Journal of Empirical Finance 47 (2018) 190–206

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Portfolio construction and crowding

Salvatore Bruno [a], Ludwig B. Chincarini [b,*], Frank Ohara [b]

[a] IndexIQ, 800 Westchester Avenue, Suite N-611, Rye Brook, NY, 10573, United States
[b] University of San Francisco, School of Management, 101 Howard Street, San Francisco, CA 94105, United States

ARTICLE INFO

JEL classification:
G0
G01
G02
G11

Keywords:
Risk management
Crowding
Herding
Crowded spaces
Copycat trading
Quantitative equity portfolio management
Optimal portfolios

ABSTRACT

We use industry data to investigate how the crowding of an equity space develops due to the portfolio construction process in the equity asset management sector. We find crowding can be reduced by slightly altering the risk management process. We also find that crowding in the financial system could be lower if the distribution of risk model usage amongst portfolio managers was more diversified.

1. Introduction

Financial crises have various causes, but they are frequently caused or amplified by trade crowding in the investment space (Chincarini, 2012). Crowding can occur when investors create similarly concentrated portfolios using correlated strategies. As investors follow similar trading models, it is likely that their resulting portfolios will be very similar. They may arrive at similar models either by coincidental or outright replication of each other's ideas.

Crowding may also occur when investors use similar techniques to construct their portfolios. For example, investors can have different models for generating their expected returns, but using the same portfolio risk models and portfolio construction techniques can also cause their portfolios to converge. As a result, crowded spaces alter the perceived risk and return dynamics of the investment space. In crowded spaces, the action of traders is the primary determinant of price movements. This interconnectedness in crowded spaces leads to systemic effects that make the risk of the entire investment space greater than traditional risk measures would suggest (Menkveld, 2014). If one investor suffers a negative shock, this can force the investor to reduce his positions, which puts pressure on the positions of other investors with similar holdings, causing the shock to spread throughout the trading community, and ultimately causing a much larger aggregate shock (Brunnermeier and Pedersen, 2005). For investors who use leverage, the problems associated with crowding can be even larger. As positions move against an investor, funding becomes more difficult as counterparties request higher margin collateral which can ultimately lead to liquidity spirals (Brunnermeier and Pedersen, 2009). The risk of a trade is endogenous to the trade itself and when traders cannot estimate the concentration of similar trades by other arbitrageurs, they may mismeasure the risk of the position (Chincarini, 1998; Stein, 2009).

* Corresponding author.
E-mail addresses: sbruno@IndexIQ.com (S. Bruno), lbchincarini@usfca.edu (L.B. Chincarini), ohara@usfca.edu (F. Ohara).
URL: http://www.ludwigbc.com (L.B. Chincarini).

https://doi.org/10.1016/j.jempfin.2018.02.003
Received 26 September 2016; Accepted 26 February 2018
Available online 9 March 2018
0927-5398/

Recent research has shown that stocks that are connected by shared ownership can have higher correlation when selling occurs (Anton and Polk, 2014; Bohlin and Rosvall, 2014; Ibbotson and Idzorek, 2014; Yan, 2013). There have been some initial attempts to measure crowding (Blocher, 2011; Pojarliev and Levich, 2011; Marmer, 2015). Some of these measures include the percentage of overlap of similar positions and the correlations between portfolios, which are now also being produced in standard investment bank reports (Amenta, 2013a, b; Cahan and Luo, 2013; Subramanian, 2013; Sneider et al., 2012).

Our paper investigates how crowding occurs through the portfolio construction process in an attempt to understand the linkages between portfolio construction and crowding. We focus on the equity industry, where many portfolio managers use similar techniques to construct portfolios.[1]

By using standard equity portfolio construction procedures and risk models, we show that even when managers have unrelated methods of picking stocks, they can have very similar portfolios (crowding) due to their use of similar risk models and portfolio construction methodology. This form of crowding which occurs through the portfolio building process, to our knowledge, has never been studied before.

We find that the extent of crowding is more severe for long only managers, than market neutral or long-short managers. We show that a relatively simple modification of the variance–covariance matrix using the Marchenko–Pastur adjustment can reduce the crowding by anywhere from 14% to 60%.

We also find that a financial system in which portfolio managers use primarily one risk model can create more crowding than a system in which portfolio managers divide their usage equally across risk models. This has important implications for the structure of the financial system.

We make three contributions to the literature on crowding. First, we identify a neglected source of crowding risk that is created from the portfolio construction process. Second, we document how the distribution of risk model usage in the financial system could increase crowding and systemic risk. Third, we suggest two methods that could reduce crowding in the financial system. One method is an action that individual portfolio managers could implement. The other is a suggestion for the equilibrium distribution of risk model usage in the equity industry.

The paper is organized as follows: we define a measure of crowding that will be used in this paper and attempt to explain the intuition behind portfolio construction and crowding using a simple optimization example. We then describe our empirical framework for examining the crowding from portfolio construction and also describe our empirical simulation results. Finally, we discuss how the distribution of risk models in the financial system could lead to overall crowding.

2. A definition of crowding

For the purposes of this paper we define crowding to be when investors own portfolios with similar holdings. Let the similarity between two portfolios be measured by s_{ij}, which is the dot product between the position weight vectors ($\mathbf{w}$) of each portfolio i and j divided by the product of the Euclidean norm of each vector. Thus,

$$s_{ij} = \frac{\mathbf{w}_i'\mathbf{w}_j}{|\mathbf{w}_i||\mathbf{w}_j|} \tag{1}$$

where the Euclidean norm is defined across N assets as

$$|\mathbf{w}_i| = \sqrt{\sum_{n=1}^{N} \mathbf{w}_{in}^2} \tag{2}$$

This measure will have a value between 0 and 1 for portfolios that can only be long securities (i.e. long only portfolios). This measure will have a value between -1 and 1 for portfolios that can have negative weights.[2]

In our paper, we will study more than just two portfolios. Thus, for studying a group of M portfolios, we define the N-by-M portfolio holdings matrix as the matrix, H, which consists of columns of position weight vectors on N assets for each of M portfolios. The similarity matrix amongst all portfolios is computed as

$$S = \left(H'H\right) \circ \hat{\hat{H}} \tag{3}$$

where $\circ$ represents the Hadamard product or the element-by-element multiplication of the matrices, and

$$\hat{\hat{H}} = \begin{bmatrix} \frac{1}{\hat{h}_{11}} & \frac{1}{\hat{h}_{12}} & \dots & \frac{1}{\hat{h}_{1M}} \\ \dots & \dots & \dots & \dots \\ \frac{1}{\hat{h}_{M1}} & \frac{1}{\hat{h}_{M2}} & \dots & \frac{1}{\hat{h}_{MM}} \end{bmatrix} \tag{4}$$

[1] For example, MSCI Barra, Northfield, and Factset produce standard risk models that are widely used by many participants.

[2] This measure is related to a more commonly used measure known as Pearson correlation. One can think of Pearson correlation as a de-meaned version of Cosine Similarity.

and $\hat{H} = |H|'|H|$, where $|H|$ contains the Euclidean norm of each manager's weight vector. The matrix S contains the similarities of each portfolio with every other portfolio. For example, element S_{12} represents the similarity of the portfolios of managers 1 and 2. For a specific set of portfolios, our measure of crowding is given by the average of the off-diagonal elements of this matrix.[3]

From the similarity matrix of M portfolios or portfolio managers, we measure the crowding, C, amongst the group of portfolios as the average similarity between portfolios.[4]

$$C = \frac{\sum_{i=1}^{M}\sum_{j=1}^{M} S_{i,j} - M}{M^2 - M} \tag{5}$$

A simple example with a universe of three portfolios holding 3 stocks each might help to illustrate the concept of crowding. Suppose our matrix H of manager holdings is given as

$$H = \begin{bmatrix} 0.4 & 0.8 & 0.45 \\ 0.4 & 0.1 & 0.45 \\ 0.2 & 0.1 & 0.10 \end{bmatrix}. \tag{6}$$

This example includes the portfolios of 3 managers. Each manager has a portfolio whose holdings sum to 1. The portfolio of manager 1 has 40% in stock 1, 40% in stock 2, and 20% in stock 3. Portfolio 2 has 80% in stock 1 and 10% in stocks 2 and 3. Portfolio 3 has 45% in stocks 1 and 2 and 10% in stock 3.

One can see that manager 1 and manager 3 have very similar or "crowded" portfolios. Manager 2's portfolio is less related to the other two. Using our formula for computing the similarity matrix, we find that[5]

$$S = \begin{bmatrix} 1 & 0.7796 & 0.9831 \\ . & 1 & 0.7930 \\ . & . & 1 \end{bmatrix}. \tag{7}$$

The resulting similarity matrix corresponds with our intuition. That is, portfolios 1 and 2 have a similarity measure of 0.7796, which is high, but not as high as portfolios 1 and 3, which have a measure of 0.9831. For this universe of portfolios, the crowding measure is $C = 0.8519$. This indicates that there is a high level of similarity or crowding in the investment space from these 3 portfolio managers.

In addition to capturing the crowding of a group of portfolios or portfolio manager holdings, we also wish to study specifically how the portfolio construction process creates additional crowding in the investment space. One way to do this is to measure the crowding of portfolios *before* the portfolio construction process and *after* the portfolio construction process. Specifically, if we were able to observe the expected return models or alpha models of portfolio managers before they assigned weights to their portfolios, we could infer the amount of crowding that is added or removed from portfolio construction techniques.

Let us define S_α as the similarity matrix of portfolio managers from their alpha models. This is the similarity of their stock picking models, whether quantitative or qualitative managers. Define S_p as the similarity of portfolios after the manager has combined his alpha model with his optimization model to construct his final portfolio. Thus, S_p is the similarity matrix of actual portfolio holdings. Both measures are computed as described previously. For both S_α and S_p, the crowding measures are also computed and given by C_α and C_p. In our analysis, we will compute the ratio of these two as

$$\Omega = \frac{C_p}{C_\alpha}. \tag{8}$$

When this ratio is greater than one, it means that the portfolio construction process has caused portfolios to become more crowded than they were just from the different portfolio manager beliefs about the attractiveness of different stocks and vice versa. In other words, this metric represents how much more similar on average the portfolios are than the expected return models.[6]

---

[3] The diagonal elements are the similarity of each portfolio with itself, which are irrelevant. Our measure of the similarity of portfolios to measure crowding is related to a more commonly known cosine similarity, which is a measure of the similarity between two vectors of an inner product space that measures the cosine of the angle between them. This measure is given as $\theta = \cos^{-1}\left(\frac{\mathbf{w}_i'\mathbf{w}_j}{\|\mathbf{w}_i'\|\|\mathbf{w}_j\|}\right)$.

[4] Essentially, the numerator represents the summation of all the similarities between every portfolio manager and every other, including its own. By subtracting m, we normalize this measure to be the average similarity in excess of a group of portfolio managers that are completely dissimilar to each other. In that case, the similarity matrix would be a diagonal of 1s.

[5] The components of S are given by,

$$H'H = \begin{bmatrix} 0.36 & 0.38 & 0.38 \\ . & 0.66 & 0.415 \\ . & . & 0.415 \end{bmatrix}.$$

$$\hat{H} = \begin{bmatrix} 0.36 & 0.4874 & 0.3865 \\ . & 0.66 & 0.5234 \\ . & . & 0.4150 \end{bmatrix}.$$

and

$$\hat{\hat{H}} = \begin{bmatrix} 2.778 & 2.0515 & 2.5872 \\ . & 1.5152 & 1.9108 \\ . & . & 2.4096 \end{bmatrix}.$$

[6] We could have also taken the average of the absolute values in this similarity matrix. This would not be as representative of crowding itself, but would also be important for a measure of financial fragility. That is if 50% of managers are long a portfolio and 50% of managers are short a portfolio, our current measure would

In our simulation analysis, we will look at the crowding of portfolios, C, as well as the ratio of crowding before and after portfolio construction, Ω.

3. The intuition linking portfolio construction and crowding

As mentioned at the outset, this paper investigates to what extent the portfolio construction process can lead to crowding. Specifically, the paper is concerned with how the use of similar risk models and portfolio construction techniques by portfolio managers might cause otherwise unrelated models of stock picking to become related. That is, whether or not the optimization process could create additional crowding of portfolios. In the empirical examination of this issue, we will use actual risk models used in the asset management industry, but in this section we will consider a simple historical variance–covariance matrix to help gain an intuition of how the optimization process might lead to crowding.

3.1. A basic portfolio optimization

Consider a set of investors who all use the same empirical covariance matrix as their risk model, but who use different and uncorrelated models to generate expected returns, and who use a standard, unconstrained mean–variance optimization (MVO) (i.e. there are no no-short constraints, sector weighting constraints, no stock weight constraints, or any other type of constraints) to construct portfolios. The solution to the unconstrained MVO is:

$$\mathbf{w} = \boldsymbol{\Sigma}_{\boldsymbol{r}}^{-1}\alpha \tag{9}$$

where $\boldsymbol{\Sigma}_{\boldsymbol{r}}^{-1}$ is the inverse of a variance–covariance matrix of asset returns, α is a vector of expected returns of the assets, and $\mathbf{w}$ is the resulting weights of the optimal portfolio. The risk aversion coefficient, λ, has been set such that no constants appear in the equation here without loss of generality.[7]

3.2. Decomposition of the covariance matrix

In the unconstrained MVO, the solution for the optimal weights is given by the inverse of the variance–covariance matrix multiplied by a vector of expected returns as in Eq. (9). One can decompose any real symmetric matrix using singular value decomposition or principal component analysis (Pearson, 1901; Hotelling, 1933; Roweis and Saul, 2000; Tenebaum et al., 2000; Jolliffe, 2002). Thus, we can decompose the inverse of our variance–covariance matrix as follows:

$$\boldsymbol{\Sigma}_{\boldsymbol{r}}{}^{-1} = \mathbf{U}\boldsymbol{\Lambda}^{-1}\mathbf{U}' \tag{10}$$

where the matrix $\mathbf{U}$ is a unitary matrix[8] whose columns are eigenvectors of $\boldsymbol{\Sigma}_{\boldsymbol{r}}^{-1}$ and $\boldsymbol{\Lambda}^{-1}$ is a diagonal matrix whose diagonal elements are the eigenvalues of $\boldsymbol{\Sigma}_{\boldsymbol{r}}^{-1}$.

We can now expand the expression for optimal portfolio weights, Eq. (9), as

$$\mathbf{w} = \boldsymbol{\Sigma}_{r}^{-1}\alpha = \left(\mathbf{U}\boldsymbol{\Lambda}^{-1}\mathbf{U}'\right)\alpha \tag{11}$$

This allows us to interpret the optimal portfolio, $\mathbf{w}$, slightly differently. The MVO solution $\boldsymbol{\Sigma}_{r}^{-1}\alpha$, projects α along the eigenvectors of $\boldsymbol{\Sigma}^{-1}$, and so the optimal portfolio that results can be seen as a mixture of the eigenvectors of $\boldsymbol{\Sigma}_{r}^{-1}$. In other words, we can interpret eigenvectors as portfolios.

Let us explore our interpretation of eigenvectors and these optimal portfolios further. First, let us establish the relationship between the eigenvectors and eigenvalues of $\boldsymbol{\Sigma}_{r}^{-1}$ and $\boldsymbol{\Sigma}_{r}$.

$$\boldsymbol{\Sigma}_{r} = (\boldsymbol{\Sigma}_{r}^{-1})^{-1} = \left(\mathbf{U}\boldsymbol{\Lambda}^{-1}\mathbf{U}'\right)^{-1} = (\mathbf{U}')^{-1}\boldsymbol{\Lambda}^{-1^{-1}}\mathbf{U}^{-1} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}' \tag{12}$$

where we have used the fact that $\mathbf{U}$ is unitary and that $\boldsymbol{\Lambda}^{-1}$ is a diagonal matrix. Specifically, $\boldsymbol{\Lambda}$ will be a diagonal matrix whose diagonal entries are the reciprocal of the diagonal entries of $\boldsymbol{\Lambda}^{-1}$. Thus we have established that the eigenvectors of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{r}}{}^{-1}$ are identical, and the eigenvalues of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$ are the reciprocals of the eigenvalues of $\boldsymbol{\Sigma}_{\boldsymbol{r}}{}^{-1}$ and vice versa.

If we can interpret eigenvectors as portfolios, then the eigenvalues can be interpreted as the variance of the eigenvector portfolios. We illustrate this below. The eigenvectors and eigenvalues of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$ are defined by

$$\boldsymbol{\Sigma}_{\boldsymbol{r}}u = \lambda u \tag{13}$$

have a lower level of crowding than the absolute measure. However, this particularly extreme case might indicate a very fragile financial system when crowding is considered in this broader context. One could also consider measuring the absolute value of stock weights when computing the similarity matrix, however, this would not represent crowding as much as it would represent activity in similar stocks.

[7] The standard solution to the mean–variance problem is $w = \frac{1}{\lambda}\Sigma^{-1}\mu$, where μ is the set of expected returns, and λ is a risk-aversion parameter. It is the solution to the optimization problem $\min_{\mathbf{w}} \mathbf{w}'\mu - \lambda\mathbf{w}'\Sigma\mathbf{w}$. Allowing $\lambda = 1$ gives our result.

[8] A unitary matrix is a square matrix $\mathbf{U}$ such that the conjugate transpose of $\mathbf{U}$ equals the inverse of $\mathbf{U}$. Thus, when working with real numbers instead of the complex number system, a unitary matrix is just an orthogonal matrix. An orthogonal matrix is a square matrix $\mathbf{U}$ such that the transpose of $\mathbf{U}$ equals the inverse of $\mathbf{U}$ (i.e. $\mathbf{U}\mathbf{U}' = \mathbf{I}$). This implies that the columns of U are orthonormal (i.e. that $u_i' u_j = 1$ if $i = j$ and 0 otherwise for all columns of $\mathbf{U}$).

where the eigenvectors, u, are the columns of the matrix $\mathbf{U}$, and λ are the scalar diagonal entries of the matrix $\boldsymbol{\Lambda}$. Multiplying both sides of Eq. (13) by u', we have that

$$u'\boldsymbol{\Sigma}_{\boldsymbol{r}}u = u'\lambda u = \lambda u'u = \lambda \tag{14}$$

Using our interpretation of eigenvectors, u, as portfolios, we now recognize the left hand side of Eq. (14) as the equation for the variance of a portfolio, which implies that we should interpret the right hand side in the same way. In other words, eigenvalues are variances of the portfolios defined by the eigenvectors.

As we saw above, we can interpret the optimal portfolio $\mathbf{w}$ as a projection of α along the eigenvectors of ${\boldsymbol{\Sigma}_{\boldsymbol{r}}}^{-1}$, which we have also established are the eigenvectors of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$. From the definition of eigenvectors and eigenvalues in Eq. (13), we can see that this projection will be the strongest along the eigenvector associated with the largest eigenvalue of ${\boldsymbol{\Sigma}_{\boldsymbol{r}}}^{-1}$ and weakest along the eigenvector associated with the smallest eigenvalue of ${\boldsymbol{\Sigma}_{\boldsymbol{r}}}^{-1}$. We now also know that the largest eigenvalue of ${\boldsymbol{\Sigma}_{\boldsymbol{r}}}^{-1}$ is the smallest eigenvalue of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$, and the smallest eigenvalue of ${\boldsymbol{\Sigma}_{\boldsymbol{r}}}^{-1}$ is the largest eigenvalue of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$.

Thus, the optimized portfolios are converging to the eigenvector associated with the smallest eigenvalue of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$. This implies that, in the limit, no other eigenvector of the variance–covariance matrix will impact the resulting portfolios. In other words, in a standard portfolio optimization problem, expected returns will matter relatively little with respect to the variance–covariance matrix and only a small slice of the risk model matters. This convergence towards one type of portfolio is what contributes to the crowding of portfolios from the portfolio construction process.

3.3. The behavior of the eigenvectors

It is disturbing that the portfolio optimization places a lower significance on expected returns. Crowding arises partially from the convergence to a portfolio that has the lowest variance in the historical data. However, if the portfolios are converging to a portfolio that is indeed optimal in the future, then maybe the tradeoff of greater crowding is warranted.

One way to analyze the importance of these historically constructed eigenvectors is to examine whether the historical eigenvectors are statistically significant or simply noise. One method to differentiate the significance of the eigenvectors from "noise" is to use Random Matrix Theory (RMT) (Marchenko and Pastur, 1967). RMT allows us to compare the distribution of eigenvalues of our covariance matrix to the theoretical distribution of eigenvalues of randomly generated matrices. This distribution of eigenvalues from random matrices is known as the Marchenko–Pastur distribution (Marchenko and Pastur, 1967). One can infer that eigenvalues that fall outside of the theoretical distribution are "non-random" whereas eigenvalues that fall inside are "random". RMT has been successfully applied to financial problems in the last few years (Laloux et al., 2000; Golubi and Guo, 2012; Plerou et al., 2000; Bai et al., 2009; Eoma et al., 2009; Sharifi et al., 2004).

The Marchenko–Pastur distribution describes the distribution of eigenvalues for the matrix $C = \frac{1}{T}X'X$ where X is a $T \times N$ random matrix where each row is chosen from the multivariate standard Gaussian: $X \sim N(0, I)$. For a standard multivariate Gaussian, the distribution is defined by $\frac{\sqrt{(\boldsymbol{\Lambda}_{+}-x)(x-\boldsymbol{\Lambda}_{-})}}{2\cdot\pi\cdot p\cdot x}$ for $\boldsymbol{\Lambda}_{-} \leq x \leq \boldsymbol{\Lambda}_{+}$ and 0 otherwise, where $p = T/N$, and $\boldsymbol{\Lambda}_{+} = (1+\sqrt{p})^2$ and $\boldsymbol{\Lambda}_{-} = (1-\sqrt{p})^2$ are known as the upper and lower supports respectively.

When studying the relationship between eigenvector significance with respect to the Marchenko and Pastur (MP) distribution, one will find that the smallest eigenvalues typically represent random noise. For example, we examined a variance–covariance matrix of the top 200 stocks in the S&P 500 and found that of the 200 eigenvectors, only 9 (one of them is very far out on the x-axis and not seen in the figure) would be found to be non-random (see Fig. 1).

Conjecture 1 (*Convergence to Noise*). *The smallest eigenvalues to which the optimal portfolio converges towards is governed by something that is indistinguishable from random.*

When studying these significant eigenvalues in a dynamic setting, researchers will find that the number of significant eigenvectors has declined over the period 2000 to the present, most strikingly between 2007 to 2010, indicating that equity markets are looking more and more random over time. One way of interpreting this result is that the non-randomness of the markets, i.e. the part of market movement that is predictable, has been declining. In other words, markets have been getting more and more efficient over the last decade.

Conjecture 2 (*Efficient Markets*). *The number of significant eigenvalues in the equity markets has been declining suggesting that the U.S. equity markets have become more efficient over the period from* 2000 *to* 2012.

Researchers that study the eigenvectors and eigenvalues over time will also find that the last eigenvector (i.e. the one with the smallest eigenvalue) declines in its ability to represent the future much faster than the higher-order eigenvectors. One will also find the smallest eigenvalues are very poor estimates of future volatility and consistently under-predict volatility, while the largest eigenvalues are relatively better estimates of future volatility (Menchero et al., 2011).[9] Our research found that simple optimization procedures

[9] This tendency of the ex-ante variance–covariance matrix to underestimate volatility was noted earlier in Chincarini and Kim (2006). Michaud did not mention this specific bias, but pointed out the tendency of mean–variance to maximize errors rather than provide optimal portfolios (Michaud, 1989).

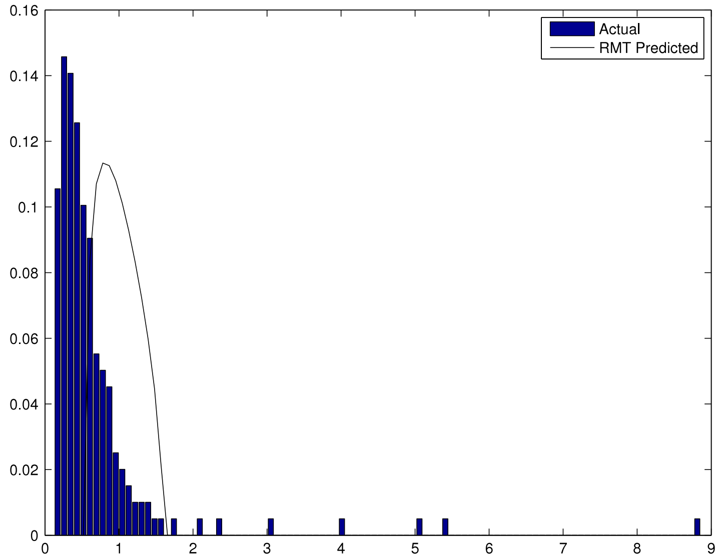

Fig. 1. Close-Up of Eigenvalues from Simple Optimization Example and Marchenko–Pastur Distribution. This figure represents a histogram of eigenvalues from a principal decomposition of the variance–covariance matrix of asset returns from a portfolio of stocks in the S&P 500 compared to the Marchenko–Pastur distribution.

underestimate the risk of the eigenvectors by over two-times and sometimes as high as forty-times when a shorter historical sample period is used to estimate the variance–covariance matrix.[10]

Conjecture 3 (*Underestimation of Risk*)**.** *Standard optimization techniques will lead to an ex-ante volatility underestimated by risk models.*

3.4. An intuitive example

A simple example with three assets may help give an intuitive feel for why expected returns are less important in constructing the optimal portfolio than the variance–covariance matrix in certain situations resulting in convergence towards similar portfolios.

Let us assume that we have a three-stock portfolio. Assume that two of the assets are highly correlated at 0.9 but that these two assets are completely uncorrelated with the third asset. Lets further assume that the variance of all of these assets is the same and is equal to 1. Our covariance matrix will thus be[11]

$$\begin{bmatrix} 1 & 0.9 & 0.0 \\ 0.9 & 1 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \tag{15}$$

and our inverse covariance matrix will be

$$\begin{bmatrix} 5.26 & -4.74 & 0.00 \\ -4.74 & 5.26 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{16}$$

The eigenvalues and corresponding eigenvectors of the covariance matrix are

$$\text{Eigenvalues} = \begin{bmatrix} 1.9 & 1.0 & 0.1 \end{bmatrix} \tag{17}$$

$$\text{Eigenvectors} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \tag{18}$$

[10] This analysis was performed as described before. We used the top 200 stocks in the S&P 500 and constructed the variance–covariance matrix using simple historical returns as of December 2013. We used different look-back periods to construct the variance–covariance matrix from 200 to 1000 days. We then computed the ratio of 50-day rolling volatility of the various eigenvectors to their eigenvalue when the eigenvector was formed. Two-hundred days after portfolio formation, this ratio was between 15 and 40 for the smallest eigenvectors and between 1 and 2 for the largest eigenvectors. Even after 50 days, it was between 7 and 20 for the smallest eigenvectors.

[11] The reader should note that when the variances of the assets equal 1, then the covariance and correlation are equal, since $\rho_{ij} = \frac{Cov(i,j)}{\sigma_i \sigma_j}$.

The riskiest eigenvector portfolio, i.e. the first eigenvector, is a portfolio consisting of equal holdings in assets 1 and 2.[12] The second riskiest eigenvector portfolio, i.e. the second eigenvector, is a portfolio consisting of only the uncorrelated asset. The variance of this portfolio is just the variance of that asset which is 1. Finally, the least risky eigenvector portfolio is a portfolio consisting of a long/short spread position between the first two assets, i.e. long asset 1 and short asset 2. This portfolio does not have much risk because with the two assets being highly correlated, any returns in the long position in asset 1 will be mostly offset by returns in the short position in asset 2.[13]

If we use this covariance matrix for portfolio optimization, we can see how it will affect the resulting portfolios. Assume that we expect a 10% return from each asset and we constrain the portfolio weights to sum to 1. The resulting portfolio will then hold 25.6% in assets 1 and 2 and 48.7% in asset 3. Suppose we make a slight adjustment to the expected return for asset 1 making its expected return 12% while leaving assets 2 and 3 at 10%. The new optimized portfolio has 73.2% in asset 1, −19.5% in asset 2, and 46.3% in asset 3. The optimization puts more weight into the spread between assets 1 and 2 because that is ex-ante the least risky trade. If we also set the expected return of asset 3 to 12%, the portfolio has 70.0% in asset 1, −17.9% in asset 2, and 50.9% in asset 3. For the same change in expected return for asset 3 that was previously made for asset 1, there is barely any change to the resulting portfolio. In fact, as long as there is any meaningful difference between the expected returns of assets 1 and 2, the resulting portfolio will always be dominated by a spread trade between these two assets, regardless of the expected return for asset 3. The reason is that the spread position between assets 1 and 2 is historically a low risk portfolio and will hence be an important part of the final portfolio.

4. The empirical framework

Our strategy to analyze the real-world implications of crowding from the portfolio construction process is to simulate the construction of portfolios using random alpha signals combined with real-world risk models and examine the extent of crowding that occurs. The data for our empirical study was obtained from several sources. We obtained all stock return data from Factset. We obtained our risk model data from the three major risk model providers in the financial industry; Barra, Axioma, and Northfield.

In this section, we discuss the techniques of our simulation process, including the creation of our expected return or α models for stocks, the risk models, and the portfolio construction techniques.

4.1. Portfolio construction

In order to examine the extent of crowding from the portfolio construction process, we created portfolios that were more common in the professional investment world. We considered two types of portfolio management techniques; a long only portfolio and a market neutral portfolio.

The long only portfolio manager maximized expected return (i.e. α) while keeping a portfolio that has a volatility equal to the historical volatility of the S&P 500, is not levered, has a maximum stock weight of 10% in any one name, and whose sector composition matches that of the benchmark.[14] We also considered the same long only portfolio manager, however, rather than maximize alpha subject to a risk target, the portfolio manager minimized risk subject to an alpha target.[15]

The market neutral manager maximizes expected return subject to having no more than 5% volatility over the risk-free rate, a dollar-neutral portfolio (i.e. the weights of the longs sum to the weights of the shorts), a leverage of 2 (i.e. the weights of the long portfolio sum to 1 and the weights of the short portfolio sum to 1), that no stock can have a weight less than −10% or more than 10%, and that the stocks are sector neutral with respect to the long and short side of the portfolio.[16] We also considered a market neutral manager that minimized the volatility of the portfolio subject to a target alpha.[17] For the market neutral manager, we also considered liquidity constraints, that is we constrained the manager to not purchase too much of a certain stock with respect to the average daily trading volume.[18]

Given these realistic portfolio construction techniques, we constructed optimal portfolios for every risk model and for every month in our sample period from a security universe of the largest 3000 publicly traded stocks in the United States.[19] We stored these weights. We used these weights to measure crowding. Our analysis covered the period 2006 to 2013.[20]

[12] This is due to the fact that those assets are held with a total portfolio weight greater than one so that the variance of a portfolio consisting of a holding in both is almost double the variance of holding just one of them. The actual weight in each is approximately 0.70. If the weights of the portfolio are normalized to sum to 1, then the resulting variance of the portfolio is 0.95.

[13] If the weights are normalized to sum to one, the variance of the portfolio is even lower at 0.05.

[14] These portfolio parameters are quite reasonable. In fact, we surveyed several portfolio managers before creating our parameters. We also experimented with other maximum and minimum weights for the portfolio. The benchmark portfolio for the purposes of sector neutralization was the top 3000 companies selected by market capitalization each month and weighted by market capitalization.

[15] The target for the randomly generated models was chosen as half of the S&P 500 5-year historical volatility. There was no specific reason for this choice, except that it seemed to be reasonable. If a specific target could not be achieved, we searched for the next reasonable target.

[16] These portfolio parameters are quite reasonable. In fact, we surveyed several portfolio managers before creating our parameters. While it is true that different managers may use slightly different parameters, the main purpose of this paper is to describe the potential crowding effects that may occur when portfolio managers use reasonable parameters and similar risk models.

[17] The target alpha was the same as with the long only case.

[18] We allowed a maximum position of 30% of average daily trading volume for the stock.

[19] This security universe was updated every month in our sample period.

[20] We have data for a longer time period, but the simulations take an enormous amount of time to compute and thus we limited our sample from 2006 to 2013. For example, the 100 random alpha portfolios can take 20 days to complete when optimizing for one year of historical data even when running on 12 processors in parallel.

4.2. The alpha model

In order to focus on the amount of crowding that is caused from the portfolio construction process, we used random alpha models for the different portfolios. That is, each portfolio receives signals about the stock universe that are random. Thus, the degree of crowding from the alpha models, prior to portfolio construction, has an average value of zero.

In order to construct the random alpha signals for each portfolio manager, we drew 100 random alpha signals for all stocks from a normal distribution, $\boldsymbol{\alpha} \sim N(0, \boldsymbol{\Sigma}_{\boldsymbol{\alpha}})$, where $\boldsymbol{\Sigma}$ is the historical variance–covariance of asset returns up to the time of portfolio selection with the off-diagonals set to 0.[21] That is, we used the historical volatility of each asset, but ignored the correlations.[22]

4.3. The risk models

Crucial to the portfolio management process is the use of a risk model for the securities. In order to understand how crowding occurs in the financial system from the portfolio construction process, we used the leading risk models in the industry to build portfolios.[23]

4.3.1. Industry risk models

Most professional money managers use standard third-party risk models to manage their portfolios. The most well-known risk models are that of MSCI-Barra, Northfield, and Axioma.[24] In this paper, we use the Barra US Equity Model (USE4) which has been active since June 30, 1995.[25] We also use Northfield's U.S. Fundamental Equity Risk Model which has been active since January 30, 1990.[26] Finally, we use Axioma's Robust Risk Model for the U.S. which has been active since January 4, 1982.[27] Barra is believed to lead most providers with around a 50% market share.

All of these risk models are multi-factor models. That is, these factor models assume that asset returns can be modeled as a linear combination of common risk factors (Ross, 1976; Chincarini and Kim, 2006). The three risk models differ by the factors chosen and other estimation techniques. We use the three prominent risk models in the industry to reconstruct the full variance–covariance matrix of asset returns that real-world portfolio managers would be using to build their optimal portfolios so that we can get an accurate estimation of the crowding that may or may not occur through the portfolio construction process.

4.3.2. The Marchenko–Pastur adjusted risk model

One of the primary reasons that risk models produce crowding is due to concentration on the smallest eigenvalues that are indistinguishable from random noise. Thus, one method to reduce the crowding would be to reduce the reliance on less informative eigenvalues. In order to do this, we compute a Marchenko–Pastur Adjusted (MPA) variance–covariance matrix. The procedure for MPA can vary, our procedure is as follows. First, decompose the variance–covariance matrix by principal component analysis.

$$\boldsymbol{\Sigma}_{\boldsymbol{r}} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}' \tag{19}$$

where $\mathbf{U}$ is a matrix whose columns represent the eigenvectors of $\boldsymbol{\Sigma}_{\boldsymbol{r}}$ in descending order of importance in explaining the covariance of asset returns, and $\boldsymbol{\Lambda}$ represents a diagonal matrix of the eigenvalues (i.e. variances) of each eigenvector.

Second, reduce the number of eigenvectors from N to K, where $K \ll N$.[28] We use Random Matrix Theory (RMT) (Marchenko and Pastur, 1967) to do this by eliminating the insignificant eigenvectors. In order to estimate the parameter, T/N in the RMT, we used the number of days used to compute the variance–covariance matrix and divided it by the number of stocks in the universe.[29] We then

---

[21] The reason for choosing 100 random draws rather than a larger number had to do with the tradeoff between sufficiently large numbers and the computation time required. To create the 100 random portfolios for twelve months of data took 20 days on a supercomputer that used twelve cores.

[22] Further research might wish to consider a random model which draws from a standard normal distribution, $\boldsymbol{\alpha} \sim N(0, 1)$, where signals for individual assets are independent of their historical volatility. Further research may also wish to consider a model that draws from a full variance–covariance matrix of asset returns rather than just the diagonals.

[23] In order to allow for comparisons across risk models, we match all data across risk model providers and our stock return every month of the analysis. We matched the data by CUSIP identification.

[24] The majority of asset managers use either Barra, Northfield, or Axioma and thus are a very representative group (Fabozzi et al., 2007; Fabozzi and Markowitz, 2011). Other providers include APT and R-squared. APT's Market Risk Model for the US has been active since January 2000. For more info, see http://www.sungard.com/campaigns/fs/alternativeinvestments/apt/solutions/apt_market_risk_models.aspx. R-Squared Customized Hybrid Risk Model (CHRM) has been active since June 29th, 2007. For more info, see http://www.rsquaredriskmanagement.com/Customised-Hybrid-Risk-and-Return-Models.

[25] For more info, see http://www.msci.com/products/portfolio_management_analytics/equity_models/barra_us_equity_model_use4.html. BARRA has another popular risk model , the Barra US Equity Model (USE3), which has been active since 1973.

[26] For more info, see http://www.northinfo.com/documents/8.pdf.

[27] For more info, see http://axioma.com/robust.htm.

[28] Many researchers use ad hoc methods to do this, including a cut-off such that, for example, 90% of the data variance can be explained by the first K eigenvalues, and some simply choose to look at the first 4 or 5 eigenvectors. Often, the 90% cutoff will correspond to the first few eigenvectors. For example, in the bond markets, the first 3 eigenvectors account for 99% of the variance of the data (Litterman and Scheinkman, 1991).

[29] The variance–covariance matrix estimated by the commercial data providers usually uses some kind of weighting of the past data to reflect the more importance of recent data. Given the half-life of their weighting scheme, we find the T relevant for our random matrix theory tests by counting "data point days". The first date point used (e.g. 1 day ago) gets a full weighting of one, while every subsequent data point is multiplied by a constant, δ, which determines the half-life. That is, $\delta = \frac{1}{2}^{1/\psi}$, where ψ is the half life in years. The total number of data point days is the infinite series, $1 + \delta + \delta^2 + \delta^3 + \cdots$, which equals $\frac{1}{1-\delta}$. Thus, if our data provider uses a 2-year half-life and use 252 for the number of trading days in a year, we get $T = 728$ or about 2.9 years for our RMT calculations.

computed the upper limit as $(1+\sqrt{(T/N)})^2$. After this, we only used significant eigenvalues to construct the new variance–covariance matrix (i.e. the eigenvalues that were larger than the upper limit).

Third, for every month, we used the reduced set of eigenvalues and eigenvectors to create a variance–covariance matrix of all asset returns as,

$$\boldsymbol{\Sigma}_{\boldsymbol{r}}^{\boldsymbol{f}} = \mathbf{U}^* \boldsymbol{\Lambda}^* \mathbf{U}^{*\prime} \tag{20}$$

where $*$ variables represent the first significant K eigenvectors and eigenvalues correspondingly, $\mathbf{U}^*$ is an $N \times K$ matrix, $\boldsymbol{\Lambda}^*$ is a $K \times K$ matrix, and $\boldsymbol{\Sigma}_{\boldsymbol{r}}^{\boldsymbol{f}}$ is the resulting $N \times N$ variance–covariance matrix after the MP transformation has been applied. This technique can be applied on any variance–covariance matrix of asset returns. For each of the industry risk models, we applied the MPA procedure to create an alternative variance–covariance matrix for portfolio construction.

5. Empirical simulation results

5.1. Overall results

The empirical results from 2006 to 2013 are contained in Tables 1 and 2 and Figs. 2 and 3.[30] Tables 1 and 2 report the crowding measures by optimization framework (e.g. long only portfolio), by risk model 1, 2, or 3, and whether or not the MP adjustment (MPA) to the variance–covariance matrix was used or not. Table 1 reports the average results for the 2006 to 2009 period and Table 2 reports the results for the 2010 to 2013 period. The alpha models all have zero crowding, as would be expected. The long only portfolios using the standard risk models have an average crowding measure of 0.85 to 0.86 depending on the risk model used for the 2006 to 2009 period and 0.70 to 0.71 for the 2010 to 2013 period. Given that our crowding measure has values between 0 and 1, this is a quite high level of crowding. We tested for the significance of the crowding and found that the crowding level is statistically different from prior to portfolio construction (i.e. from the crowding generated by the alpha models) at the 99% level. The Ω values which represent the crowding of the long only portfolio to the random model are extremely large, due to the fact that the alpha models have close to zero crowding.

The crowding measures for the market neutral portfolios are extremely small. In fact, they are zero to two decimal places. However, one can see that the relative crowding to the random model, Ω, ranges from 1.10 to 1.65 for the 2006 to 2009 period and even higher for the 2010 to 2013 period. Despite this, the tests for significance of the crowding are all rejected. Thus, the crowding level amongst market neutral portfolios is statistically indistinguishable from those of the alpha models. When liquidity constraints are added to the market neutral portfolio construction process, crowding increases for all of the models. This makes intuitive sense because liquidity constraints might lead to the exclusion of a large group of smaller companies. Consequently more weight is forced into other companies. Thus crowding becomes amplified, although it is still statistically insignificant.

The tables also present the average of the ratio of the cross-sectional sample standard deviation of the crowding of the portfolio to the sample standard deviation of the crowding of the alpha portfolio ($\overline{\sigma(C_p)/\sigma(C_\alpha)}$). For the random alpha signals, the number is the average cross-sectional standard deviation of the crowding over time. For the market neutral portfolios, where the difference in mean crowding prior and after portfolio construction is not different, it does show that in any given cross-section, the variance of the crowding is larger by anywhere from 5% to 8%. The overall conclusion is that the portfolio construction process seems to have little effect on market neutral portfolios, but in any given manager universe, there are more managers that are either less crowded or more crowded.[31] An example of these distributional differences is shown in Fig. 4.

In all of the cases, the optimization with the MPA procedure reduces the average crowding. This is true for long only portfolios, market neutral portfolios, and market neutral portfolios with liquidity constraints. For example, for the long only portfolio over the period 2006 to 2009, the crowding measure declines from 0.85 to 0.73 for risk model 1, 0.86 to 0.73 for risk model 2, and 0.86 to 0.72 for risk model 3. This is a decrease in crowding of 14%. This reduced crowding is statistically significant at the 99% level. The reduction in crowding is even more apparent in the 2010 to 2013 period.

For the market neutral portfolios, Ω declines from 1.65 to 1.24 for risk model 1, from 1.76 to 1.23 for risk model 2, and 1.10 to 1.05 for risk model 3. However, this level of crowding is statistically insignificant from the alpha case.

Fig. 2 shows how the crowding of long only portfolio evolves over time from 2006 to 2013. Once again, one can see how the MPA method reduces the crowding in all months. The other interesting feature is that crowding rises dramatically prior to August 2007, the month of the quant crisis (Chincarini, 2012). Crowding continues to rise during and after the quant crisis as well. Most of the focus of the quant crisis was on similar expected return or alpha models, however, this research finds mild evidence that portfolio construction techniques may have played a role as well.

Fig. 3 shows the crowding from long only portfolios for different industry risk models. This figure illustrates that crowding is a phenomenon that occurs with all of the risk models.

[30] In order to dynamically simulate the portfolios, we had to create an entire program to do professional portfolio optimization. When we started this research, we used MATLAB 2009, which took an extraordinary amount of time to solve simple optimizations with 3000 securities. Ironically enough, MATLAB was unable to do certain types of optimizations. We eventually switched to MATLAB 2014a, where the optimizer speed had been increased to a reasonable time. We also used CPLEX from IBM through the MATLAB API. The random simulations took an enormous amount of time to run. For example, to obtain the results for one year of data took 20 days when running the program on 12 parallel processors.

[31] We also computed Sharpe ratios for the portfolios, which were all close to zero, as would be expected given the random alpha signals.

Table 1
Summary of Crowding of Random Alpha Models from 2006 to 2009.

	Risk Model 1			Risk Model 2			Risk Model 3		
	C	Ω	S.D.R.	C	Ω	S.D.R.	C	Ω	S.D.R.
Alpha	0.00		0.14			0.14			0.14
Long Only									
Regular	0.85*	1251.17	0.438	0.86*	1140.19	0.407	0.86*	1250.08	0.434
MPA	0.73**	1123.99	0.881	0.73**	872.10	0.892	0.72**	976.13	0.891
Market Neutral									
Regular	0.00	1.65	1.016	0.00	1.76	1.020	0.00	1.10	1.029
MPA	0.00	1.24	1.000	0.00	1.23	1.000	0.00	1.05	1.000
Market Neutral Liq.									
Regular	0.00	2.02	1.038	0.00	4.23	1.056	0.00	1.20	1.070
MPA	0.00	0.78	1.008	0.00	0.73	1.009	0.00	0.84	1.008

Note: This table presents various crowding measures from the constructed portfolios using various portfolio optimization structures that minimize volatility using various risk models over the period 2006 to 2009. Risk Model 1, 2, and 3 represent leading risk models used in the industry. Regular represents the portfolios constructed with each risk model as provided by the company, while MPA represents the Marchenko–Pastur method applied to create a new variance–covariance matrix that is used to construct the portfolio. All numbers in the table are averages of various variables constructed from monthly portfolios. The computations are based on 100 portfolios formed from random alpha signals. C represents our crowding measure as described in the paper, $C = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{m^2 - m}$. Ω measures the relative crowding between random signals and actual portfolios, $\Omega = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{\alpha:i,j} - m}$. A higher value means that risk model creates more crowding. S.D.R. represents the average of the ratio of the cross-sectional sample standard deviation of the crowding of the portfolio to the sample standard deviation of the crowding of the alpha portfolio $\overline{\sigma(C_p)/\sigma(C_\alpha)}$. For the random alpha signals, the number is the average cross-sectional standard deviation of the crowding over time.

* Indicates that the crowding is statistically different from the alpha model crowding at the 99% confidence level.

** Indicates that the MPA procedure's crowding is statistically different from the regular optimization crowding at the 99% confidence level.

Table 2
Summary of Crowding of Random Alpha Models from 2010 to 2013.

	Risk Model 1			Risk Model 2			Risk Model 3		
	C	Ω	S.D.R.	C	Ω	S.D.R.	C	Ω	S.D.R.
Alpha	−0.00		0.14			0.14			0.14
Long Only									
Regular	0.71*	1101.07	0.742	0.71*	617.27	0.705	0.70*	689.01	0.753
MPA	0.57**	822.54	1.045	0.57**	711.50	1.035	0.56**	607.04	1.063
Market Neutral									
Regular	−0.00	−0.80	1.027	−0.00	3.80	1.048	−0.00	5.50	1.053
MPA	0.00	1.82	1.000	0.00	−1.18	1.001	0.00	0.34	1.000
Market Neutral Liq.									
Regular	−0.00	1.61	1.045	−0.00	5.04	1.083	−0.00	1.50	1.084
MPA	−0.00	1.49	1.016	0.00	−0.51	1.017	0.00	−0.30	1.016

Note: This table presents various crowding measures from the constructed portfolios using various portfolio optimization structures that minimize volatility using various risk models over the period 2010 to 2013. Risk Model 1, 2, and 3 represent leading risk models used in the industry. Regular represents the portfolios constructed with each risk model as provided by the company, while MPA represents the Marchenko–Pastur method applied to create a new variance–covariance matrix that is used to construct the portfolio. All numbers in the table are averages of various variables constructed from monthly portfolios. The computations are based on 100 portfolios formed from random alpha signals. C represents our crowding measure as described in the paper, $C = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{m^2 - m}$. Ω measures the relative crowding between random signals and actual portfolios, $\Omega = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{\alpha:i,j} - m}$. A higher value means that risk model creates more crowding. S.D.R. represents the average of the ratio of the cross-sectional sample standard deviation of the crowding of the portfolio to the sample standard deviation of the crowding of the alpha portfolio $\overline{\sigma(C_p)/\sigma(C_\alpha)}$. For the random alpha signals, the number is the average cross-sectional standard deviation of the crowding over time.

* Indicates that the crowding is statistically different from the alpha model crowding at the 99% confidence level.

** Indicates that the MPA procedure's crowding is statistically different from the regular optimization crowding at the 99% confidence level.

5.2. The source of crowding

So far, we have established that the process of portfolio construction, independent of the expected return models of equity managers, could lead to crowding. In our simulation procedure, we chose very reasonable methods of portfolio construction. In this section, we examined which parts of the construction procedure led to the most crowding and whether the general qualitative results of the paper continue to hold. Thus, we re-ran the simulation results and altered the constraints to see how crowding changed with different constraints. For the long only and the market neutral portfolios we repeated all of the simulations under four scenarios. In scenario 1, for the long portfolio, we removed sector constraints, and allowed weights for each security to be as large as 1. We also removed the condition that the weights must sum to 1. The only real condition we imposed is that the weights of the optimal portfolio had to be greater than 0. For the market neutral portfolio , we placed no restrictions on the long and short weights, and removed the sector constraints, leverage constraints, and dollar neutrality constraints.

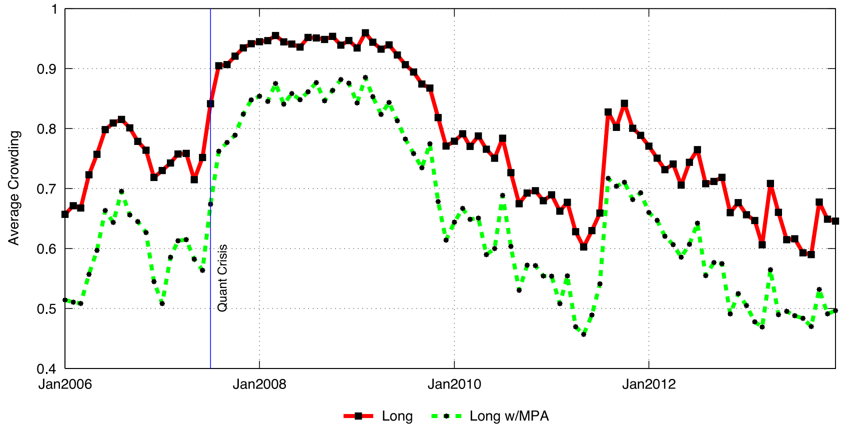

Fig. 2. Crowding over Time for Random Alpha Models using Risk Model 1. This figure represents the crowding measures for long only portfolios constructed every month from 2006 to 2013 using industry risk model 1. The solid line represents the crowding using the standard optimization practice, while the dashed line represents the crowding when optimizations are based upon the MPA method. At every point in time, actual data was used to construct 100 random alpha signals and consequently 100 optimized portfolios.

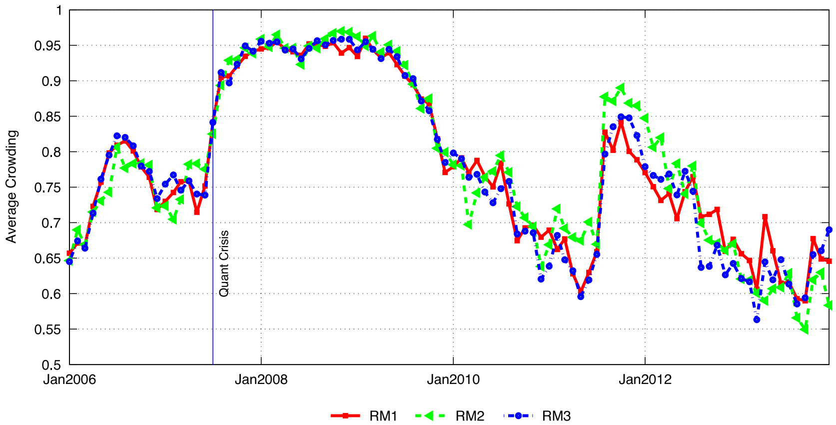

Fig. 3. Crowding over Time for Random Alpha Models' Long Portfolio and Different Risk Models. This figure represents the crowding measures for long only portfolios constructed every month from 2006 to 2013 using three industry risk models. RM1, RM2, and RM3 represent risk models 1, 2, and 3. At every point in time, actual data was used to construct 100 random alpha signals and consequently 100 optimized portfolios. The vertical line represents the starting month of the quantitative crisis of 2007.

In scenario 2, the optimization setup was the same as in scenario 1, except that the long portfolio had a constraint that the weights must sum to 1. For the market neutral portfolios we added the dollar neutrality constraint and a maximum leverage of 2. In scenario 3, the optimization setup was the same as in scenario 2, except for the long portfolio we added the sector constraint that the optimal portfolio sectors must equal that of the benchmark. For the market neutral portfolio, the weighted average of each sector in the long portfolio had to equal the weighted average of each sector of the short portfolio. In scenario 4, we added the maximum weight constraints of 0.10 (or 10%) for the long portfolio and −0.10 to 0.10 for the market neutral portfolio. We also added the constraint that the market neutral portfolio had a weighted average beta of zero.

The results of these four different scenarios are shown in Table 3. These simulations shed light on the process of crowding that is obtained through portfolio construction. In the optimization with no constraints (Scenario 1), there is very little crowding for both the long only and market neutral portfolios, although it is still statistically different for the long only portfolios compared to prior to portfolio construction. Also, the relative crowding to the alpha models is still very high for long only portfolios and about double for the market neutral portfolios. Once again, however, the market neutral portfolios' crowding is not statistically different than the alpha models.

Scenario 2 shows that the raw crowding measure for long only portfolios dramatically increases when the constraint that weights sum to 1 is imposed. Thus, most of the crowding from the portfolio construction process in long only portfolios occurs due to this basic constraint contained in long only portfolios.

Scenarios 3 and 4 show that other constraints do not matter as much for raw crowding measures, although sector constraints do increase the amount of relative crowding for long only and market neutral portfolios.

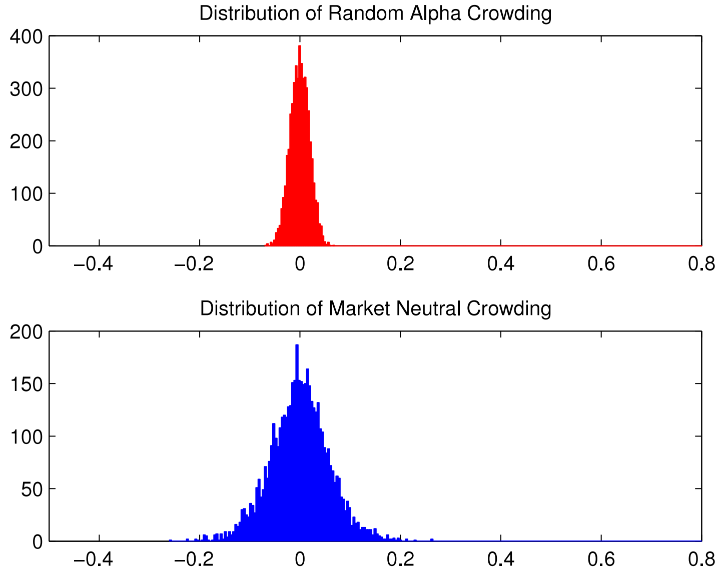

Fig. 4. Histogram of Crowding from Random Alpha Models and Market Neutral Portfolios. This figure represents a histogram of crowding for the random alpha signals and the market neutral portfolios for July 2011.

From this examination of various constraints, we find that the qualitative aspect of the previous analysis is still valid. Crowding can occur through the portfolio construction process. The MPA method continues to reduce crowding for both long only and market neutral portfolios even when no constraints are applied.

6. The systemic effects of the induced crowding

In the previous sections, we have documented the crowding of portfolios that results from the risk management or portfolio construction process. This crowding occurred with all of the industry risk models used to manage portfolio risk. However, even though industry risk models are similar, they are not the same. Each risk model has its own uniqueness. Thus, it might be interesting to examine whether it matters whether portfolio managers use one model or another in terms of its effect on overall market crowding. That is, would it be better to have an industry of equity portfolio managers that share their usage amongst risk models or would this do very little for the crowding problem?

In this section, we take all of the results from the random simulated portfolios and examine whether the crowding amongst this sample group of simulated portfolios would be less, the same, or greater if the distribution of risk models was different. In every month of our sample period, we generated random alpha signals for each stock for 100 portfolios. Portfolios were then formed based upon the portfolio construction parameters described earlier in the paper for each risk model. All of the risk models used the same random alpha signals for each stock. In order to study the effects of the distribution of risk models, we choose a distribution of usage amongst the three risk models. For example, one such distribution was 90% risk model 1, 10% risk model 2 and 0% risk model 3. Based on this, each month we chose 90 randomly selected portfolios from risk model 1 and 10 from risk model 2 and their corresponding random alpha vectors. We then computed crowding measures for this distribution and so on and so forth.

Table 4 contains the results for different scenarios of risk model usage. The Table shows selected degrees of risk model usage by different portfolio managers, from 100% of a risk model (i.e. 100–0–0 for 100% of risk model 1 and none of the others) to an equal percentage of risk models used (33–33–34). For the long only portfolios, the results are intriguing. From 2006 to 2009, if the entire portfolio manager universe used only one risk model, the amount of average crowding would be around 0.85 to 0.86. The lowest amount of crowding, 0.51, would be achieved when the universe used all risk models in the same proportion. This represents a systemic reduction in crowding by 40%. For the market neutral portfolios, the results are mixed. First, we cannot examine the crowding measure alone, since for the alpha models and the market neutral models they are equal to 0 to two decimal places. Thus, we observe the Ω, which represents the crowding of the portfolios compared to the random expected return models. Although the equal weighted distribution of risk model usage has a low ratio at 1.92, even 100% in any of the risk models has a lower ratio than this. The equal-weighted distribution is still one of the lower crowding situations, but there are also others that have lower overall crowding. Over the period 2006 to 2009, risk model 3 resulted in a very low level of relative crowding.

Even though many of the commercial risk models are very similar in structure and nature, there are differences among them. These differences alone are enough to warrant distributing the concentration of risk model usage in the entire financial system so as to reduce the resulting crowding from risk models. This is especially true for long only portfolio managers.

Conjecture 4 (*Distribution of Risk Models and Systemic Risk*)**.** *Crowding in the financial system will be less when there is a diversification of risk models used in the system.*

Table 3
Crowding from Various Optimization Constraints.

	Risk Model 1			Risk Model 2			Risk Model 3		
	C	Ω	S.D.R.	C	Ω	S.D.R.	C	Ω	S.D.R.
Scenario 1: No Constraints									
Alpha	0.00		0.05			0.05			0.05
Long Only									
Regular	0.03*	2155.92	1.96	0.04*	3296.43	1.95	0.05*	3810.21	2.35
MPA	0.01**	1062.38	2.24	0.02**	1801.67	2.43	0.04**	3076.09	3.54
Market Neutral									
Regular	0.00	2.41	1.29	0.00	−0.25	1.53	0.00	8.74	1.58
MPA	0.00	1.08	1.00	0.00	1.44	1.00	0.00	1.13	1.00
Scenario 2: Sum of Weights									
Alpha	−0.00		0.05			0.05			0.05
Long Only									
Regular	0.91*	−18653.55	1.08	0.91*	−18539.17	0.98	0.92*	−18859.92	1.05
MPA	0.70**	−13882.32	5.16	0.75**	−14646.39	4.04	0.75**	−14668.99	4.64
Market Neutral									
Regular	−0.00	0.48	1.28	−0.00	−1.91	1.47	−0.00	−0.61	1.56
MPA	−0.00	0.70	1.02	−0.00	0.77	1.02	−0.00	0.95	1.01
Scenario 3: Sector Weights									
Alpha	−0.00		0.05			0.05			0.05
Long Only									
Regular	0.89*	−80198.27	1.21	0.89*	−80267.64	1.20	0.89*	−80795.61	1.18
MPA	0.77**	−70544.77	2.31	0.76**	−69363.29	2.64	0.78**	−70837.40	2.29
Market Neutral									
Regular	−0.00	8.07	1.29	−0.00	12.96	1.47	−0.00	16.28	1.58
MPA	−0.00	0.85	1.01	−0.00	0.51	1.01	−0.00	0.98	1.01
Scenario 4: Max Weights									
Alpha	0.00		0.05			0.05			0.05
Long Only									
Regular	0.89*	15344.92	1.22	0.89*	15313.87	1.20	0.89*	15416.11	1.20
MPA	0.78**	13400.08	2.26	0.78**	13245.92	2.27	0.78**	13751.99	2.20
Market Neutral									
Regular	0.00	1.44	1.25	−0.00	2.26	1.46	−0.00	1.17	1.54
MPA	0.00	0.96	1.01	0.00	1.06	1.01	0.00	1.09	1.01

Note: This table presents various crowding measures from the constructed portfolios using various portfolio optimization structures that minimize volatility using various risk models over the period of 2009. Four types of optimization constraints are used in the table. Risk Model 1, 2, and 3 represent leading risk models used in the industry. Regular represents the portfolios constructed with each risk model as provided by the company, while MPA represents the Marchenko–Pastur method applied to create a new variance–covariance matrix that is used to construct the portfolio. Scenario 1 has no constraints on the portfolio optimizations. Scenario 2 forces the long portfolio to have weights that sum to 1 and forces the market neutral portfolio to have the long portfolio weights to sum to the short portfolio weights. Scenario 3 forces the long portfolio's sector weights to equal that of the benchmark sector weights and forces the market neutral portfolio to have the long portfolio sector weights equal the short portfolio sector weights. Scenario 4 constrains the maximum weight in any given stock to be 0.10 for long weights and -0.10 for short weights. All numbers in the Exhibit are averages of various variables constructed from monthly portfolios. The computations are based on 100 portfolios formed from random alpha signals. C represents our crowding measure as described in the paper, $C = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{m^2 - m}$. Ω measures the relative crowding between random signals and actual portfolios, $\Omega = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{\alpha:i,j} - m}$. A higher value means that risk model creates more crowding. S.D.R. represents the average of the ratio of the cross-sectional sample standard deviation of the crowding of the portfolio to the sample standard deviation of the crowding of the alpha portfolio $\overline{\sigma(C_p)/\sigma(C_\alpha)}$. For the random alpha signals, the number is the average cross-sectional standard deviation of the clowding ovel time.

* Indicates that the crowding is statistically different from the alpha model crowding at the 99% confidence level.

** Indicates that the MPA procedure's crowding is statistically different from the regular optimization crowding at the 99% confidence level.

7. Conclusion

The great recession and financial crisis of the years 2007 to 2009 was amplified and may have even been caused by the interconnections within the system. Many investment areas were crowded by investors of different types that made similar trades who did not consider the behavior of other investors with similar positions. Thus, crowding could occur when investors engage in mimetic behaviors by using the same model to predict asset returns. Our paper investigated another source of potential crowding that could occur when investors use similar processes to construct equity portfolios. This is the only research that we know of that examines crowding in this context.

Table 4
Systemic Crowding Risk from Distribution of Risk Model Usage.

Percentage of models used	Long only		Market neutral	
	C	Ω	C	Ω
100–0–0	**0.85**	1251.17	0.00	1.65
0–100–0	**0.86**	1140.19	0.00	1.76
0–0–100	**0.86**	1250.08	0.00	1.10
80–20–0	0.65	869.71	0.00	2.96
80–0–20	0.76	1176.42	0.00	1.38
20–80–0	0.65	799.36	0.00	2.37
0–80–20	0.66	788.17	0.00	2.33
20–0–80	0.76	1181.01	0.00	1.29
0–20–80	0.66	859.13	0.00	2.29
45–45–10	0.52	623.48	0.00	3.02
10–45–45	0.52	620.27	0.00	3.03
45–10–45	0.63	939.13	0.00	2.28
60–40–0	0.55	672.34	0.00	3.54
60–20–20	0.58	802.99	0.00	3.05
40–60–0	0.55	644.00	0.00	2.74
0–60–40	0.56	633.06	0.00	3.00
40–0–60	0.72	1152.52	0.00	1.79
0–40–60	0.56	660.20	0.00	2.73
33–67–0	0.58	673.88	0.00	2.31
67–0–33	0.58	710.80	0.00	3.12
0–67–33	0.58	661.92	0.00	3.02
33–33–34	**0.51**	681.27	0.00	1.92
10–90–0	0.74	961.72	0.00	1.77
10–0–90	0.80	1200.84	0.00	0.78
90–10–0	0.74	1028.33	0.00	2.35
0–10–90	0.75	1029.26	0.00	1.67
90–0–10	0.74	1032.74	0.00	2.37

Note: The table presents the crowding measures for different combinations of risk models used by the group of 100 simulated portfolio managers over the period 2006 to 2009. For example, 45–45–10 indicates that 45 of the portfolios are constructed using risk model 1, 45 are constructed using risk model 2, and 10 are constructed using risk model 3. C represents our crowding measure as described in the paper, $C = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{m^2 - m}$. Ω measures the relative crowding between random signals and actual portfolios, $\Omega = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{\alpha:i,j} - m}$. A higher value means the portfolio construction process creates more crowding.

We contribute to the understanding of systemic risk by explaining why *ex-ante* risk may underestimate realized risk, why inadvertent crowding might occur during the process of portfolio construction, even when the investment forecasts are unrelated to each other, and suggest two methods to reduce crowding in the financial system.

There are many directions for future research. It has been documented that crowding can lead suddenly to a distorted risk–return space (Chincarini, 2012). Future research might further examine the extent to which crowding in an investment space leads to inferior subsequent returns. This could be done using a subset of funds in the mutual fund or hedge fund space for a variety of investment strategies. It is likely that crowding may lead to subsequent inferior returns on a risk-adjusted basis (Ibbotson and Idzorek, 2014).

We examined risk models that are commonly used amongst portfolio managers. However, there are other systems that might create crowding through interconnections. For example, the Office of Financial Research of the U.S. Treasury recently wrote that "...consulting or pricing services to other asset managers [are] creating interconnections and dependencies that increase their importance in financial markets." (The Economist, 2013). One such system is Aladdin, a risk management product used by many of Blackrock's customers representing about \$11 trillion or 7% of the \$225 trillion of financial assets in 2013. It might be worth studying these interdependent risk management systems.

Another direction for future research is to study international or global portfolios. Portfolio managers are increasingly becoming more global and it would be interesting to know what type of crowding is created in smaller, emerging markets.

Recently, there have been worries about crowded trading towards the end of the trading day (Strumpf, 2015). Our study is related to crowding over one-month holding periods and it would be interesting to study the possibilities for intra-day crowding in the financial markets. Crowding and interdependence amongst investors is an extremely important but under-researched topic. This paper contributes to the understanding of how system processes, such as standard portfolio construction, might contribute to the crowding problem. Our research is an addition to the important research into the area of crowding.

Acknowledgments

We would like to thank Sikai Wang for research assistance. We would especially like to thank Stephen Ross, Frank Fabozzi, Francis Longstaff, Robert Whitelaw, and Dan deBartolomeo for their comments and David Card, Rowilma del Castillo, and Chris Paciorek and the Econometrics laboratory at UC Berkeley for making this research possible. We would like to thank Gabriel Baracat, Mark Carhart, Jesse Davis, Matt Dixon, Steve Gaudett, Ralph Goldsticker, Steve McQueen, Steve Greiner, Paul Intrevado, Jeff Hamrick, Christopher Martin, Terence Parr, Deborah Berebichez, Zach Shockley, Shamin Parikh, Tuan Anh Le, Joseph Lieto, Vikas Kalra, Jose Menchero, Chris Martin, Daehwan Kim, David Uminsky, Michael Chigirinskiy, Saurabh Harsh, Thomas Grossman, Steve Dyer, Dessislava Pachamanova, Manuel Tarrazo, Fabio Moneta, Joseph Losonczy, Chris Canova, and an anonymous referee. We also thank the Western Economic Association International conference participants.

Appendix. Applied optimization details

This appendix describes the optimization problem set up. All of our optimizations were performed in MATLAB using MATLAB's optimization routines, in addition to user-adjusted optimization routines, and the CPLEX optimization tools from IBM.[32]

A.1. The long portfolio

Our approach is to maximize the expected return (or alpha signal) of the portfolio subject to a variety of constraints, including that the portfolio volatility be equal to the 60-month historical volatility of the S&P 500,[33] the weights of the portfolio sum to 1, the weights of any individual stock are between 0 and 10%, and that the portfolio has the same exposure to each sector as the benchmark universe of 3000 stocks.

$$\max_{\mathbf{w}} \mathbf{w}'\boldsymbol{\mu} \tag{21}$$

$$\text{s.t.} \tag{22}$$

$$\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} = \sigma^2_{S\&P500} \tag{23}$$

$$\mathbf{w}'\boldsymbol{\iota} = 1 \tag{24}$$

$$0 \le \mathbf{w} \le 0.10 \tag{25}$$

$$\mathbf{S}\mathbf{w} = \mathbf{w}_s^{BM} \tag{26}$$

where $\mathbf{w}$ are the weights of the stocks in the portfolio, $\boldsymbol{\mu}$ is a vector of alpha signals for each stock, $\boldsymbol{\Sigma}$ is the variance–covariance matrix of stock returns, $\boldsymbol{\iota}$ is a vector of ones, $\mathbf{S}$ is an M-by-N matrix of zeros and ones representing the M sectors of the economy with a 1 if the security is in that sector and a 0 if not, and $\mathbf{w}_s^{BM}$ is an M-by-1 vector of sector weights for the benchmark universe.

A.2. The market neutral portfolio

Our approach is to maximize the expected return or alpha of the portfolio, while constraining the portfolio to have a target volatility equal to 5% over the risk-free rate, have leverage of 2 and be dollar-neutral (that is, sum of long weights sum to 1 and sum of short weights sum to 1), the long portfolio is sector neutral to the short portfolio, the weights of an individual stock cannot be less than −10% or greater than 10%,[34] and beta neutral (that is, the weighted average beta of the long portfolio equals the weighted average beta of the short portfolio).

$$\max_{\mathbf{w}} \mathbf{w}'\boldsymbol{\mu} \tag{27}$$

$$\text{s.t.} \tag{28}$$

$$\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} = 0.05 \tag{29}$$

$$\mathbf{w}'_L\boldsymbol{\iota} = 1 \ \forall w_i \ge 0 \tag{30}$$

$$\mathbf{w}'_S\boldsymbol{\iota} = -1 \ \forall w_i \le 0 \tag{31}$$

$$-0.10 \le \mathbf{w} \le 0.10 \tag{32}$$

$$\mathbf{w}'\boldsymbol{\beta}|_{w_i \ge 0} = -\mathbf{w}'\boldsymbol{\beta}|_{w_i \le 0} \tag{33}$$

$$\mathbf{S}\mathbf{w}_L = -\mathbf{S}\mathbf{w}_S \tag{34}$$

where $\mathbf{w}$ are the weights of the stocks in the portfolio, $\boldsymbol{\mu}$ is a vector of alpha signals for each stock, $\boldsymbol{\Sigma}$ is the variance–covariance matrix of stock returns, $\boldsymbol{\iota}$ is a vector of ones, $\boldsymbol{\beta}$ is a vector of the CAPM beta for each stock estimated on 5-year historical return data, $\mathbf{S}$ is an M-by-N matrix of zeros and ones representing the M sectors of the economy with a 1 if the security is in that sector and a 0 if not, $\mathbf{w}_L$ and $\mathbf{w}_S$ represents the weights of the long and short portfolio respectively.

[32] Some of the optimizations were not solvable in feasible time with versions of MATLAB older than 2014a.

[33] In cases where it was not feasible to achieve the S&P 500 historical volatility, the closest feasible volatility was used in the optimization.

[34] We initially started with smaller weight restrictions of 0.03 and −0.03, but many of the optimizations could not be solved, thus we expanded the weight constraint.

A.3. The market neutral portfolio with liquidity constraints

Crowding is a real-world phenomena involving real-world transactions, thus we also construct market neutral portfolio that incorporate reasonable self-imposed liquidity constraints. The optimization approach is exactly the same as for the market neutral portfolios, however we add a liquidity purchase constraint that is a fraction of the average daily trading volume.

The constraint takes the form of a portfolio manager not wishing to trade more than some percentage of the average daily trading volume of the stock. That is, the constraint is $V_t w_{it} \leq c ADTV_{it}$ or $w_{it} \leq \frac{c}{V_t} ADTV_{it}$, where c represents the constant indicating the threshold percentage that the portfolio manager wishes to trade in any given stock, V_t is the dollar value of the portfolio, and $ADTV_{it}$ is the average daily trading volume of stock i at time t in dollars. A typical value for c in the investing world is 15%.[35]

Since the liquidity constraint is essentially an upper bound weight constraint, it makes sense to adjust the existing upper bound weight constraint for each stock rather than adding a new series of constraints. Thus, the upper bound and lower bound weight constraint for every stock is adjusted using the following algorithm. If the liquidity constraint is higher than the existing stock constraint (i.e. 10%), then the stock's weight constraint is unaltered. If smaller, change the upper and lower bound constraint to be equal to the liquidity constraint value for each stock. We did this for both the long and short side of the portfolio.[36]

A.3.1. Market neutral construction

One of the challenges of the market neutral optimization is to set up the problem so that leverage can be limited. The method we employ is for every one of the N stocks in our buy list, we create an additional set of weights called buy weights and an additional set of sell weights. Thus, if we have N stocks, we create weights, $w_1 \ldots w_N$, $w_1^b \ldots w_N^b$, and $w_1^s \ldots w_N^s$. We then construct our entire optimization with these $3N$ weights. In preparing our inputs for optimization, we formulate the following:

$$\boldsymbol{\mu} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{35}$$

where there are $2N$ zero values in the column. The variance–covariance matrix has also to be modified as follows:

$$\boldsymbol{\Sigma} = \begin{bmatrix} V(r_1) & C(r_1, r_2) & \cdots & C(r_1, r_N) & 0 & \cdots & 0 \\ C(r_2, r_1) & V(r_2) & \cdots & C(r_2, r_N) & 0 & \cdots & 0 \\ \vdots & & & & 0 & \cdots & 0 \\ C(r_N, r_1) & C(r_N, r_2) & \cdots & V(r_N) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & & & & \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{36}$$

We then alter the constraints in such a way as to keep all the main constraints on the final weights that we are interested in (i.e. $w_1 \ldots w_N$), while achieving our market neutral leverage and dollar-neutral constraints. Thus,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & -1 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 0 & 0\cdots & 0 & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1\cdots & 1 & \end{bmatrix} \tag{37}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 1 \end{bmatrix} \tag{38}$$

These constraints create an optimization whereby $w_i = w_i^b - w_i^s$, $\sum_i^{n_b} w_i^b = 1$, and $\sum_i^{n_s} w_i^s = 1$. We also add constraints that $\mathbf{w}_B \geq 0$ and $\mathbf{w}_S \geq 0$ in our upper and lower bound constraints. This insures that our market neutral portfolio is dollar neutral and has a leverage limited to 2. One could modify this for other forms of leverage very easily. The weights we ultimately care about from this optimization are still the $\mathbf{w}$. Any additional constraints on these weights, such as upper and lower bounds or sector constraints can be added to the constraint matrix, A, simply by adding rows and replacing zeros wherever the w^b and w^s occur.

Although this solution allows us to enable leverage and dollar-neutral constraints on our market neutral portfolio, it does not guarantee that we do not have wasteful solutions such that we purchase and sell pieces of the same stock. In order to reduce this chance, we introduce a penalty function into our objective function. That is,

$$\max_{\mathbf{w}} \mathbf{w}' \boldsymbol{\mu} - \lambda(\boldsymbol{\iota}' \mathbf{w}_B + \boldsymbol{\iota}' \mathbf{w}_S) \tag{39}$$

---

[35] The portfolio manager might also have a total limit on the ultimate size of any position, for example, 3 times the ADTV. We ignore this additional consideration for this study.

[36] In reality, the short portfolio may also have stocks that are difficult to borrow. This particular issue is not considered in this paper.

References

Amenta, Michael, 2013a. Factset hedge fund ownership. Factset Publ. 1–11.
Amenta, Michael, 2013b. Factset institutional ownership. Factset Publ. 1–27.
Anton, Miguel, Polk, Christopher, 2014. Connected stocks. J. Finance 69, 1099–1128.
Bai, Zhidong, Huixia, Liu, Wong, Wing-Keung, 2009. Enhancement of the applicability of Markowitz's portfolio optimization by utilizing random matrix theory. Math. Finance 19.
Blocher, Jesse, 2011. Contagious Capital: A Network Analysis of Interconnected Intermediaries, Vol. 1. (Unpublished Ph.D. Thesis), pp. 1–53.
Bohlin, L., Rosvall, M., 2014. Stock portfolio structure of individual investors infers future trading behavior. PLoS ONE 9, 1–8.
Brunnermeier, Markus K., Pedersen, Lasse Heje, 2005. Predatory trading. J. Finance 1825–1863.
Brunnermeier, Markus K., Pedersen, Lasse Heje, 2009. Market liquidity and funding liquidity. Rev. Financ. Stud. 2201–2238.
Cahan, Rochester, Luo, Yin, 2013. Standing out from the crowd: Measuring crowding in quantitative strategies. J. Portfolio Manage. 71.
Chincarini, Ludwig B., 1998. The failure of long term capital management. BIS Banking Paper 1–15.
Chincarini, Ludwig B., 2012. The Crisis of Crowding. Quant Copycats, Ugly Models, and the New Crash Normal. Wiley/Bloomberg.
Chincarini, Ludwig B., Kim, Daehwan, 2006. Quantitative Equity Portfolio Management. An Active Approach to Portfolio Construction and Management. McGraw-Hill.
Eoma, Cheoljun, Gabjin, Ohb, Jungc, Woo-Sung, Hawoong, Jeonge, Kim, Seunghwan, 2009. Topological properties of stock networks based on minimal spanning tree and random matrix theory in financial time series. Physica A 388, 900–906.
Fabozzi, Frank J., Kolm, Petter N., Pachamanova, Dessislava, Focardi, Sergio M., 2007. Robust Portfolio Optimization and Management. John Wiley & Sons.
Fabozzi, Frank J., Markowitz, Harry M., 2011. Equity Valuation and Portfolio Management. John Wiley & Sons.
Golubi, Anton, Guo, Zhen, Correlation stress tests using the random matrix theory: an empirical implementation to the chinese market, Unpublished Paper, 2012.
Hotelling, H., 1933. Analysis of a complex of statistical variables into principal components. Educ. Psychol. 498–520.
Ibbotson, Roger, Idzorek, Thomas, 2014. Dimensions of popularity. J. Portfolio Manage. 40, 68–74.
Jolliffe, I.T., 2002. Principal Component Analysis. Springer.
Laloux, Laurent, Pierre, Cizeau, Marc, Potters, Bouchaud, Jean-Philippe, 2000. Random matrix theory and financial correlations. Int. J. Theor. Appl. Finance 03.
Litterman, Robert, Scheinkman, J., 1991. Common factors affecting bond returns. J. Fixed Income 1, 54–61.
Marchenko, V.A., Pastur, L.A., 1967. Distribution of eigenvalues for some sets of random matrices. Sb. Math. 4, 507–536.
Marmer, Harry S., 2015. Fire! Fire! Is U.S. low volatility a crowded trade?. J. Invest. 17–37.
Menchero, Jose, Jun, Wang, Orr, D.J., Eigen-adjusted covariance matrices: improving forecasts for optimized portfolios, Research Insight MSCI Barra Presentation, 2011.
Menkveld, Albert, Crowded trades: an overlooked systemic risk for central clearing counterparties, VU University of Amsterdam Working Paper, 2014.
Michaud, Richard O., 1989. The Markowitz optimization enigma: Is 'optimized' optimal?. Financ. Anal. J. 45 (1), 31–42.
Pearson, K., 1901. On lines and planes of closest fit to systems of points in space. Phil. Mag. 6, 559–572.
Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral, L.A.N., Stanley, H.E., 2000. A random matrix theory approach to financial cross-correlations. Physica A 287, 374–382.
Pojarliev, Momtchil, Levich, Richard M., 2011. Detecting crowded trades in currency funds. Financ. Anal. J. 67, 26–39.
Ross, Stephen, 1976. The arbitrage theory of capital asset pricing. J. Econom. Theory 13, 341–360.
Roweis, S.T., Saul, L.K., 2000. Nonlinear dimensionality reduction by locally linear embedding. Science 5500, 2323–2326.
Sharifi, S., Crane, M., Samaie, A., Ruskin, H., 2004. Random matrix theory for portfolio optimization: A stability approach. Physica A 335, 629–643.
Sneider, Amanda, David, Kostin, Stuart, Kaiser, Ben, Snider, Peter, Lewis, Reddy, Rima, 2012. Hedge fund trend monitor. Goldman Sachs Publ. 1–39.
Stein, Jeremy C., 2009. Presidential address: Sophisticated investors and market efficiency. J. Finance 64, 1517–1548.
Strumpf, Dan, 2015. Stock market traders pile in at the close. Wall Street J..
Subramanian, Savita, 2013. US quantitative primer 2013. Bank Am. Merrill Lynch Publ. 1–220.
Tenebaum, J.B., deSilva, V., Langford, J.C., 2000. A global geometric framework for nonlinear dimensionality reduction. Science 5500, 2319–2323.
The Economist, 2013. The monolith and the markets. The Economist.
Yan, Philip, Crowded trades, short covering, and momentum crashes, http://ssrn.com/abstract=2404272, 2013.