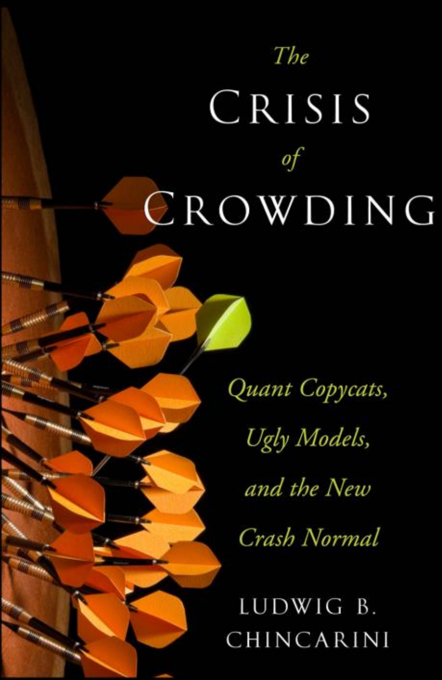
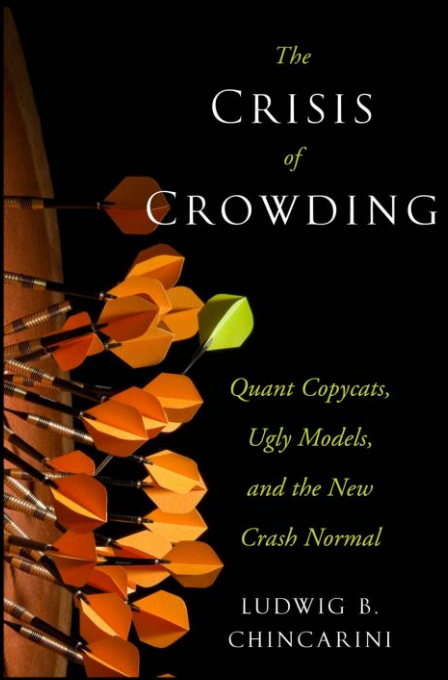


Enhanced Index & Selectivity Theory

Bolshakov, Chincarini, and Lewis

June 27, 2021



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- Thank you Ivelina and the SWFA for the invitation.

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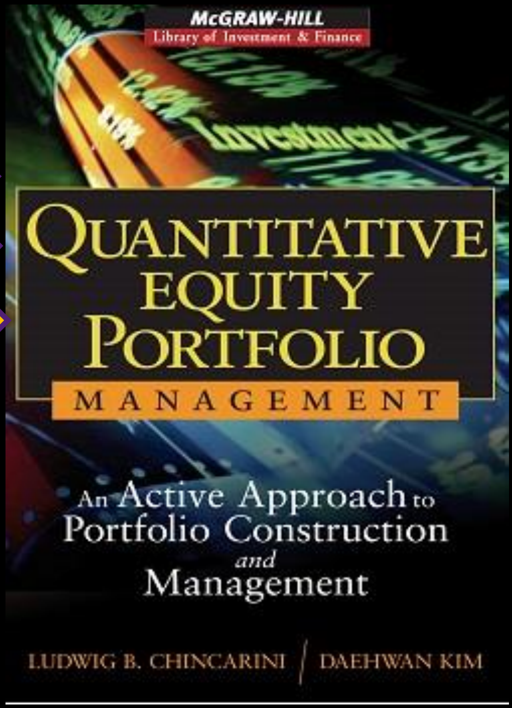


1. Brief Review of Selectivity Theory

- Selectivity Theory (Bolshakov and Chincarini (2020))

Chincarini, Ludwig (w/ Andrei Bolshakov). "Manager Skill and Portfolio Size with Respect to a Benchmark." European Financial Management, February, 2020.

2. The model

Assumption 1: In any given index, 50% of the stocks will outperform and 50% will underperform.

Assumption 2: Stock either outperforms or underperforms (1 or 0), magnitude is unimportant.

Assumption 3: A portfolio manager’s constant skill lies in the probability to pick a “winner” versus a “loser”.

Assumption 4: The benchmark and portfolio are equally-weighted.

2. The model

We introduce the notion of omega (ω), where ω >1 if a portfolio manager is more likely to pick a good stock versus a bad stock.

To get a rough idea of how ω is related to probabilities, if $\omega = 1.1$ and a manager is picking the 1st stock, the probability of picking a good one is about 0.5238.

2. The model

Two Possible Selection Methods for a group of n stocks out of a universe of N stocks.

Method 1: Bulk Selection

Method 2: Sequential Selection

2. The model

Bulk Selection: This means that the portfolio manager selects the stocks into the portfolio ALL AT ONCE using his/her skill.

Mathematically, this is governed by the Fisher Noncentral Hypergeometric Distribution.

Sequential Selection: The portfolio manager decides ex-ante how many of the stocks in the benchmark to chose. Then he/she selects them ONE AT A TIME using his/her skill.

Mathematically, this is governed by the Wallenius Noncentral Hypergeometric Distribution

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- **Bulk Selection** – no path dependency
- No skill (ω=1), then probability of getting 3 good: 39.68%
- Skill (ω=1.1), then probability of getting 3 good: 41.49%
- For 3: Numerator: $\binom{5}{3}\binom{5}{2}\omega^3$
- For 3: Denominator: $\binom{5}{0}\binom{5}{5}\omega^0+\binom{5}{1}\binom{5}{4}\omega^1+\binom{5}{2}\binom{5}{3}\omega^2+\binom{5}{3}\binom{5}{2}\omega^3+\binom{5}{4}\binom{5}{1}\omega^4+\binom{5}{5}\binom{5}{0}\omega^5$

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking more good stocks than bad stocks in a portfolio of 5 stocks?

- We need to sum up the probabilities of selecting 3, 4 and 5 stocks. The result is 53.4%

TABLE 1 A simple example of picking 5 stocks from the noncentral fisher distribution

This table shows a simple example of the noncentral Fisher distribution in the context of portfolio selection. The table shows the probability of selecting 0, 1, . . . , 5 good stocks in a portfolio of 5 stocks chosen from a 10-stock benchmark with an equal amount of good and bad stocks.

Number of Good Stocks	Numerator	Denominator	Probability of Event (%)
0	1.00	320.81	0.31
1	27.50	320.81	8.57
2	121.00	320.81	37.72
3	133.10	320.81	41.49
4	36.60	320.81	11.41
5	1.61	320.81	0.50

2. The model

Simple Example: Benchmark has 10 stocks, 5 good, 5 bad. What's the probability of picking 3 good stocks in a portfolio of 5 stocks?

- **Sequential Selection** – path dependency, thus slightly more difficult calculation

- So once all combinations have been computed, you add them – in this case probability of 3 good stocks = 41.98%

- Similar steps for 4, 5 stocks to derive the probability of picking more good than bad stocks (54.39%).

TABLE 2 The different possible paths to picking 5 stocks

This table shows the different paths that can occur when selecting five stocks from a universe of 10 stocks. A ''1'' indicates that a good stock has been picked, while a ''0'' indicates a bad stock was picked. There are 10 possible combinations of picking five stocks consisting of three good stocks from a universe of 10 stocks containing an equal number of good and bad stocks. The probability of picking any given stock in the sequence is shown in the second section of the table, below the paths, and the probability of any individual sequence is shown at the bottom of the table.

Path 1	Path 2	Path 3	Path 4	Path 5	Path 6	Path 7	Path 8	Path 9	Path 10
1	1	1	0	0	0	1	0	1	1
1	0	1	1	1	0	0	1	1	0
1	1	0	1	0	1	1	1	0	0
0	1	1	1	1	1	0	0	0	1
0	0	0	0	1	1	1	1	1	1
Probabilities of Individual Picks									
52.38	52.38	52.38	47.62	47.62	47.62	52.38	47.62	52.38	52.38
46.81	53.19	46.81	57.89	57.89	42.11	53.19	57.89	46.81	53.19
39.76	52.38	60.24	52.38	47.62	64.71	52.38	52.38	60.24	47.62
69.44	45.21	45.21	45.21	59.46	59.46	54.79	54.79	54.79	59.46
64.52	64.52	64.52	64.52	52.38	52.38	52.38	52.38	52.38	52.38
Probabilities of Entire Sequence									
4.37	4.26	4.31	4.21	4.09	4.04	4.19	4.14	4.24	4.13

2. The model

Portfolio Manager selects n stocks from a benchmark of N stocks. There are 50% “good” stocks and 50% “bad” stocks. Good stocks provide a 10% return and bad stocks a -10% return.

We will then compare a portfolio manager’s performance against the benchmark via the Information Ratio.

When the portfolio manager draws from Fisher or Wallenius, we will know the expected number of good stocks. Thus, the expected return and standard deviation of the portfolio are given by:

$$E(r_{\mathrm{P}}) = p^* r_g + (1 - p^*) r_b,$$

$$S(r_{\mathrm{P}}) = (r_g - r_b)\frac{\sqrt{\sigma_x^2}}{n},$$

2. The model

We can show that the Information Ratio of the portfolio will be:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P)}{S(r_P)}.$$

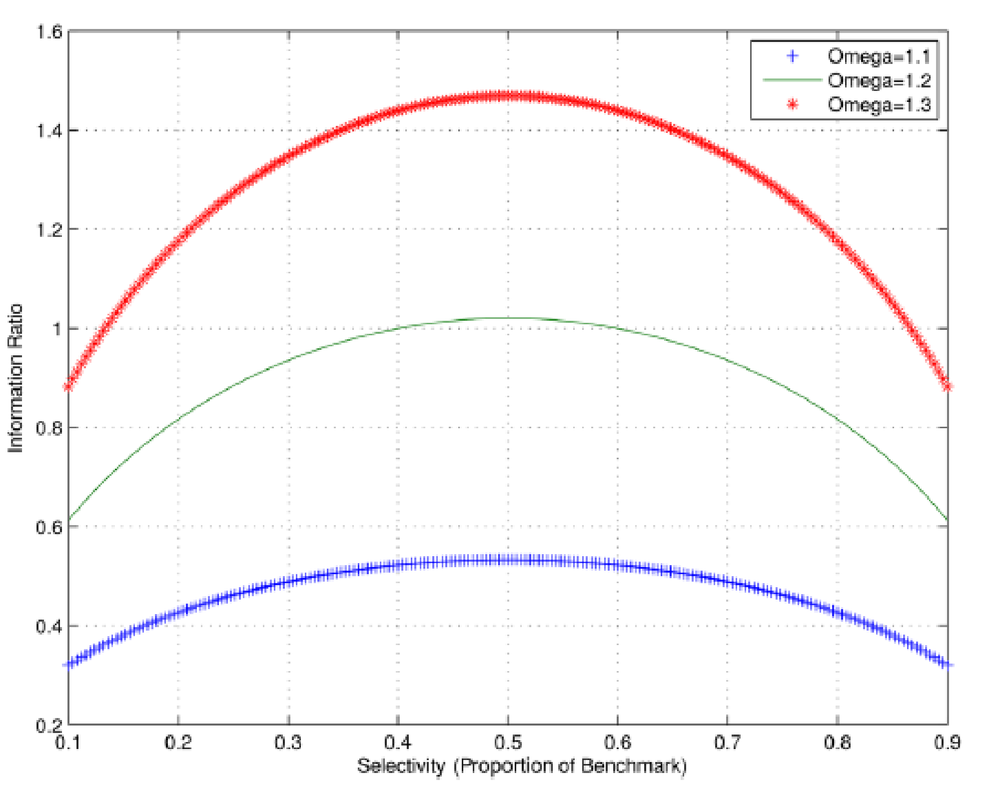
We also look at the Downside Information Ratio:

$$IR(n/N, N, \omega, n_g, n_b) = \frac{E(r_P) - E(r_{BM})}{SS(r_P - r_{BM})}$$

$$SS(r_P - r_{BM}) = \sqrt{\psi \sum_{i=1}^{n_l} \left[\min\left(0, r_{P,i} - r_{BM}\right)\right]^2 \cdot f(r_{P,i} - r_{BM})},$$

3. Behavior of model

Example: N=500, n(g) = 250 n(b) = 250, ω=1.1 What is optimal selectivity ratio?

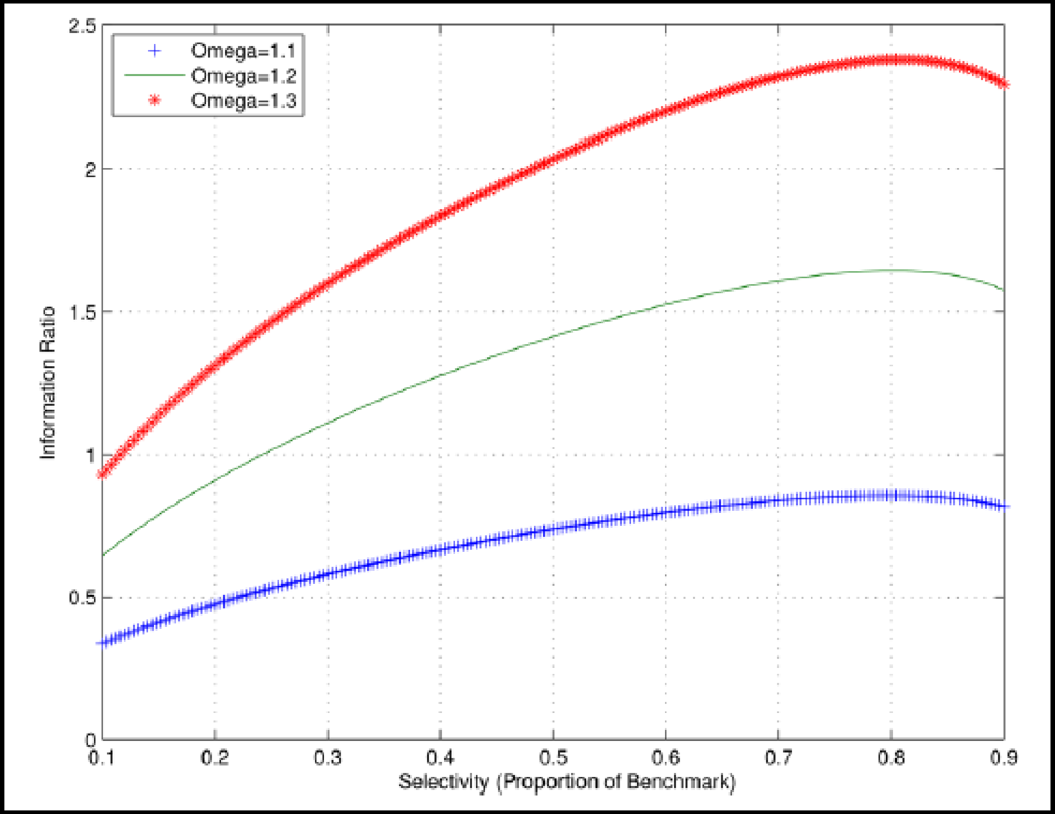


Bulk Selection = 50%

Note: For all ω, it's 50%!

3. Behavior of model

Example: N=500, n(g) = 250 n(b) = 250, ω=1.1 What is optimal selectivity ratio?

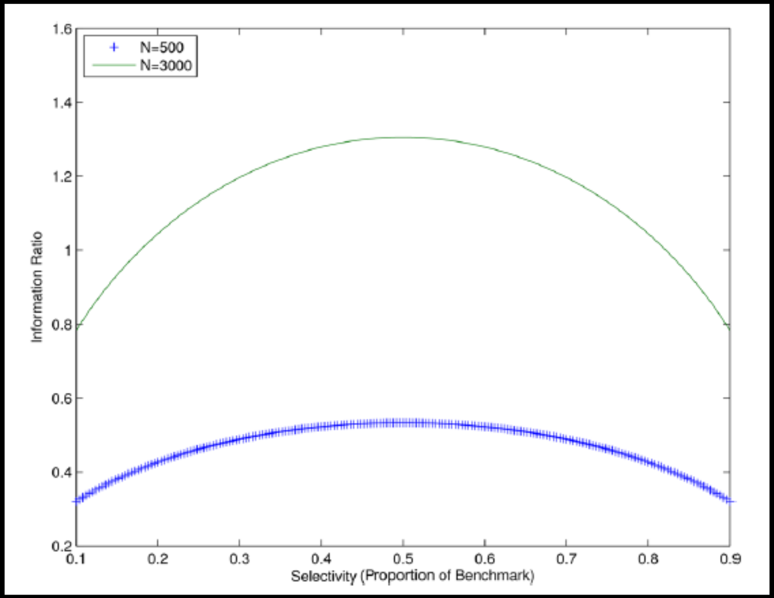


Sequential ~ 80%

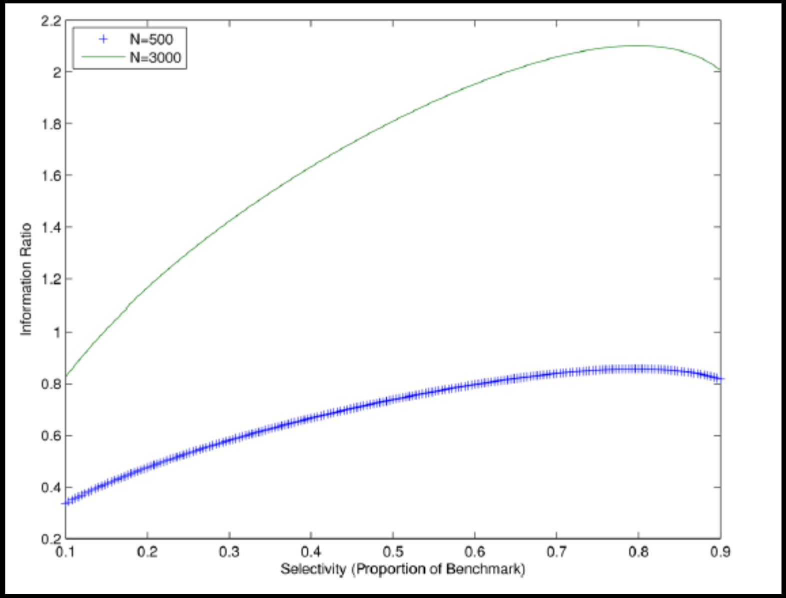
Note: For all ω, it's 80% (for reasonable values of ω)!

3. Behavior of model

Question: How do more stocks in benchmark affect the result?



Same selectivity ratio, but higher IR.



4. Characteristics of model

There are some general characteristics about the model's predictions.

Characteristic 1. Given a benchmark universe of stocks, N, the highest Information Ratio for a manager with skill level ω is obtained at a selectivity ratio (n/N) between 50% and 80%. For the bulk selection method, it is always at 50%. For the sequential selection method, it is near 80% for reasonable values of ω.

Characteristic 2. Given a manager with skill level ω that stays constant as the universe increases, a larger universe, M, will result in a larger Information Ratio, which is approximately $\sqrt{M/N}$ larger.

Characteristic 3. Given a certain selectivity ratio, the Information Ratio for the sequential selection method (Wallenius) will always be higher than the Information Ratio for the bulk selection (Fisher) method given a constant level of skill level, ω.

5. The imperfection of IR

For most applications, the Information Ratio (IR) is thought to be a reliable measure of performance versus a benchmark.

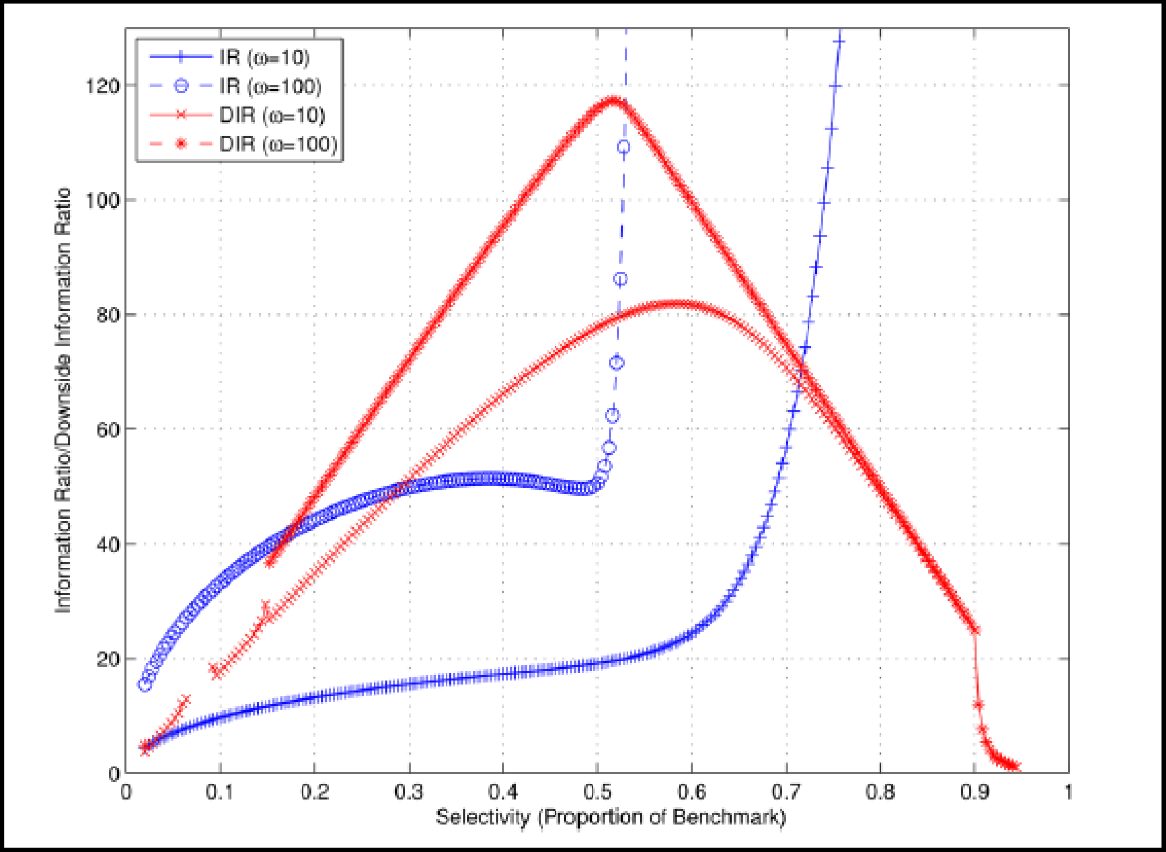
In our theoretical framework, when skill is very large, this measure performs very poorly.

For sequential picking, at very high levels of skill, the optimal IR is at 100% or complete indexing (TE declines faster than E(r)).

The problem is that at high levels of skill, although the probability of underperforming the benchmark is tiny, because the distribution of returns isn't centered around zero – IR is much less relevant, but DIR becomes appropriate criterion.

5. The imperfection of IR

- However, the Downside Information Ratio (DIR) resolves this problem as can be seen in graph.



Bottom Line: With the more appropriate DIR, as skill goes to infinity, sequential chooses 50% of portfolio.

7. Relaxing model assumptions

The model has certain simplifying assumptions about the investment universe.

Assumption 1: In any given index, 50% of the stocks will outperform and 50% will underperform.

Assumption 2: Stock either outperforms or underperforms (1 or 0), magnitude is unimportant.

Assumption 3: A portfolio manager’s skill lies in the probability to pick a “winner” versus a “loser.”

Assumption 4: The benchmark and portfolio are equally-weighted.

7. Relaxing model assumptions

The paper for this conference deals with this issue.

The results of the assumption relaxation are available on request in *Enhanced Indexing and Selectivity Theory* (Bolshakov, Chincarini, and Lewis) (2020). Here, I will just summarize.

One way to think of the results is in terms of the Information Ratio. IR = Excess Return / Tracking Error

7. Relaxing model assumptions

Summary of Results

Relax Assumption 1: Still near 80% Selectivity for Sequential
Relax Assumption 2: Still near 80% Selectivity for Sequential
Relax Assumption 4: Still above 70% Selectivity for Sequential

Why? In all cases, the average excess return doesn't really change, but the tracking error increases. But that doesn't change the optimal point much.

Relax Assumption 3 (more complicated): a. Steadily Decline = 50% (converges to Fisher (bulk). b. Jack Knife (skill for x% of universe) – changes to x% c. Ability Saturation (can only identify x% of good stocks, not all of them), close to 80% again d (unless super ability – high omega). Uncertainty in Skill (omega has a mean and vol), still close to 80%.

7. Relaxing model assumptions

Simulation Overview:

1. **Concentrated Winners** – there are not 50% winners, but the winner concentration can vary in the universe.
2. **Non Binary Returns** – returns are not 10% or -10% for winners/losers, rather can vary according
3. **Steadily Declining Omega** – manager has skill, but omega declining from >1 to 1 over the stock universe
4. **Jack Knife** – manager has skill but only up to a percentage of existing universe (e.g. 30% of universe), then no skill
5. **Ability Saturation** – manager has skill for subset of population, but doesn’t know exactly which part
6. **Uncertain Omega** – manager has a mean positive omega, but there is variance around it, in one method we choose omega on each pick, in another we keep for entire stock picking exercise
7. **Non Equal Weight** – we allow market cap weighted benchmark versus equal-weighted
8. **Dynamic** – allow manager to select stocks every rebalancing period, rather than just buy and hold

7. Relaxing model assumptions

Simulation Overview

Table 1: Summary of Parameters in the Different Simulations

	Parameters								
Simulation Name	N	Winners (m_1)	Losers (m_2)	Returns	Weight	Holding Period	ω	N_{sims}	S^*
Baseline	500	50%	50%	+/- 10%	EW	One Horizon	1.1	100,000	Near 79%
Concentrated Winners	500	Varies	Varies	+/- 10%	EW	One Horizon	1.1	100,000	Near 79%
Non Binary Returns	500	Varies	Varies	S&P 500 Z-Scores	EW	One Horizon	1.1	100,000	Near 79%
Steadily Declining ω	500	50%	50%	+/- 10%	EW	One Horizon	1.1 to 1	100,000	Near 50%
Jack Knife	500	50%	50%	+/- 10%	EW	One Horizon	1.1	100,000	Near 30%†
Ability Saturation	500	50%	50%	+/- 10%	EW	One Horizon	1.1	100,000	Near 75%††
Uncertainty	500	50%	50%	+/- 10%	EW	One Horizon	$\omega \sim N(1.1, \sigma_\omega^2)$	100,000	Near 79%
Non Equal Weight	500	50%	50%	+/- 10%	Varies	One Horizon	1.1	100,000	Near 79%† † †
Dynamic	500	Varies	Varies	S&P 500 Z-Scores	EW	Multi-Horizon	1.1	10,000	Near 79%

Note: This table presents the parameters for each simulation that relaxes the assumptions of selectivity theory. N represents the number of stocks in the benchmark portfolio, Winners (m_1) represents the number of winner stocks in the benchmark, Losers (M_2) represents the number of loser stocks in the benchmark, Returns describes whether the simulation uses 10% for winners and -10% for losers or uses the actual stock returns of S&P 500 stocks normalized as a Z-score, Weight is for the mechanism to weight stocks, where EW is for equal-weighted, Holding Period is the typical holding period after a portfolio selection is made, ω represents the skill of the portfolio manager, N_{sims} is the number of simulations, and S^* is the optimal selectivity. † This number will vary depending on the cut-off point where the manager's skill ends. We chose to 30% as the cutoff point. †† This is only true for reasonable values of ω. For a very high ω, where the portfolio manager is extremely skilled, this optimal selectivity ratio will converge to the saturation point. † † † The optimal selectivity will converge to 79% if the appropriate benchmark is chosen based on the weighting of the benchmark.

7. Relaxing model assumptions

Baseline Simulation: 10,000 simulations with basic of original model.

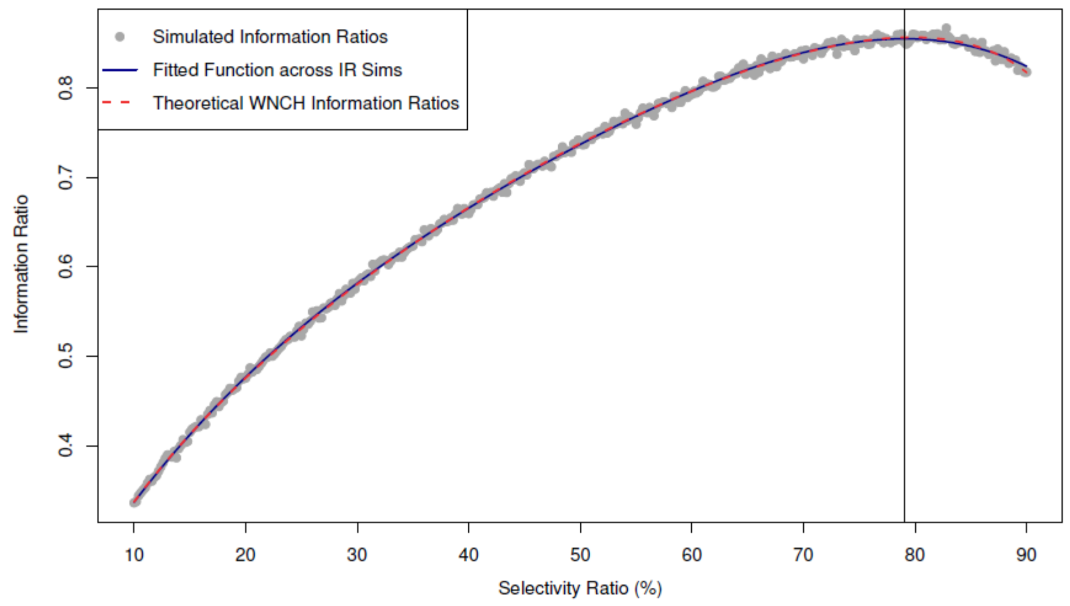


Figure 1: **Baseline Simulations.** This figure shows the baseline simulations of each selectivity ratio and its corresponding information ratio. The dots represent the average information ratio from 10,000 simulated portfolios at that particular selectivity ratio. Also shown is a fitted line across the simulation averages and the theoretical Wallenius information ratios for each selectivity ratio. The optimal selectivity ratio is at 79%. The parameters for the simulations are $N = 500$, $\omega = 1.1$, $r_g = 0.10$, $r_b = -0.10$, and $r_{BM} = 0$.

7. Relaxing model assumptions

Relax Assumption 1: In practice benchmarks are not required to have an equal number of winners and losers; the benchmark return will be calculated for whatever proportions exist.

Example: r(1) = +10%, r(2) = -10%, r(3) = -10% r(bm)= -3.33%. Only 1/3 of stocks are "winners."

In order to investigate the robustness of this assumption, we performed 100,000 Wallenius (sequential selection) simulations at each selectivity ratio, under different winner/loser proportions. The conclusions in this presentation have been also affirmed for the Fisher (bulk selection) simulations.

Each simulation randomly picks one stock at a time and recalculates the probabilities of the next pick based on how many winners/losers have been picked before. The simulation stops once the desired selectivity ratio has been reached.

7. Relaxing model assumptions

As the % of winners in the benchmark shrinks, the manager has fewer chances for their skill to shine. Their IR declines as a result.

Table 2: Portfolio Characteristics as Concentration of Winners Changes

	Percentage of Winners in Benchmark				
	50%	48%	40%	30%	20%
Benchmark Return (%)	0.000	-0.400	-2.000	-4.000	-6.000
Optimal Selectivity Ratio	79.0	79.0	79.4	79.4	79.6
A.R. at Optimal Selectivity	0.197	-0.203	-1.813	-3.839	-5.878
A.R. at 79% Selectivity	0.197	-0.203	-1.812	-3.836	-5.876
I.R. at Optimal Selectivity	0.855	0.854	0.851	0.791	0.697
I.R. at 79% Selectivity	0.855	0.854	0.846	0.795	0.694

Note: This table shows the results from 100,000 simulations for each selectivity level for the bulk selection method. The parameters for the simulation are $N = 500$, $\omega = 1.1$, $r_g = 10\%$, and $r_b = -10\%$. The percentage of winners in the benchmark are altered in each of the simulations. Thus, 50% corresponds to a benchmark with 250 winner stocks and 250 loser stocks, whereas 20% corresponds to a situation where the benchmark has only 100 winner stocks and 400 loser stocks and so on. The benchmark return is provided for convenience showing that when there are less than 50% winner stocks, the benchmark return is naturally negative. A.R. is for average return and I.R. is for information ratio.

However their optimal behavior is relatively unchanged:
IR is still maximized by holding ~80% of the benchmark.

7. Relaxing model assumptions

Relax Assumption 2: The empirical distribution of stock returns is much more continuous than the assumed binary, ±10% outcomes. It also typically has excess kurtosis and skewness.

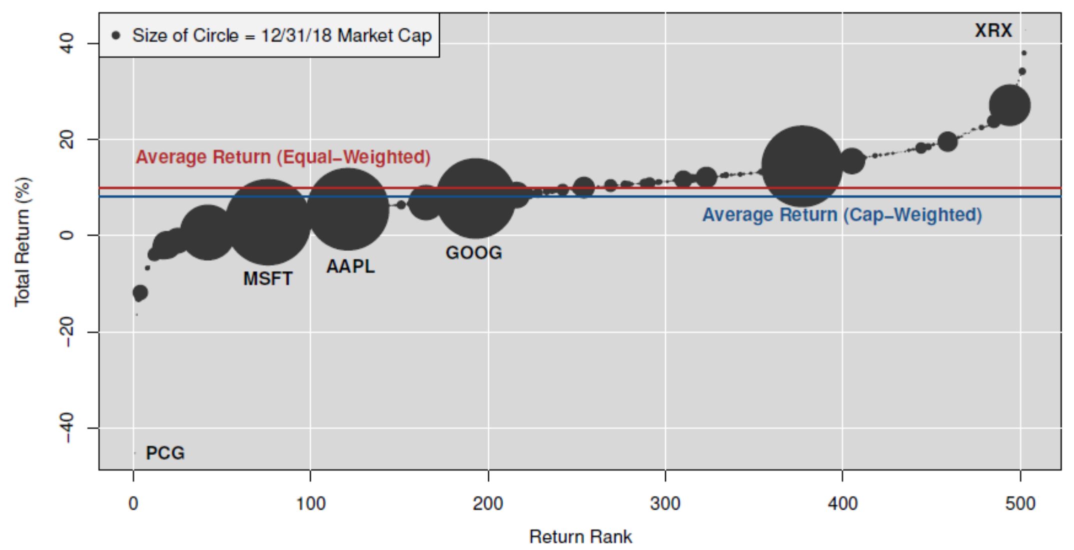


Figure 6: **Cross-Section of Individual S&P 500 Stock Returns for January 2019.** This figure shows the returns of individual stocks in the S&P 500 for the month of January 2019. The stocks are ordered from lowest return (1) to highest return (500). The companies are also represented by a circle which is proportional to their relative market capitalization as of December 31, 2018.

7. Relaxing model assumptions

We could relax this assumption in one given period (i.e. month) or across time and cross-section. Either way, it would not matter. We chose to compute Z-scores of each stock return (see Chincarini & Kim (2006)) for S&P 500 stocks each period and across time to get an "average" distribution of Z-scores (similar to relative returns).

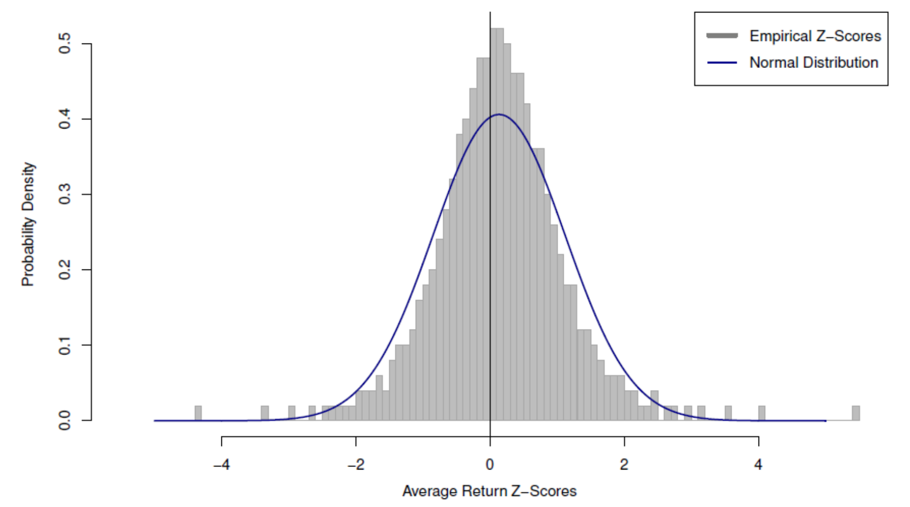


Figure 2: **Average Cross-Sectional Z-scores for S&P 500 Stock Returns.** This figure shows the distribution of Z-scores of stock returns for stocks in the S&P 500 from December 1988 to December 2018. In each period, cross-sectional Z-scores are created based on the returns of each stock in the S&P 500. This is repeated for every month in the sample period and the average Z-scores compute along with the frequency of occurrence. A normal distribution is also fitted to this histogram

7. Relaxing model assumptions

To simulate this effect, each benchmark stock has a Z-score "return" assigned to it based on the frequency distribution from the prior slide. As stocks are sampled from the benchmark their unique "returns" follow them into the manager's portfolio.

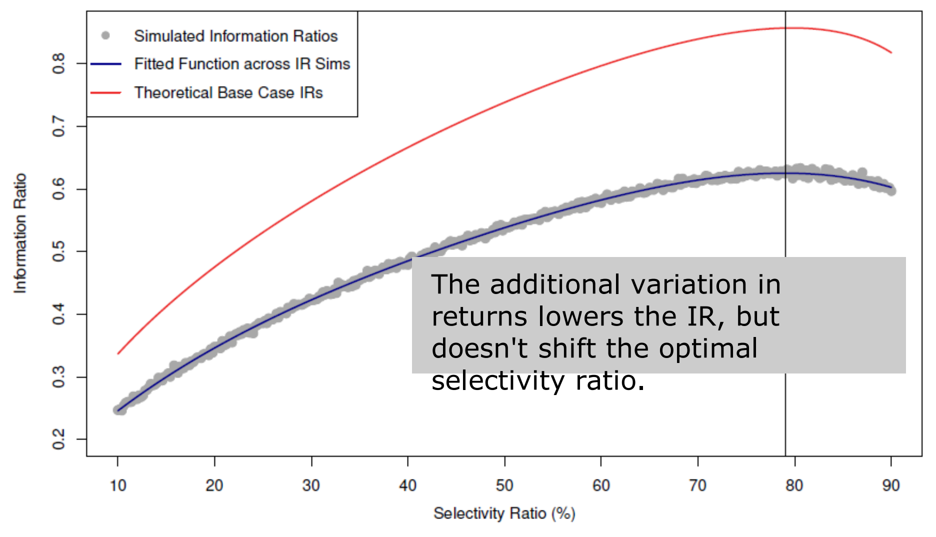


Figure 3: **Selectivity Ratio and information ratio for Empirical S&P 500 Returns.** This figure shows each selectivity ratio and its corresponding information ratio when returns are not binary for good and bad stocks, but are based on the cross-sectional Z-scores of actual S&P 500 individual stock returns. The dots represent the average information ratio from 10,000 simulated portfolios at that particular selectivity ratio. Also shown is a fitted line across the simulation averages and the theoretical Wallenius information ratios for each selectivity ratio. The parameters for the simulations are $N = 500$, $\omega = 1.1$, $r_i = \bar{z}_i$, and $r_{BM} = 0$.

7. Relaxing model assumptions

Relax Assumption 3: One might posit that a portfolio manager's skill in picking good stocks either (1) starts high and slowly declines; (2) only applies to a subset of the benchmark that they know well; (3) reaches some saturation point; or (4) follows a stochastic process that might reward or punish an expected level of skill.

Four scenarios have been studied through Monte Carlo simulation:

1. Sequentially declining skill.
2. Skill for only a subset of the winner population.
3. Skill saturation point.
4. Skill as a random variable.

7. Relaxing model assumptions

If the manager's skill declines linearly after each stock pick, the probability boost that typically comes from a sequential process (Wallenius) is overwhelmed by that deterioration. As a result the Information Ratios converge towards the bulk selection method (Fisher). **Steadily Declining Skill**

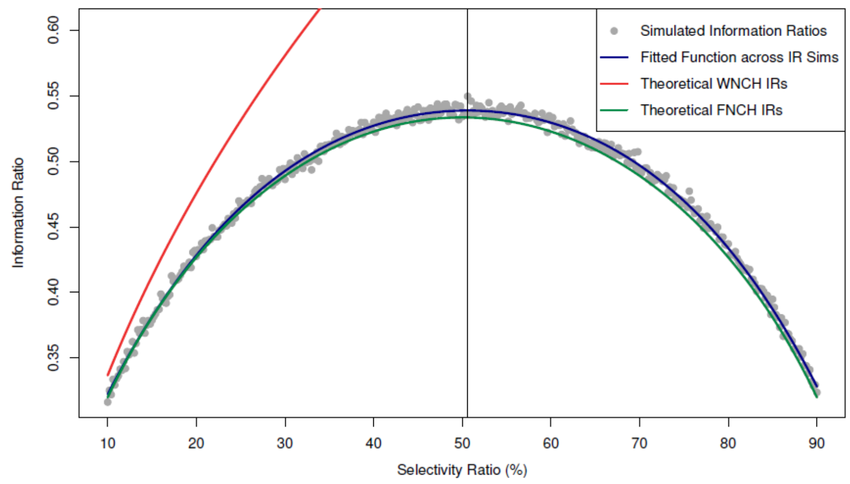


Figure 4: **Selectivity with Steadily Declining Manager Skill.** This figure shows the information ratios and selectivity levels when the manager's skill is steadily declining from $\omega = 1.1$ on the first stock pick to $\omega = 1$ by the last stock pick. The dots represent the average information ratio from 10,000 simulated portfolios at that particular selectivity ratio. Also shown is a fitted line across the simulation averages and the theoretical Wallenius information ratios for each selectivity ratio. The optimal selectivity ratio is at 50%. The parameters for the simulations are $N = 500$, $r_g = 0.10$, $r_b = -0.10$, and $r_{BM} = 0$.

7. Relaxing model assumptions

The other three scenarios produce intuitive results, for the most part maintaining the Wallenius properties. **Jack Knife**

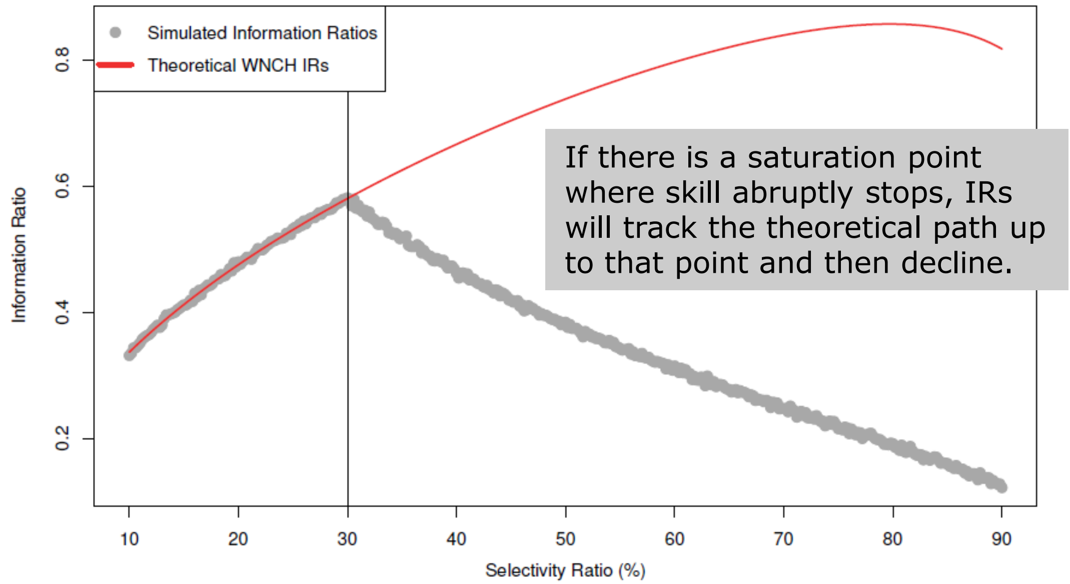


Figure 5: **Selectivity with Jack Knife Skill.** This figure shows each selectivity ratio and its corresponding information ratio when the portfolio manager's skill completely vanishes at a certain selectivity ratio. That is, the manager has skill until a certain selectivity and then it reverts to random. The dots represent the average information ratio from 10,000 simulated portfolios at that particular selectivity ratio. Also shown is a fitted line across the simulation averages and the theoretical Wallenius information ratios for each selectivity ratio. The parameters for the simulations are $N = 500$, $\omega = 1.1$ for $\phi = n/N < 0.30$, $\omega = 1$ for $\phi = n/N \geq 0.30$, $r_g = 0.10$, $r_b = -0.10$, and $r_{BM} = 0$.

7. Relaxing model assumptions

The other three scenarios produce intuitive results, for the most part maintaining the Wallenius properties. **Ability Saturation** (although if ability is strong enough, will converge to saturation point)

Table 3: Portfolio Characteristics with Winner Saturation

	Percentage of Winners Manager Can Identify				
	100%	80%	60%	40%	20%
Optimal Selectivity Ratio	79.0	78.2	77.0	76.2	75.0
A.R. at Optimal Selectivity	0.197	0.161	0.124	0.084	0.042
A.R. at 79% Selectivity	0.197	0.157	0.116	0.079	0.040
I.R. at Optimal Selectivity	0.855	0.681	0.508	0.334	0.161
I.R. at 79% Selectivity	0.855	0.683	0.506	0.342	0.174

Note: This table shows the results from 100,000 simulations for each selectivity level for the bulk selection method. The parameters for the simulation are $N = 500$, $\omega = 1.1$, $r_g = 10\%$, and $r_b = -10\%$. The percentage of winners in the benchmark are altered in each of the simulations. Thus, 50% corresponds to a benchmark with 250 winner stocks and 250 loser stocks, whereas 20% corresponds to a situation where the benchmark has only 100 winner stocks and 400 loser stocks and so on. The benchmark return is provided for convenience showing that when there are less than 50% winner stocks, the benchmark return is naturally negative. A.R. is for average return and I.R. is for information ratio.

7. Relaxing model assumptions

The other three scenarios produce intuitive results, for the most part maintaining the Wallenius properties. **Uncertain Omega**

Table 4: Portfolio Characteristics when Manager Skill is Stochastic

	Standard Deviation of Skill (σ)				
	0.00	0.05	0.10	0.15	0.20
Panel A: Skill Varies for Each Stock Selection					
Optimal Selectivity Ratio	79.0	78.8	79.0	79.0	79.2
A.R. at Optimal Selectivity	0.197	0.196	0.188	0.177	0.161
A.R. at 79% Selectivity	0.197	0.196	0.188	0.177	0.163
I.R. at Optimal Selectivity	0.855	0.841	0.816	0.766	0.701
I.R. at 79% Selectivity	0.855	0.851	0.816	0.766	0.705
Panel B: Skill Varies for Each Portfolio Simulation					
Optimal Selectivity Ratio	79.0	79.0	79.0	79.2	81.0
A.R. at Optimal Selectivity	0.197	0.195	0.187	0.173	0.150
A.R. at 79% Selectivity	0.197	0.195	0.187	0.178	0.160
I.R. at Optimal Selectivity	0.855	0.788	0.627	0.474	0.354
I.R. at 79% Selectivity	0.855	0.788	0.627	0.483	0.354

Note: This table shows the results from 100,000 simulations for each selectivity level for the bulk selection method when the skill of the manager is uncertain. The parameters for the simulation are $N = 500$, $\omega \sim N(1.1, \sigma^2)$, where σ varies in the table from 0.00 to 0.20, $r_g = 10\%$, and $r_b = -10\%$. The benchmark return is 0% for information ratio calculations. The standard deviation of skill, σ, is used in two ways. In Panel A, a random draw of skill occurs before a manager selects his group of stocks for each selectivity level. In Panel B, the manager's skill is selected before each simulation of all selectivity levels. A.R. is for average return and I.R. is for information ratio.

Skill volatility lowers the IRs that can be achieved, but it doesn't alter the Wallenius curvature.

Shown here: $E(\omega) = 1.1$; $\sigma(\omega) = 0.1$

7. Relaxing model assumptions

Relax Assumption 4: Many benchmarks (e.g. the S&P 500) are weighted according to market capitalizations. Such an approach can lead to a handful of stocks having an outsized gravitational pull on the benchmark's return.

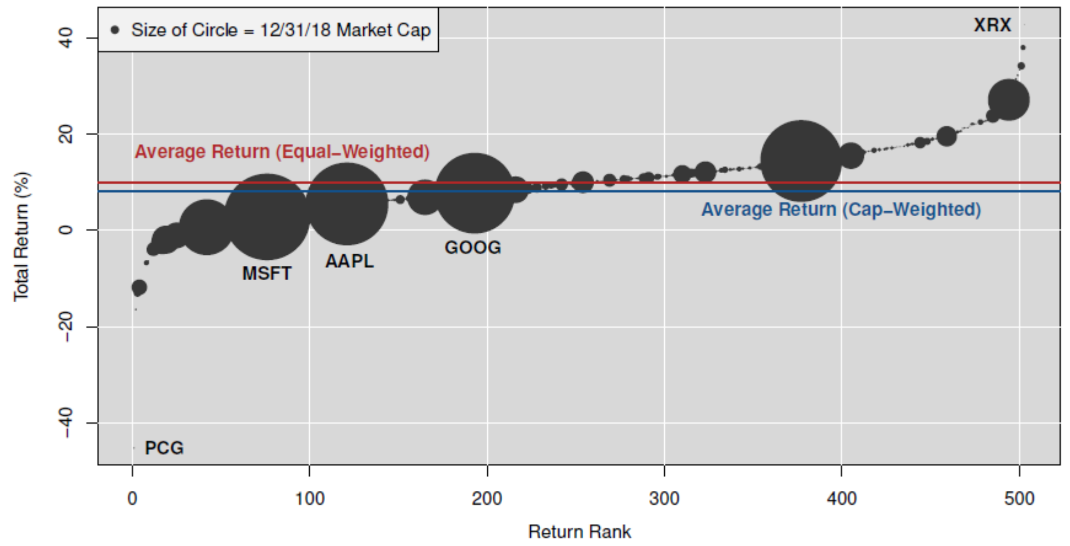


Figure 6: **Cross-Section of Individual S&P 500 Stock Returns for January 2019.** This figure shows the returns of individual stocks in the S&P 500 for the month of January 2019. The stocks are ordered from lowest return (1) to highest return (500). The companies are also represented by a circle which is proportional to their relative market capitalization as of December 31, 2018.

7. Relaxing model assumptions

Introducing cap-weighted effects into the simulation framework reduces the Information Ratios that the manager can achieve.

Table 7: Simulation Results for Market Capitalization Weighted Benchmarks

	Type of Market Cap Benchmark				
	Strong Large-Cap Tilt	Large-Cap Tilt	Neutral Size	Small-Cap Tilt	Strong Small-Cap Tilt
Panel A: Equally Weighted Portfolios					
Optimal Selectivity Ratio	10.0	10.0	78.8	90.0	90.0
A.R. at Optimal Selectivity	0.448	0.449	0.198	0.122	0.121
A.R. at 79% Selectivity	0.196	0.198	0.197	0.196	0.197
I.R. at Optimal Selectivity	-5.071	-0.595	0.861	9.246	48.947
I.R. at 79% Selectivity	-30.138	-4.559	0.851	6.260	31.848
Panel B: Market Capitalization Weighted Portfolios					
Optimal Selectivity Ratio	82.6	78.0	78.4	78.6	74.2
A.R. at Optimal Selectivity	7.231	1.448	0.203	-1.050	-7.032
A.R. at 79% Selectivity	7.241	1.444	0.201	-1.055	-7.046
I.R. at Optimal Selectivity	0.586	0.397	0.407	0.406	0.611
I.R. at 79% Selectivity	0.586	0.404	0.411	0.401	0.620
Benchmark Return	7.151	1.250	0.000	-1.250	-7.151

Note: This table shows the results from 100,000 simulations for each selectivity level for the bulk selection method using a market capitalization weighted index. The parameters for the simulation are $N = 500$, $\omega = 1.1$, $r_g = 10\%$, and $r_b = -10\%$. The portfolios are compared against five types of market capitalization benchmark, one in which all winner stocks are the largest 250 stocks in the index ("Strong Large-Cap Tilt"), one in which all of the winner returns are the smallest 250 stocks in the index ("Strong Small-Cap Tilt"), one in which the returns are distributed such that the market capitalization weighted returns are equal to 0 (i.e. $\left(\sum_{k=1}^{n_g} MC_k \approx \sum_{j=1}^{n_b} MC_j\right)$), and one which winners are slightly tilted towards big stocks ("Large-Cap Tilt") and one in which winner stocks are slightly tilted towards smaller companies ("Small-Cap Tilt"). Two type of portfolios are analyzed with respect to selectivity; an equally weighted portfolio and a market capitalization weighted portfolio. A.R. is for average return and I.R. is for information ratio.

Once again though, the optimal selectivity ratio to attain the best IR varies only slightly from the theoretical ~80%.

7. Relaxing model assumptions

SUMMARY: Relaxing the simplifying assumptions required for the theory does not significantly alter its theoretical conclusion of which selectivity ratio maximizes the manager's Information Ratio.

In fact, the consistency of results across different assumptions reinforces the validity of the theory.

7. Summary

- Traditionally, enhanced portfolio management has considered the tracking error with a benchmark and mean-variance type optimization, as well as other ad hoc techniques to find OPTIMAL portfolios.

- Our approach is a new one. First, it asks what percentage of a benchmark should the manager choose to maximize Information Ratio. Second, it uses some concepts never applied to these portfolio problems (according to the best of our knowledge).

- The results are extremely interesting, in that, many enhanced managers do not hold such a large portion of their benchmark (50 to 80%).

8. FURTHER RESEARCH

- For Selectivity Theory, working on showing practical use with manager selection.

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3. Chincarini, Ludwig B. and Daehwan Kim. *Quantitative Equity Portfolio Management*. New York, McGraw-Hill, 2006. *Note*: New Edition should be released in 2021 with lots of new stuff.

Thank you

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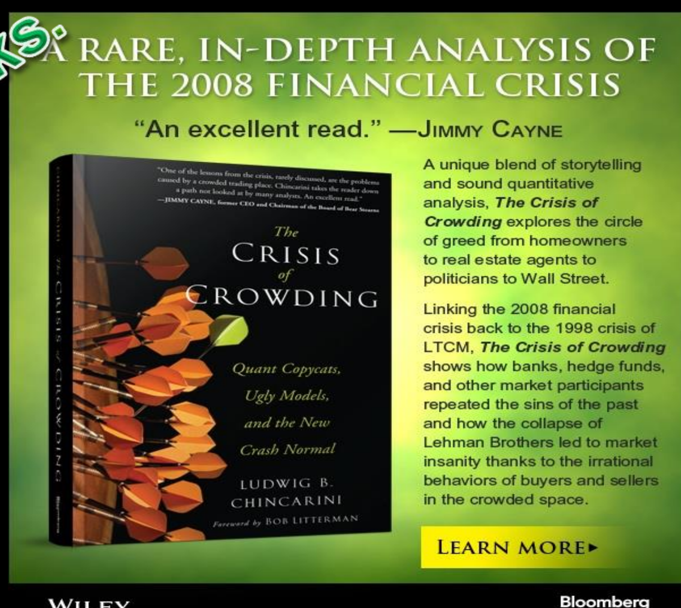

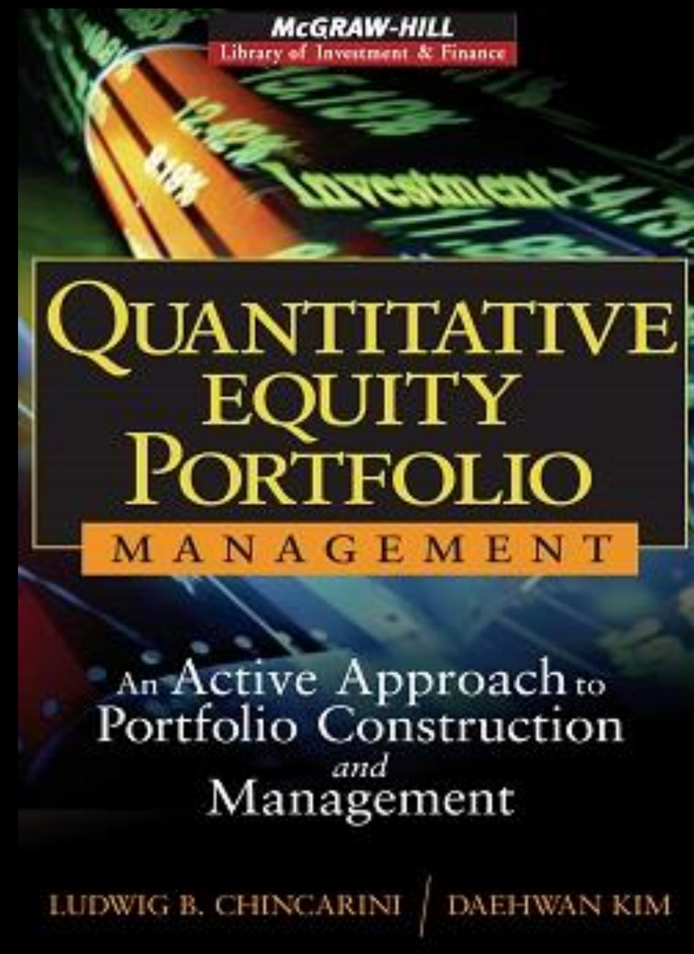

Open Discussion for all Participants