolio ALL AT ONCE using his/her skill.

Mathematically, this is governed by the **Fisher Noncentral Hypergeometric Distribution**.

Sequential Selection: The portfolio manager decides ex-ante how many of the stocks in the benchmark to chose. Then he/she selects them ONE AT A TIME using his/her skill.

Mathematically, this is governed by the **Wallenius Noncentral Hypergeometric Distribution**

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad.
What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- **Bulk Selection** – no path dependency
- No skill ($\omega=1$), then probability of getting 3 good: 39.68%
- Skill ($\omega=1.1$), then probability of getting 3 good: 41.49%
- For 3: Numerator: $\binom{5}{3}\binom{5}{2}\omega^3$
- For 3: Denominator:
$$\binom{5}{0}\binom{5}{5}\omega^0 + \binom{5}{1}\binom{5}{4}\omega^1 + \binom{5}{2}\binom{5}{3}\omega^2 + \binom{5}{3}\binom{5}{2}\omega^3 + \binom{5}{4}\binom{5}{1}\omega^4 + \binom{5}{5}\binom{5}{0}\omega^5$$

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking more good stocks than bad stocks in a portfolio of 5 stocks?

- We need to sum up the probabilities of selecting 3, 4 and 5 stocks. The result is 53.4%

TABLE 1 A simple example of picking 5 stocks from the noncentral fisher distribution

This table shows a simple example of the noncentral Fisher distribution in the context of portfolio selection. The table shows the probability of selecting 0, 1, ..., 5 good stocks in a portfolio of 5 stocks chosen from a 10-stock benchmark with an equal amount of good and bad stocks.

Number of Good Stocks	Numerator	Denominator	Probability of Event (%)
0	1.00	320.81	0.31
1	27.50	320.81	8.57
2	121.00	320.81	37.72
3	133.10	320.81	41.49
4	36.60	320.81	11.41
5	1.61	320.81	0.50

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad.
What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- **Sequential Selection** – path dependency, thus slightly more difficult calculation
- So once all combinations have been computed, you add them – in this case probability of 3 good stocks = **41.98%**
- Similar steps for 4, 5 stocks to derive the probability of picking more good than bad stocks (**54.39%**).

TABLE 2 The different possible paths to picking 5 stocks

This table shows the different paths that can occur when selecting five stocks from a universe of 10 stocks. A “1” indicates that a good stock has been picked, while a “0” indicates a bad stock was picked. There are 10 possible combinations of picking five stocks consisting of three good stocks from a universe of 10 stocks containing an equal number of good and bad stocks. The probability of picking any given stock in the sequence is shown in the second section of the table, below the paths, and the probability of any individual sequence is shown at the bottom of the table.

Path 1	Path 2	Path 3	Path 4	Path 5	Path 6	Path 7	Path 8	Path 9	Path 10
1	1	1	0	0	0	1	0	1	1
1	0	1	1	1	0	0	1	1	0
1	1	0	1	0	1	1	1	0	0
0	1	1	1	1	1	0	0	0	1
0	0	0	0	1	1	1	1	1	1
Probabilities of Individual Picks									
52.38	52.38	52.38	47.62	47.62	47.62	52.38	47.62	52.38	52.38
46.81	53.19	46.81	57.89	57.89	42.11	53.19	57.89	46.81	53.19
39.76	52.38	60.24	52.38	47.62	64.71	52.38	52.38	60.24	47.62
69.44	45.21	45.21	45.21	59.46	59.46	54.79	54.79	54.79	59.46
64.52	64.52	64.52	64.52	52.38	52.38	52.38	52.38	52.38	52.38
Probabilities of Entire Sequence									
4.37	4.26	4.31	4.21	4.09	4.04	4.19	4.14	4.24	4.13

2. The model

Portfolio Manager selects n stocks from a benchmark of N stocks. There are 50% “good” stocks and 50% “bad” stocks. Good stocks provide a 10% return and bad stocks a -10% return.

We will then compare a portfolio manager’s performance against the benchmark via the Information Ratio.

When the portfolio manager draws from Fisher or Wallenius, **we will know the expected number of good stocks**. Thus, the expected return and standard deviation of the portfolio are given by:

$$E(r_P) = p^* r_g + (1 - p^*) r_b,$$

$$S(r_P) = (r_g - r_b) \frac{\sqrt{\sigma_x^2}}{n},$$

2. The model

We can show that the Information Ratio of the portfolio will be:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P)}{S(r_P)}.$$

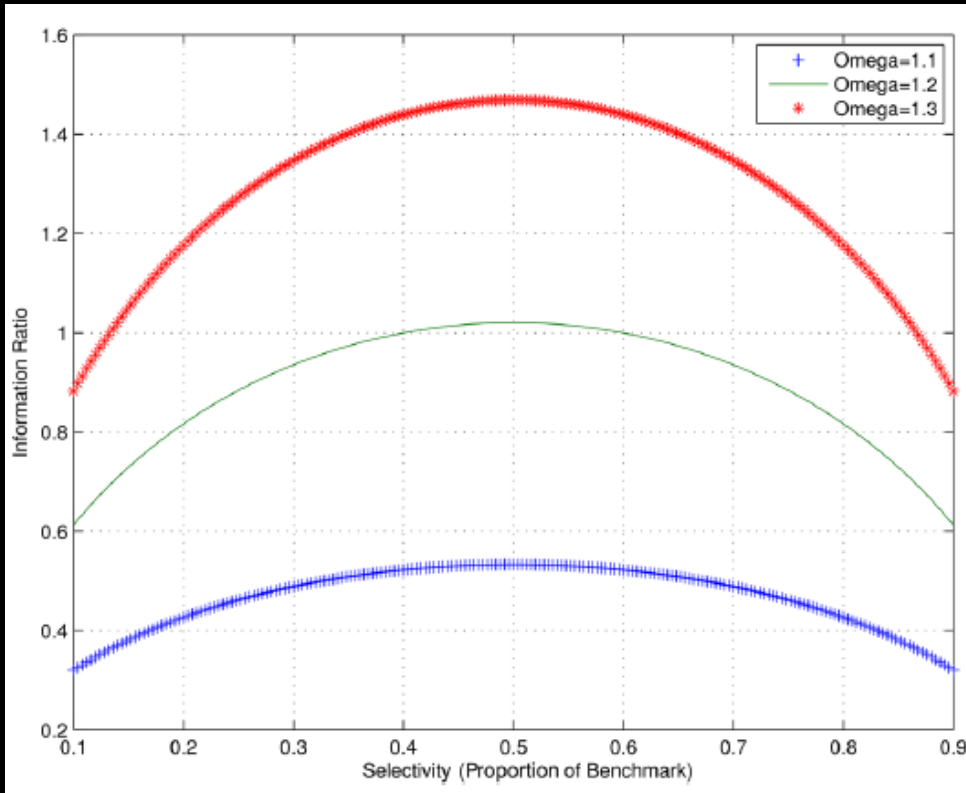
We also look at the Downside Information Ratio:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P) - E(r_{BM})}{SS(r_P - r_{BM})}$$

$$SS(r_P - r_{BM}) = \sqrt{\psi \sum_{i=1}^{n_l} [\min(0, r_{P,i} - r_{BM})]^2 \cdot f(r_{P,i} - r_{BM})},$$

3. Behavior of model

Example: $N=500$, $n(g) = 250$ $n(b) = 250$, $\omega=1.1$ What is optimal selectivity ratio?

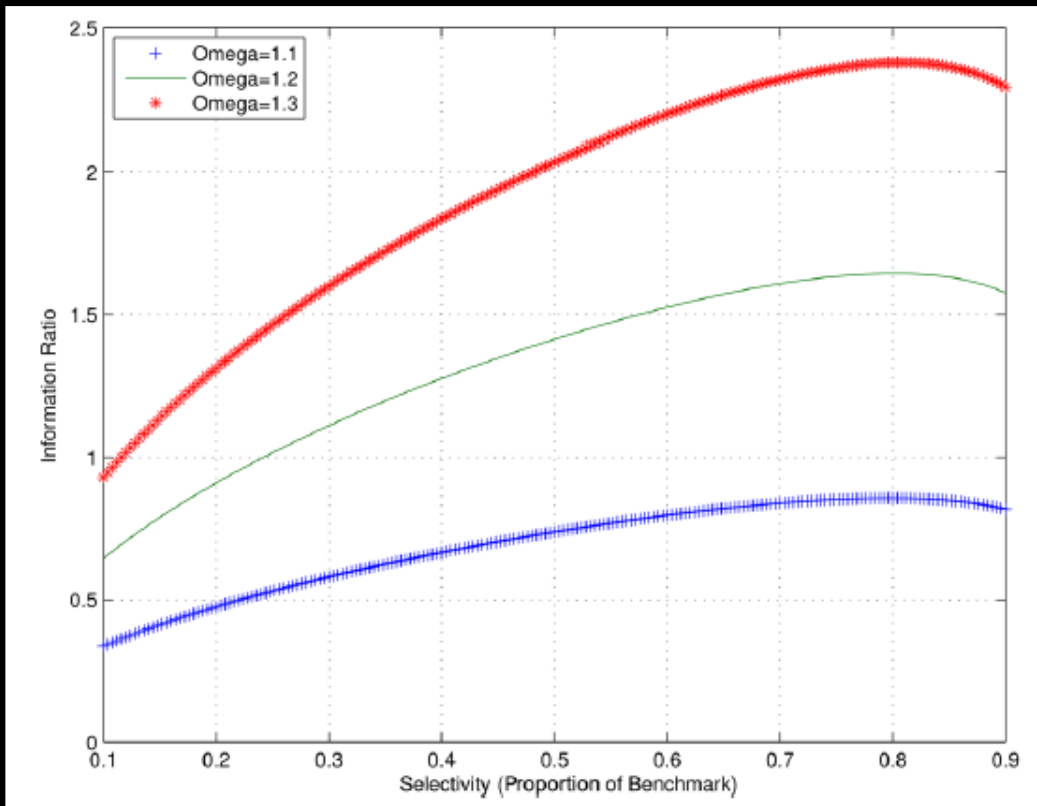


Bulk Selection = 50%

Note: For all ω , it's 50%!

3. Behavior of model

Example: $N=500$, $n(g) = 250$ $n(b) = 250$, $\omega=1.1$ What is optimal selectivity ratio?

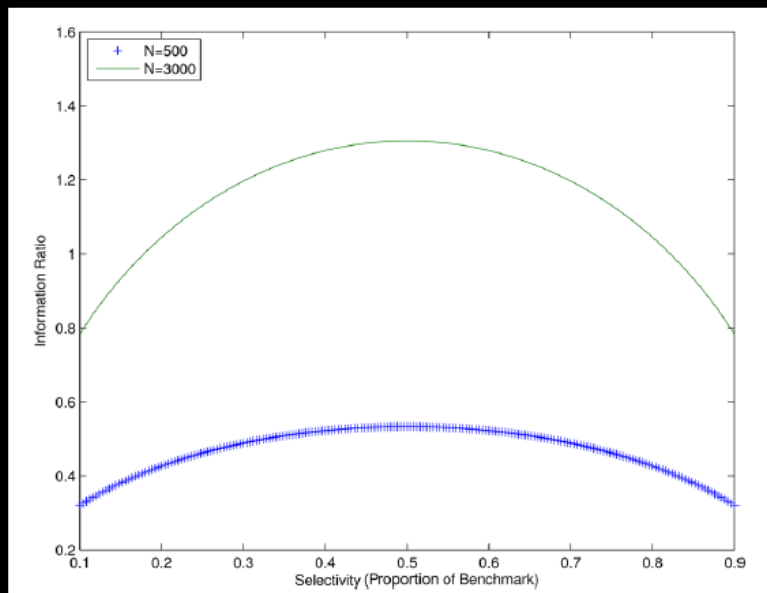


Sequential $\sim 80\%$

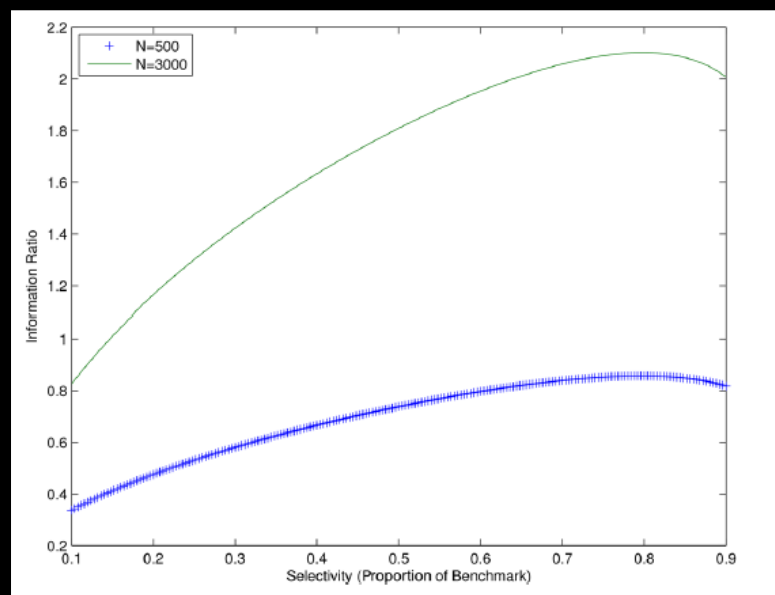
Note: For all ω , it's 80% (for reasonable values of ω)!

3. Behavior of model

Question: How do more stocks in benchmark affect the result?



**Same selectivity
ratio, but higher IR.**



4. Characteristics of model

There are some general characteristics about the model's predictions.

Characteristic 1. Given a benchmark universe of stocks, N , the highest Information Ratio for a manager with skill level ω is obtained at a selectivity ratio (n/N) between 50% and 80%. For the bulk selection method, it is always at 50%. For the sequential selection method, it is near 80% for reasonable values of ω .

Characteristic 2. Given a manager with skill level ω that stays constant as the universe increases, a larger universe, M , will result in a larger Information Ratio, which is approximately $\sqrt{M/N}$ larger.

Characteristic 3. Given a certain selectivity ratio, the Information Ratio for the sequential selection method (Wallenius) will always be higher than the Information Ratio for the bulk selection (Fisher) method given a constant level of skill level, ω .

5. The imperfection of IR

For most applications, the Information Ratio (IR) is thought to be a reliable measure of performance versus a benchmark.

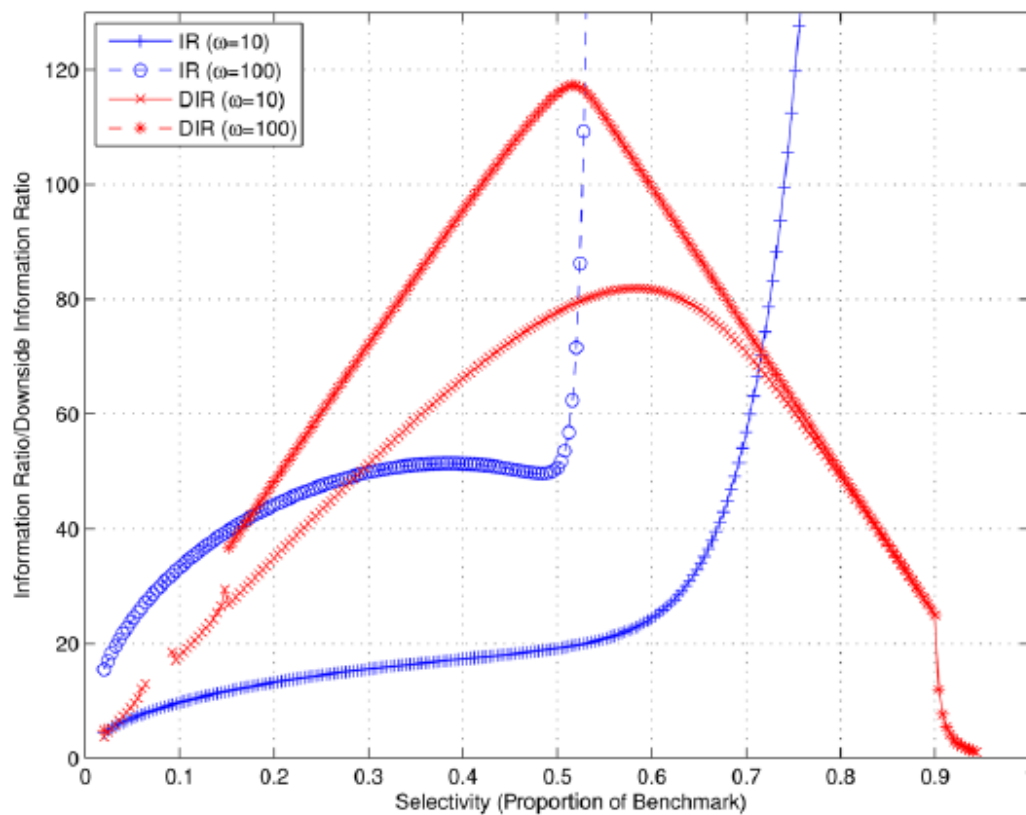
In our theoretical framework, when skill is very large, this measure performs very poorly.

For sequential picking, at very high levels of skill, the optimal IR is at 100% or complete indexing (TE declines faster than $E(r)$).

The problem is that at high levels of skill, although the probability of underperforming the benchmark is tiny, because the distribution of returns isn't centered around zero – IR is much less relevant, but DIR becomes appropriate criterion.

5. The imperfection of IR

- However, the Downside Information Ratio (DIR) resolves this problem as can be seen in graph.



Bottom Line:
With the more appropriate DIR, as skill goes to infinity, sequential chooses 50% of portfolio.

6. Current Research

Assumption 1 and 2 make it hard to take the model to real data.

Assumption 1: In any given index, 50% of the stocks will outperform and 50% will underperform.

Assumption 2: Stock either outperforms or underperforms (1 or 0), magnitude is unimportant.

Question: Is there a way to model the mathematics with a realistic distribution so that we might use it with real data.

7. Why This Paper?

- Original selectivity theory predicts optimal portfolio breadth (50%-80%).
- But assumptions are too restrictive for empirical work.
- Returns are not binary (winner/loser); magnitudes matter.
- Need a mapping from observable Information Ratio (IR) to latent skill (ω).
- Goal: make selectivity theory testable with real-world data.

8. Main Contribution

- Replace binary returns with continuous return distributions.

Independent normal returns

Correlated Returns through Single Index Model

- • Derive analytical formulas for:
 - • • Expected Active Return
 - • • Tracking Error
 - • • Information Ratio
- • Create a direct mapping: $IR \rightarrow \omega$ (manager skill).
- • Framework works beyond Fisher and Wallenius selection.

9. Intuition of the Framework

- Step 1: Draw stock returns from a realistic distribution.
- Step 2: Divide stocks into 'good' and 'bad' relative performers (compared to median)
- Step 3: Manager selects more good than bad stocks according to skill ω .
- Step 4: Derive $E(\text{Active Return})$, TE , and IR analytically.
- Step 5: Link observed IR implies an underlying level of stock-selection skill.
- Key: Works with selectivity theory (Wallenius or Fischer) or any other stock picking mechanism that can be represented by mean and standard deviation of good stocks.

10. What the Mathematics Delivers

- The paper derives closed-form approximations for:
- Equation (3): Expected Portfolio Return
- Equation (4): Portfolio Variance
- Equation (5): Expected Active Return
- Equation (6): Tracking Error
- Equation (7): Information Ratio

These formulas depend on N , NP , μ_x and σ_x^2 .

10. Mathematics Continued

$$\mathbf{E}(r_P) \approx \mu + \sigma \left(\frac{2\mu_x}{N_P} - 1 \right) \left(\sqrt{\frac{2}{\pi}} + \frac{1}{N} \cdot \frac{4-\pi}{2\sqrt{2\pi}} - \frac{1}{N^2} \cdot \frac{-96+40\pi-3\pi^2}{16\sqrt{2\pi}} \right). \quad (3)$$

$$\begin{aligned} \text{Var}(r_P) &\approx \sigma^2 \left[\frac{1}{N_P} \cdot \frac{\pi-2}{\pi} + \frac{\sigma_x^2}{N_P^2} \cdot \frac{8}{\pi} \right] \\ &+ \frac{\sigma^2}{N} \left[\frac{2}{\pi} + \frac{1}{N_P} \cdot \frac{2(\pi-3)}{\pi} + \frac{\sigma_x^2}{N_P^2} \cdot \frac{4(4-\pi)}{\pi} \right] \\ &+ \frac{\sigma^2}{N^2} \left[-\frac{16(\pi-3)-(4-\pi)^2}{4\pi} - \frac{1}{N_P} \cdot \frac{120-60\pi+7\pi^2}{4\pi} \right. \\ &\quad \left. - \frac{\mu_x}{N_P} \cdot \frac{(4-\pi)^2}{\pi} + \frac{\mu_x^2}{N_P^2} \cdot \frac{(4-\pi)^2}{\pi} \right. \\ &\quad \left. + \frac{\sigma_x^2}{N_P^2} \cdot \frac{3(4-\pi)^2+(96-40\pi+3\pi^2)}{2\pi} \right] \end{aligned} \quad (4)$$

$$\mathbf{E}(r_P - r_B) \approx \sigma \left(\frac{2\mu_x}{N_P} - 1 \right) \left(\sqrt{\frac{2}{\pi}} + \frac{1}{N} \cdot \frac{4-\pi}{2\sqrt{2\pi}} - \frac{1}{N^2} \cdot \frac{-96+40\pi-3\pi^2}{16\sqrt{2\pi}} \right). \quad (5)$$

$$\begin{aligned} \text{Var}(r_P - r_B) &\approx \sigma^2 \left[\frac{1}{N_P} \cdot \frac{\pi-2}{\pi} + \frac{\sigma_x^2}{N_P^2} \cdot \frac{8}{\pi} \right] \\ &+ \frac{\sigma^2}{N} \left[-1 + \frac{2}{\pi} + \frac{1}{N_P} \cdot \frac{2(\pi-3)}{\pi} + \frac{\sigma_x^2}{N_P^2} \cdot \frac{4(4-\pi)}{\pi} \right] \\ &+ \frac{\sigma^2}{N^2} \left[-\frac{2(\pi-3)}{\pi} - \frac{1}{N_P} \cdot \frac{120-60\pi+7\pi^2}{4\pi} \right. \\ &\quad \left. + \frac{1}{4} \left(\frac{2\mu_x}{N_P} - 1 \right)^2 \frac{(4-\pi)^2}{\pi} \right. \\ &\quad \left. + \frac{\sigma_x^2}{N_P^2} \cdot \frac{3(4-\pi)^2+(96-40\pi+3\pi^2)}{2\pi} \right]. \end{aligned} \quad (6)$$

$$\text{IR} \approx \frac{\left(\frac{2\mu_x}{N_P} - 1 \right) \left(\sqrt{\frac{2}{\pi}} + \frac{1}{N} \cdot \frac{4-\pi}{2\sqrt{2\pi}} - \frac{1}{N^2} \cdot \frac{-96+40\pi-3\pi^2}{16\sqrt{2\pi}} \right)}{\sqrt{\blacksquare}} \quad (7)$$

11. Extension to a Single-Factor World

- Embed selectivity theory inside a CAPM / single-index framework.
- Manager skill is assumed to identify positive idiosyncratic shocks (ε).
- Market risk enters through β differences.
- If $\beta_P \approx \beta_B$, mapping between IR and ω remains largely intact.
- This provides a much more realistic asset-pricing setting.

12. Simulations and Empirical Use

- 100,000 Monte Carlo simulations validate the formulas.
- Analytical IR closely matches simulated IR.
- Mapping tables can convert an observed IR into implied ω .

Table 1: Mapping from Error Returns and Omega to Information Ratio (Portfolio Drawn from Benchmark - Approximate Method)

		N= 100.00			N= 500.00			N= 1000.00		
	ω	10%	50%	80%	10%	50%	80%	10%	50%	80%
Theory	1.05	0.0616	0.1349	0.1558	0.1376	0.3017	0.3499	0.1946	0.4266	0.4951
Sims.	1.05	0.0612	0.1318	0.1630	0.1356	0.3063	0.3453	0.1970	0.4242	0.5002
Theory	1.10	0.1204	0.2635	0.3045	0.2688	0.5893	0.6837	0.3801	0.8334	0.9674
Sims.	1.10	0.1180	0.2606	0.3045	0.2669	0.5866	0.6808	0.3800	0.8302	0.9617
Theory	1.15	0.1766	0.3864	0.4467	0.3942	0.8641	1.0030	0.5573	1.2220	1.4193
Sims.	1.15	0.1717	0.3858	0.4443	0.4016	0.8638	1.0001	0.5564	1.2273	1.4170
Theory	1.20	0.2303	0.5040	0.5831	0.5142	1.1271	1.3092	0.7270	1.5940	1.8525
Sims.	1.20	0.2374	0.5051	0.5751	0.5138	1.1280	1.3077	0.7283	1.5903	1.8522
Theory	1.25	0.2818	0.6168	0.7141	0.6292	1.3793	1.6035	0.8897	1.9507	2.2689
Sims.	1.25	0.2838	0.6171	0.7116	0.6346	1.3854	1.6042	0.8873	1.9463	2.2717
Theory	1.30	0.3313	0.7252	0.8403	0.7397	1.6216	1.8868	1.0459	2.2933	2.6698
Sims.	1.30	0.3301	0.7169	0.8379	0.7368	1.6234	1.8928	1.0371	2.2870	2.6612
Theory	1.35	0.3789	0.8294	0.9620	0.8460	1.8546	2.1602	1.1962	2.6228	3.0566
Sims.	1.35	0.3744	0.8303	0.9577	0.8480	1.8525	2.1588	1.1935	2.6286	3.0529

13. Real World Application

- Estimate manager skill more accurately than realized IR alone.
- Separates skill from return luck.
- This occurs, even when we estimate sample statistics. That is, we measure the cross-sectional stock returns from the simulated data, rather than assume we know it.

13. Real World Application

Figure 1: The Distribution of Information Ratios for Wallenius Process. This figure shows the empirically measured information ratios of Wallenius portfolio managers with 100 stocks in the benchmark, a selectivity ratio of 30%, an $\omega = 1.3$, the cross-sectional mean of stock returns of 12% with a standard deviation of 20%. The traditional estimates are obtained by taking the historical returns of the portfolio manager and benchmark and computing their sample information ratio. The mapping technique estimate takes the empirical number of winner stocks and the standard deviation of those winner stocks in each period for the portfolio manager and enters these values into the formula for the information ratio derived in this paper and uses that as the calculation of the information ratio. In this particular simulation, the mean absolute error (MAE) of the information ratio compared to the theoretical information is 0.08504 for the traditional method and 0.062945 for the mapping technique (or 14.905% error versus 11.0324% error). The mean squared error (MSE) information ratio compared to the theoretical information is 0.011451 for the traditional method and 0.0062332 for the mapping technique. The black line is the theoretical information ratio of the manager at 0.57055.

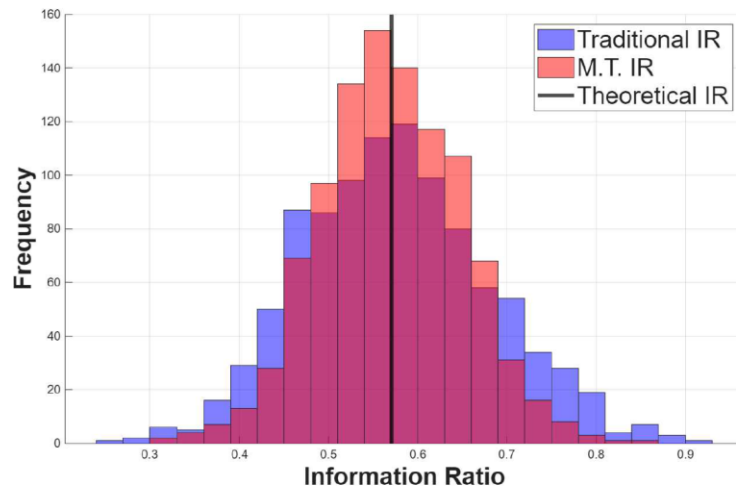
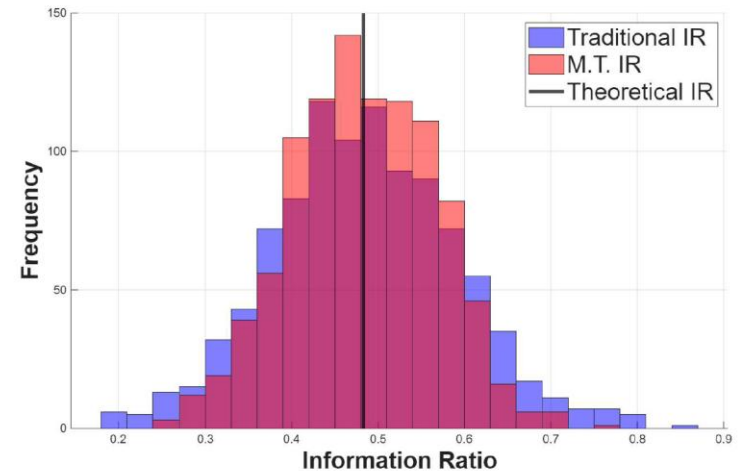


Figure 2: The Distribution of Information Ratios for Fisher Process. This figure shows the empirically measured information ratios of Fisher portfolio managers with 100 stocks in the benchmark, a selectivity ratio of 30%, an $\omega = 1.3$, the cross-sectional mean of stock returns of 12% with a standard deviation of 20%. The traditional estimates are obtained by taking the historical returns of the portfolio manager and benchmark and computing their sample information ratio. The mapping technique estimate takes the empirical number of winner stocks and the standard deviation of those winner stocks in each period for the portfolio manager and enters these values into the formula for the information ratio derived in this paper and uses that as the calculation of the information ratio. In this particular simulation, the mean absolute error (MAE) of the information ratio compared to the theoretical information is 0.085101 for the traditional method and 0.067448 for the mapping technique (or 17.6171% error versus 13.9628% error). The mean squared error (MSE) information ratio compared to the theoretical information is 0.011461 for the traditional method and 0.0068684 for the mapping technique. The black line is the theoretical information ratio of the manager at 0.48306.



13. Key Appendix Results

- Appendix A: Full derivation of Equations (3)-(7).
 - Uses truncated normal distributions and median-order statistics.
 - Appendix B: Extension to the single-index model.
 - Appendix C: Exact solutions under independent benchmark/portfolio draws.
 - Shows robustness of the mapping framework.

14. Summary

Selectivity theory can now be connected to real-world performance measures.

- Information Ratio becomes interpretable as implied stock-selection skill.
- Framework generalizes earlier selectivity theory.
- Provides foundation for future empirical tests using manager holdings data.
- Creates a bridge between theory and practice.

15. Extensions

It would be nice to extend to non-equal, but fixed weights. In principle, this should solve, it will not change the expected return, but will change the tracking error.

It would be nice to expand to non-normal returns, but this may not have an “analytical” solution.

15. Extensions - Ideas

Let

$$a_i = w_i^P - w_i^B$$

be the active weight of stock i . Define

$$H_A = \sum_{i=1}^N a_i^2$$

and

$$S = \sum_{i=1}^N a_i G_i,$$

where $G_i = 1$ if stock i is good and 0 otherwise.

15. Extensions - Ideas

Then the weighted analogues are:

$$E(r_P - r_B) \approx 2\sigma\bar{S} \left(\sqrt{\frac{2}{\pi}} + \frac{1}{N} \frac{4 - \pi}{2\sqrt{2\pi}} - \frac{1}{N^2} \frac{-96 + 40\pi - 3\pi^2}{16\sqrt{2\pi}} \right).$$

$$\begin{aligned} \text{Var}(r_P - r_B) \approx & \sigma^2 \left[H_A \frac{\pi - 2}{\pi} + \sigma_S^2 \frac{8}{\pi} \right] + \frac{\sigma^2}{N} \left[H_A \frac{2(\pi - 3)}{\pi} + \sigma_S^2 \frac{4(4 - \pi)}{\pi} \right] \\ & + \frac{\sigma^2}{N^2} \left[-H_A \frac{120 - 60\pi + 7\pi^2}{4\pi} + \bar{S}^2 \frac{(4 - \pi)^2}{\pi} + \sigma_S^2 \frac{3(4 - \pi)^2 + (96 - 40\pi + 3\pi^2)}{2\pi} \right]. \end{aligned}$$

Therefore,

$$IR \approx \frac{2\bar{S} \left(\sqrt{\frac{2}{\pi}} + \frac{1}{N} \frac{4 - \pi}{2\sqrt{2\pi}} - \frac{1}{N^2} \frac{-96 + 40\pi - 3\pi^2}{16\sqrt{2\pi}} \right)}{\sqrt{\Omega}},$$

where Ω is the variance expression above without σ^2 .

15. Extensions - Ideas

To compute $\bar{S}$ and σ_S^2 , use:

$$\bar{S} = \mu_x \bar{a}_P + \left(\frac{N}{2} - \mu_x\right) \bar{a}_Q,$$

where $\bar{a}_P$ is the average active weight among portfolio stocks and $\bar{a}_Q$ is the average active weight among non-portfolio stocks.

The variance is

$$\sigma_S^2 = (\bar{a}_P - \bar{a}_Q)^2 \sigma_x^2 + \frac{N_P \mu_x - (\mu_x^2 + \sigma_x^2)}{N_P - 1} s_P^2 + \frac{N_Q \left(\frac{N}{2} - \mu_x\right) - \left[\left(\frac{N}{2} - \mu_x\right)^2 + \sigma_x^2\right]}{N_Q - 1} s_Q^2,$$

where $N_Q = N - N_P$, s_P^2 is the cross-sectional variance of active weights among portfolio stocks, and s_Q^2 is the cross-sectional variance of active weights among non-portfolio stocks.

So the equal-weight substitutions are:

$$\begin{aligned} \frac{\mu_x}{N_P} - \frac{1}{2} &\rightarrow \bar{S} \\ \frac{\sigma_x^2}{N_P^2} &\rightarrow \sigma_S^2 \\ \frac{1}{N_P} - \frac{1}{N} &\rightarrow H_A. \end{aligned}$$

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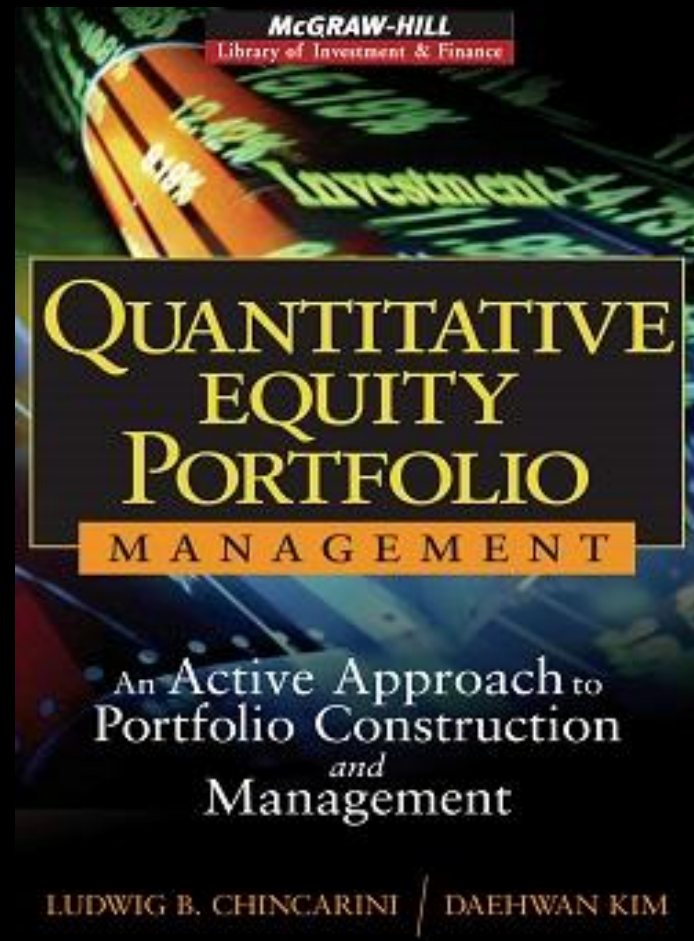
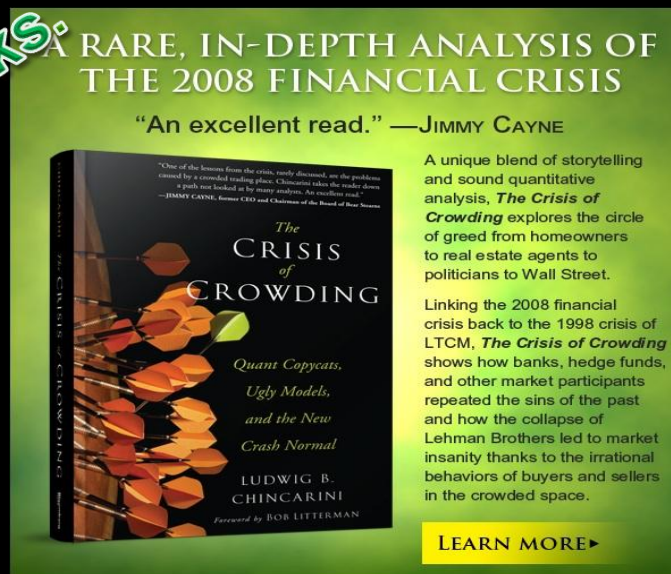
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