Stephen A. Ross: *Excellence Beyond Recognition*

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On March 3, 2017, Professor Stephen A. Ross of MIT died of unexpected coronary failure. He earned his Bachelor of Science in physics from Cal Tech in 1965 and went on to obtain his PhD in economics from Harvard University. He was a professor of economics and finance at the Wharton School from 1970 to 1977, a professor of economics and finance at Yale University from 1977 to 1998, and the Franco Modigliani Professor of Financial Economics at MIT from 1998 until his passing. He published his agency theory and arbitrage pricing theory (APT) while at the University of Pennsylvania. He published the binomial option pricing theory with coauthors John Cox and Mark Rubinstein while at Yale. He published his term structure model with Jonathan Ingersoll and John Cox in 1985 and his work on survivorship bias in 1992. His most recent work, "The Recovery Theorem," was controversial, yet it created a new dialogue in the field of finance and was published in 2015 while he was at MIT.

Over the years, in addition to his well-known contributions, he also published a host of interesting articles as well as a standard textbook, *Corporate Finance* (Ross, Jaffe, and Westerfield [2013]).[1] The book was one of the two most adopted textbooks in corporate finance. Almost more impressive than his intellectual pursuits was his devotion to his students. Stephen A. Ross created an army of successful students.[2] These students decided to publish a collection of their work in his honor entitled *Stephen A. Ross, Mentor: Influence through Generations*. This is an accomplishment that not all well-known economists achieve.

Professor Ross made a huge contribution to the finance and economics profession that goes beyond any academic awards, yet we believe that he would have won the Nobel Prize had he not died. One natural time to have awarded him the Nobel Prize might have been with Eugene Fama in 2013. It could have been given to Professor Ross

[1] The first edition of the book was published in 1990 and the 11th edition was published in 2013.

[2] Several of Ross's students have written articles in this special issue. Brown and Goetzmann [2018] write about Ross's relatively unknown paper "Regression to the Max," which laid a framework for cautioning empirical researchers on their ex post analysis of historical events and was the precursor to much of the work on survivorship bias that came later. Dybvig [2018] discusses Ross's work on agency theory, performance evaluation, and incentives and compensation. Campbell [2018] summarizes Ross's most important contributions and how he laid theoretical foundations for empirical testing. Berk [2018] discusses what he learned from Ross, including what he views as Ross's most important work, the use of state prices to price actual security prices, which we now call stochastic discount factors in finance.

for the development of the APT and Eugene Fama for the application of the APT with the three-factor Fama–French model, or to both of them for their contributions to asset pricing. Nevertheless, that is not what happened. He was the second person in asset pricing to not receive the prize despite his major contributions. The other person was Fischer Black, who passed away in August 1995 while his coauthor, Myron Scholes, and colleague, Robert Merton, jointly received the award for their works in option pricing in 1997.[3] In this article, we discuss Professor Ross's contributions to the field of economics and finance.[4]

STEPHEN ROSS'S MAIN CONTRIBUTIONS

By the time he died at age 73, Stephen Ross of MIT had made many contributions to economics and finance. In this section, we will discuss some of his major achievements and how they represent excellence beyond recognition.[5]

APT

The contribution. In a 1976 article published in the *Journal of Economic Theory* entitled "The Arbitrage Theory of Capital Asset Pricing," Professor Ross proposed an alternative to the capital asset pricing model (CAPM) (Treynor [1961, 1962]; Sharpe [1964]; Lintner [1965]; Mossin [1966]).[6] The new theory required fewer assumptions than the CAPM, allowing for more factors than the market to explain stock returns. The argument was simple: If investors agree that security returns are driven by common factors, then because of arbitrage in security markets, the expected return of every security can be expressed as a linear combination of factor premiums.[7]

One practical drawback to the theory is that it did not specify the identity of the common factors and the number of factors needed. In fact, much of academia and the practitioner world has spent time trying to empirically identify these factors. Perhaps the most well-known factor model is that of Fama and French [1992].[8]

Originality, importance, and impact of the contribution. First, how did this discovery affect the industry? In terms of impact, the multifactor model that deviates from the CAPM is the standard in the practitioner's world of portfolio management.[9] All of the major software providers of equity risk management use some form of a multifactor model.[10] The use of these multifactor models may have taken place without the development of the APT because they can be regarded simply as explanatory factors for the returns of stocks. Within academia, the Fama–French three-factor model has been extended to four- and five-factor models and has been used in asset management for portfolio construction and risk control. Thus, the factor model has had an enormous impact on both the practitioner and the academic world. Whether that impact would have been as large without an underlying theory is impossible to know, but a brief discussion of the history of factor models and the APT may help us put Ross's discovery in context.

As William Sharpe [1984] noted, although factor models existed prior to the APT, it is important to highlight how those models were applied. Perhaps the earliest

[3] Although it is true that the Nobel Committee acknowledged Black's contribution in the award declaration, they still failed to award him the Nobel Prize while he was alive. By the time of his death in 1995, it had been clear for many years what an immense contribution the Black–Scholes formula had been. This seems a bit unfair. One might argue that for related work, the actual prize could be awarded posthumously, even if the cash award is not.

[4] We circulated a draft of this article to many people. Some of them thought the idea of connecting Ross's work to the Nobel Prize was an excellent idea. Others felt that it was a disservice to even mention the Nobel Prize in this essay, primarily because he pursued his intellectual interest without ever caring for prizes. Bengt Hölmstom told us, "I knew him for more than 30 years. Not once did he indicate that he was disappointed or bitter about not getting the prize. Indeed, the Nobel Prize was not a topic that he liked to talk about."

[5] A more technical version of Ross's contributions will be available on SSRN in the future.

[6] In this special issue, Elton and Gruber [2018] discuss the APT in more detail. Although Ross's work in the *Journal of Economic Theory* is the most quoted piece on the APT, the concept was first published in a book chapter in 1976 under the title "Return, Risk, and Arbitrage" (Ross [1976]). We thank Phillip Dybvig for pointing this out to us. Of course, the APT idea also floated around in a 1972 Wharton working paper (Ross [1972]).

[7] For more information, see Chincarini and Kim [2006].

[8] For more information, see Chapter 15 of Chincarini and Kim [2006].

[9] Ross's last published work was an invited editorial in this journal in a special issue on factor-based investing (Ross [2017]).

[10] The multifactor models of the following commercial providers are examples: MSCI Barra, Axioma, Northfield, and APT.

introduction of the factor model specification was in an article by King [1966]. However, King considered these "additional factors" as industry factors and estimated them on real data. Subsequent work continued to consider factors as industry factors (Cohen and Pogue [1967]) or as the derivation of factors from a principal component decomposition of returns (Feeney and Hester [1967]; Elton and Gruber [1973]). Neither of these is really how factor models are used today. Farrell [1974] created a portfolio of stocks by characteristics, such as growth, cyclical, stable, and oil-related, and showed that the stock market return does not fully capture these category returns. Barr Rosenberg, the founder of the most well-known factor-model software, Barra (now MSCI Barra), wrote several published and working papers in the early 1970s about factor models (Rosenberg [1974]; Rosenberg and Marathe [1976]; Rosenberg and McKibben [1973]; Rosenberg et al. [1975]).[11] These models, like the one proposed by King [1966], expressed the factor models and discussed potential factors, including industry factors and factors such as earnings growth, the dividend payout ratio, and many others. However, a central theme in Barr Rosenberg's papers was an attempt to show how the CAPM beta would be related to these accounting variables or factor variables. Thus, in some sense, the authors wanted to explain the beta of the CAPM in terms of these other factors.

Reading these papers, one can see that there seemed to be a struggle to justify the approach to be consistent with the CAPM, which at the time was the only equilibrium model of security returns. The importance of Ross's APT is that, with a few simplifying assumptions, he was able to take a factor world into an equilibrium model of security returns. That is, he was able to say that the expected returns of all stocks could be represented as an expression of the stocks' factor exposure to the factors and the expected factor realizations. From this point on, it was not necessary to refer to the beta of the CAPM directly.

Some analogies might provide further clarification on Ross's contribution. When William Sharpe (and Lintner, Treynor, and Mossin) formulated the CAPM, Markowitz [1959] had already spoken about a single index market model instead of a covariance model.[12] However, it was not until the CAPM that an equilibrium model was created that asserted that the expected return of a stock would be related to the stock's beta multiplied by the market's expected return. When Einstein [1905] proposed the theory of relativity, the basic equations for special relativity were already known and had been proposed by Lorentz [1904]. However, Einstein offered meaning to these equations and changed an entire field. We do not want to imply that we think the APT was as grand as the theory of relativity, but we do want to point out that it offered meaning to the use of factor models. One might criticize the APT for its assumptions or for its failure to identify the number and identity of the common factors. However, this is the same criticism leveled against the CAPM, which has incredibly unrealistic assumptions and fails to take into account the actual world of asset management with widely varying beliefs, overvaluations, crowding, and active management (Chincarini [2012]).

In this light, the innovation by Professor Ross, combined with the previous introduction of factor models, had a substantial impact on both the practitioner and academic world. The work was also original in the sense that the equilibrium approach allowed the model to be used as a stand-alone model without the need for the CAPM, which factor models alone could not do.

The Binomial Option Pricing Model

The contribution. In 1973, Fischer Black and Myron Scholes discovered how to price options and other derivative securities (Black and Scholes [1973]). This was one of the greatest discoveries in financial economics, not only because of the impact it had on the field of financial economics but also because of the pure elegance of the model they created. One drawback to the model is that it required a sophisticated knowledge of continuous-time finance, such as Ito's Lemma, and was not very flexible for dealing with derivatives that were more complex than traditional ones. This all changed in 1979, when Professor Ross and his two colleagues, Mark Rubinstein and John Cox, introduced the concepts of binomial trees in their article "Option Pricing: A Simplified Approach" (Cox, Ross, and Rubinstein [1979]). This new model required elementary mathematics, was simple to implement, lent itself to more generalizations and the valuation of more complex options, and contained a limiting case whereby

[11] Barra stands for Barr Rosenberg Associates and is now owned by Morgan Stanley as part of MSCI Barra.

[12] For more information, see Markowitz [1959].

the formula would converge to the Black–Scholes formula. The model was simple to implement because users would construct a tree of possibilities. Starting from the current date, the stock price could either go up or down. One could then know the value of the option for the case in which the stock went up and for the case in which the stock went down. One could then create a portfolio of the underlying stock and the option—that is, one could buy Δ shares of the stock and short the option in such a way as to create a riskless portfolio, and hence the value of the option could be calculated.

Originality, importance, and impact of the contribution. In terms of assessing its originality and impact, the binomial model is probably the most commonly used method to value options. It is explained in every textbook on derivatives and is a much friendlier way to teach option pricing. The model has had an enormous impact in the private sector and has also led to many new models using similar techniques, including the Black–Derman–Toy model for valuing fixed-income options and interest-rate derivatives and Edgeworth binomial trees allowing for an analyst-specified skew and kurtosis in spot returns for valuing commodity derivatives.

The concepts have also been used in corporate finance to evaluate real options. The real option valuation framework provides a way to quantify the benefits and costs for different scenarios in business, and they can be used by corporate management in a variety of applications. The binomial model can be applied instead of the Black–Scholes model because of its greater flexibility, including mergers and acquisitions analysis or evaluating the expansion or abandonment option embedded in a prospective capital investment project.

Although the approach was original, there are two qualifying statements. The first is that the origin of the idea was born when John Cox was an assistant professor at Stanford and chatted with his colleague William Sharpe. He had given Sharpe a paper written by Cox and Ross [1975] on the possibility of pricing options when the underlying stock distribution was a Poisson process.[13] Professor Sharpe suggested to him that a simple tree of up and down movements of the stock price might be able to price options. That is, Sharpe thought that their paper would be easier to understand if they modeled the stock movements in discrete time rather than in continuous time.[14] John Cox took this insight to Mark Rubinstein and Stephen Ross, and the three of them developed the full-scale machinery for the binomial model.[15] Even though it was Sharpe who made the initial suggestion, it is still true that the three of them greatly expanded and formalized the idea into a working theory.

The second qualifier is that the binomial model was not the first model to solve option pricing. The real breakthrough came with the idea of a riskless portfolio that Fischer Black and Myron Scholes discovered. This is important because Robert Merton also won the Nobel Prize for option pricing theory. Although Merton added a more complete method of looking at the problem, it was Black and Scholes who discovered the key ingredient.[16] Thus, in this way, one might also consider the binomial option pricing model another modeling method on top of Black–Scholes. It was easier and more intuitive and in that sense was an original discovery.

[13] According to Sharpe, he had thought of this idea prior to reading their paper.

[14] It was also at this time that Mark Rubinstein and John Cox were asked to teach options to MBA students. Both of them were struggling to find a way to do this. As John Cox told us, "The discrete time model was naturally a better way to teach MBAs." In fact, before the famous paper was published (Cox, Ross, and Rubinstein [1979]), Cox and Rubinstein had used the model to teach MBAs. Ross had not used the model to teach because he was teaching primarily economics. In fact, Cox remembers that finance was a "private hobby" for Ross, and many of his economics colleagues did not understand why he was wasting time on something so frivolous as finance. Eventually, Michael Jensen saw some of the teaching notes and asked the trio to submit the paper to the *Journal of Financial Economics*.

[15] In their acknowledgments in the paper, the authors wrote, "My best thanks go to William Sharpe, who first suggested to us the advantages of the discrete-time approach to option pricing developed here." Cox, Rubinstein, and Ross offered Sharpe co-authorship, but he kindly declined.

[16] The first use of the term *Black–Scholes option pricing model* was by Robert Merton, who was Black's colleague at the time at MIT (Merton [1973]). Merton added the continuous time dynamic replication mechanics to the problem. In Merton's own words, "My contribution was to show in the limit of continuous trading, the B-S dynamic zero-beta strategy was actual a zero-sigma strategy. That is, all of the risk would be eliminated if continuous trading were possible. Thus, a dynamic strategy in the stock and cash would replicate the option payoff exactly and hence the realized return would equal the risk-free rate." Funny enough, toward the end of his life, Fischer Black preferred the CAPM derivation, partly because he did not believe in continuous trading.

Agency Theory

The contribution. It is difficult to exactly pinpoint the origins of agency theory because the conflict between principal and agent has probably been known since the beginning of business. In fact, Adam Smith described it in his *Wealth of Nations* (Smith [1776]) as follows:

> In the modern language of the Theory of Incentives, the masters are principals and the workmen are their agents. What are the common wages of labor, depends everywhere upon the contract usually made between those two parties, whose interests are not the same. The workmen desire to get as much, the masters to give as little as possible.

However, it is certainly plausible that one of the early papers outlining the discussion of how principals might create an incentive contract to induce agents to act on their behalf was that of Stephen Ross, in a 1973 paper entitled "The Economic Theory of Agency: The Principal's Problem." This paper was cited by Bengt Hölmstrom, 2016 Nobel Prize winner, in his first publication on the topic (Hölmstrom [1979]). Hölmstrom also cited articles by Wilson [1968] and by Spence and Zeckhauser [1971]. The former article has more to do with a syndicate of people making a decision under uncertainty, and the latter article has more to do with constructing insurance contracts when the insurance company and the individual might have different incentives. Neither of them seems to explicitly deal with the principal–agent problem.

The theory of agency that Ross brought forward described how principal–agent problems in general might be amenable to incentive contracts. He showed that under certain conditions, an incentive fee contract is consistent with Pareto optimality. The theory of corporate structure and agency is most notably credited to Jensen and Meckling [1976]. In fact, their paper also cited Ross [1973]. The Jensen and Meckling work differs from that of Ross in that it tries to explain a whole variety of items related to corporate structure that are based on property rights and agency theory. Ross's work was much more specific and focused on issues related to compensation in solving a principal–agent problem.

Ross added an interesting line in the conclusion of his paper: "To mention one more path of interest—in a world of true uncertainty where adequate contingent markets do not exist, the manager of the firm is essentially an agent of the shareholders."

Originality, importance, and impact of the contribution. Stephen Ross's contribution to agency theory was important. He was the first person to lead researchers toward this important question. The actual techniques used to solve principal–agent problems changed from a technical perspective in the way Ross had formulated the question, but sometimes a question is much more important than the answer. Ross did not continue his work in this field after this important contribution. There were also related papers before his paper, that by Wilson [1968] being one of them. Many in the field of financial economics would argue that the paper most closely associated with agency is that by Jensen and Meckling [1976], which specified the link between corporate structure as related to the principal–agent problem. However, some believe that Professor Ross was the first to set up this problem explicitly as a relational problem.

In terms of impact, principal–agency theory ballooned in economics as a whole. We believe it is probably fair to conclude that this was another interesting area to which Stephen Ross made an important contribution.

The Cox–Ingersoll–Ross Term Structure Model

Interest rate modeling was another area in which Stephen Ross contributed along with his co-authors, John Cox and Jonathan Ingersoll, with their paper "A Theory of the Term Structure of Interest Rates" (Cox, Ingersoll, and Ross [1985]).[17] To value certain types of derivatives on interest rates, it is important for pricing to know how the term structure of interest rates evolves over time. The term structure of interest rates is the value of interest rates with different maturities. The simultaneous behavior of interest rates of varying maturities over time is critically important to value many types of derivatives, such as swaptions and interest rate floors and caps, as well as debt obligations with embedded options such as callable bonds, putable bonds, convertible bonds, and floaters with caps and floors. It is also critical to

[17] According to John Cox, the first draft of the paper was written around 1976. However, Cox wanted to integrate the model with the real economy. Thus, work on the paper dragged on, and eventually it was published as two separate papers in *Econometrica*, almost 10 years later (Cox, Ingersoll, and Ross [1985a, b]).

understanding monetary policy, the economy, and the consequences of issuing government debt. More specifically, an area known as term structure modeling attempts to model the behavior of zero-coupon bonds of varying maturities over time.[18] Today, there are many mathematical models for modeling the dynamic behavior of the yield curve. These models typically attempt to make sure there is no arbitrage as interest rates of varying maturities evolve and that the return distribution can be non-normal if needed.

One of the earliest papers on this topic was by Vasicek [1977], who modeled short-term interest rates as a mean-reverting process with a stochastic component. Although the Vasicek model is well known as one of the first interest-rate models, it has a shortcoming in that it can generate negative interest rates. This is generally not thought of as possible, although in recent years, the negative yields of government bonds have challenged even that empirical wisdom. Ross and his colleagues introduced what is now known as the CIR model of interest rates, which avoids negative interest rates.

STEPHEN ROSS'S ADDITIONAL CONTRIBUTIONS

Risk-Neutral Pricing

Today it is very common for financial professionals to use what is known as risk-neutral pricing, which means that to price a derivative instrument one can simply discount all the payoffs by the risk-free rate and then multiply each payoff by the probability of its occurrence. This is a convenient and useful concept, especially for valuing complicated derivatives.

This simple idea was born in the early days of option pricing. The Black–Scholes formula had recently been published. Stephen Ross was an economics professor at the University of Pennsylvania, and John Cox was a student of finance at Wharton. They began collaborating on how to price an option if stock prices followed a Poisson distribution rather than a normal distribution. Finding the problem too mathematically difficult to solve, Cox and Ross sought the assistance of Penn's mathematics department. The problem was indeed complex because the mathematicians could not solve it either. Not knowing what to do, they realized that if their math skills were limited, they could surely use economic analysis. It was then that they realized that if options could be dynamically hedged, then they must be priced regardless of investor preferences. In other words, the price of an option had to be the same regardless of a particular investor's preferences. Thus, the price must be the same for a risk-neutral investor (i.e., an investor who cares only about the expected return and not the risk). This realization allowed them to price the option with jumps (i.e., the Poisson distribution of stock prices). They then realized that any option that can be dynamically replicated can be priced using the notion of risk neutrality.

Cox and Ross sent the paper to the *Journal of Political Economy* in April 1975. The journal was slow to respond.[19] In fact, it took two years before they heard back from the journal with a rejection. The reason given for the rejection was that, in the interim, Cox and Ross had published two papers with similar ideas (Cox and Ross [1976a, 1976b]).[20]

Empirical Work

Although much of Ross's work was theoretical, he did make some interesting empirical contributions. In 1980, a few years after his APT was published, he wrote a paper that explained how the APT might be tested and performed these tests on actual stock market data (Roll and Ross [1980]). The research found that at least three factors were important for explaining stock returns.[21] Ross did not identify the factors; rather, he

[18] Of course, this is all that matters because one can construct the value of coupon-paying fixed-income securities from the underlying theoretical zero-coupon rates. A zero-coupon interest rate is the interest rate for a bond that pays no coupons and pays both principal and interest at maturity.

[19] It should also be noted that this was the same journal that originally rejected the paper by Black and Scholes on option pricing, and it was only published later with the encouragement of Merton Miller. New ideas are oftentimes met with resistance; as Fischer Black wrote, the option pricing model started with tinkering and ended up with "delayed recognition." (Black [1989]).

[20] One could argue that the idea of pricing any options with risk-neutral pricing was implicit in the Black–Scholes formula or that others would have soon realized or had already realized the same idea. Although this may be true, John Cox told us that he explained the idea to Fischer Black, and Fischer Black was not convinced it was universally true. Cox showed Black how it could be used to price an option with a different type of diffusion that was difficult to solve, and that convinced Black.

[21] Interestingly enough, this paper was published years before the well-known Fama–French three-factor model (Fama and French [1992, 1996]).

used a statistical decomposition similar to principal component analysis to identify the important factors explaining the variance in returns.

In another paper, Ross attempted to use unexpected macroeconomic variables to explain stock returns (Chen, Roll, and Ross [1986]). This was Ross's first attempt to actually name the potential factors in the arbitrage pricing model. He believed that the factors should be theoretically associated with broad economic forces. This model used five factors to explain stock returns, including growth in industrial production, unexpected inflation, credit spreads, and the term structure of interest rates. Chen, Roll, and Ross found that the stock market index had an insignificant influence on expected returns compared with these economic variables. In other words, they were arguing that the CAPM version of the world was incomplete when a more expanded factor model was considered.

Two other of Ross's empirical contributions are worth mentioning. He developed a test for the efficiency of a portfolio using stock return data, known as the Gibbons–Ross–Shanken test, or GRS for short (Gibbons, Ross, and Shanken [1989]). He also offered one of the early rational explanations for the closed-end discount puzzle (Ross [2002]), in which, for many closed-end mutual funds, the price per share of the fund is lower than the net asset value of the fund. In other words, the investor can buy the fund for less than its worth. Behavioral economists immediately pointed to inefficiency and reasons for that inefficiency. Stephen Ross came up with a simple, rational explanation—management fees. That is, if investors knew that management fees soon would be charged on the assets under management, then they should pay less for the fund. How much less? The present discounted value of the fees. Using actual closed-end fund data, Ross was able to show that this simple, rational explanation addressed much of the closed-end fund discount puzzle. In fact, as Ross put it, "We have seen that a simple fee based neoclassical argument can explain the closed end fund puzzle. This puts a great burden on those who would advocate the need for theories based on irrational models of investor behavior. Appealing to investor sentiment seems to me to be almost limitless in its ability to explain just about anything. There are studies that say people are overconfident and studies that say they are timid—for every zig there is a zag."

Survivorship Bias

In studies of historical finance data, researchers must be cognizant of survivorship bias. In the past, it was oftentimes neglected by researchers; even in the present day, it is not always fully appreciated. Ball and Watts [1979] were among the first to mention this sort of bias, but academics studying the performance of mutual funds and hedge funds seemed to have forgotten this issue in their quest to analyze whether fund managers could outperform the market and whether this outperformance could be consistently repeated. Many studies published between 1980 and the early 1990s showed that not only did many mutual fund managers outperform the market (e.g., the S&P 500), but that they could repeat this performance (Grinblatt and Titman [1992]; Elton et al. [1993]; Hendricks, Patel, and Zeckhauser [1993]; Goetzmann and Ibbotson [1994]; Brown and Goetzmann [1995]; Elton, Gruber, and Blake [1996]; Wermers [1997]). Of course, this is vastly important for investors because if investors can identify successful managers, they will have a strategy for picking which managers to use. In 1992, Ross, Brown, Goetzmann, and Ibbotson showed that these results could be an artifact of survivorship bias (Brown et al. [1992]).[22] That is, fund managers might not be beating their benchmark, and their performance might not be persistent.

They showed that even if none of the managers possessed skill, a world with managers that vary by the risk they take, such that some take more risk and others take less risk, can generate the appearance of winner persistence. The reason is that very volatile funds will sometimes do so badly that they will have to close their funds and hence disappear from the database. The researcher who analyzes the historical data of fund performance without knowing about the missing funds will conclude that there is persistence in winner funds even though there is not.

Since the publication of this article, academics have gone to great lengths to account for survivorship bias by creating survivorship-bias-free databases in the

[22] William Goetzmann shared with us a manuscript by Stephen Ross from 1987 called "Regression to the Max" that was a precursor to the survivorship bias work and illuminated the dangers of using past data to infer economic facts. (For more information see https://www.researchgate.net/publication/247378278_Regression_to_the_max.) In this special issue, Brown and Goetzmann [2018] discuss this important contribution by Ross in more detail.

mutual fund and hedge fund world.[23] As Stephen Ross said, "Financial research is like paleontology in that both fields are delving into how things were in the past. Paleontologists have the easier job, however, because they can dig under the ground and find the bones of the past from which to make inferences" (Ross [1994]).

The Recovery Theorem

In April 2013, Stephen Ross came to the University of San Francisco to present a new idea he called the recovery theorem.[24] It was eventually published in 2015 (Ross [2015]). The paper was intriguing in that Ross believed that option prices could be used to infer both the probability that a future state of the world would occur and the magnitude of that event. Economists have been trying for years to do this, but they have not been able to do so for logistical reasons.[25] Simply stated, it may be impossible to separate risk preferences from probabilities of events using only observable security prices. If what Ross claimed was possible, this would be very important in that it would allow us to infer from option prices what the market believes future probabilities of events really are. This is immensely important in economics because almost all of economics and finance has to do with expectations. To provide realistic expectations today for the future, one needs the probabilities of different returns in the future. This would consequently affect all tests of asset pricing models if reliable estimates for expected returns were available. We could also answer questions such as how likely a recession is.

Within a short amount of time, several people had written papers related to the recovery theorem.[26] The recovery theorem states that under certain assumptions about the probability of transitioning from one state of the world to another, a researcher can use option prices to obtain the representative agent's beliefs about the probabilities of different future events.

In practice, the recovery theorem is harder to apply. First, one has to determine a sufficiently large range of states. For example, if one is concerned with the U.S. equity market, one might consider values of the S&P 500 in the future as a range of possible states. Second, one must find liquid options trading at different maturities and at different strike prices so as to create a sufficiently rich set of time-state periods to solve for the state price matrix. Third, there are also various criticisms of the Ross approach, including the time-homogeneity assumption. For example, the probabilities of moving from one state to the other might depend on time, in the sense that high volatility in the market in one time period might change the transition probabilities over time.[27] Fourth, the analysis assumes that all investors are identical to the composite representative agent. This is a departure from the reality of different endowments of individuals, different attitudes toward risk, different beliefs about the true return-generating process, different exposures to risk factors, different institutional constraints, and issues related to interconnectivity and crowding (Chincarini [2012]). These deviations from the basic representative agent are problematic for the assumptions inherent in the recovery theorem. Also, they may create path dependence in the transition probabilities, which Ross assumes are independent.

However, what is probably most important about Ross's work on the recovery theorem is that this work sparked a whole new interest in recovering probability distributions from option prices—and this contribution was made by Ross at 71 years old, very late in his already successful career.[28] There are some economists who believe that the recovery theorem is one of the most exciting new developments in finance in the last

[23] Survivorship bias can still be an issue because some funds fail to report their last month of returns before closing. Thus, a researcher must decide whether to assign a −100% weight to that month or to ignore that month.

[24] The talk can be found on the Internet at http://ludwigbc.com/the-recovery-theorem-and-thoughts-about-the-field-of-finance-guest-dr-stephen-ross/.

[25] Economists have been able to infer risk-neutral probabilities. For example, see Bates [1991] and Figlewski [2008].

[26] See Jensen, Lando, and Pederson [2018]; Carr and Yu [2012]; Audrino, Huitema, and Ludwig [2014]; and Borovička et al. [2016].

[27] The biggest critics of this assumption are Borovička, Hansen, and Scheinkman [2016]. Jensen, Lando, and Pedersen [2018] removed this assumption and proposed a generalized recovery theorem. Carr and Yu [2012] extended the recovery theorem to continuous time and demonstrated why there might be a problem with the time homogeneity assumption. Some of the empirical work on the recovery theorem used Bloomberg's volatility surface data, which some people have found to have problems. Many of the papers that have been written on the recovery theorem do a poor job of explaining the theorem itself and the empirical methods used. This is unfortunate, and we hope better work is done in the future.

[28] John Cox told us, "It amazed me that he [Ross] was still working at that level in his 70s."

10 years and was another important contribution by Stephen Ross to asset pricing. There is more work to do, but if we can agree on the results, the possibilities for future research will be enormous.[29]

ONE OF THE RECOGNITIONS OF ACHIEVEMENT: THE NOBEL PRIZE

It is extremely difficult to know precisely the criteria used by the Nobel Prize Committee in making its selection of the prize winner. However, based on historical data and an essay by Assar Lindbeck posted on the Nobel Prize web site, some reasonable deductions can be made.[30] Professor Lindbeck was chairman of the Prize Committee for The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel from 1980 to 1994.

Generally speaking, Alfred Nobel originally wanted the prize to be given for a specific outstanding achievement. In his last will and testament of November 27, 1895, he wrote that his estate should be distributed as "prizes to those who, during the preceding year, shall have conferred the greatest benefit to mankind." Of course, this has generally not been followed in most disciplines, just as it has not been followed for The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel. The prize for economics is unique in that it is not contained in the original topic matters that Alfred Nobel had listed; it was added in 1969.[31] In 1968, to honor the 300th anniversary of the Sveriges Riksbank, the central bank of Sweden, the bank endowed a new prize honoring achievements in the field of economics. For this reason, the prize is referred to as the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel or The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel. An independent, nongovernmental scientific organization, The Swedish Academy of Sciences (one of the Royal Academies of Sweden that selects the prize winners in chemistry and physics) was chosen to select the winner of the Nobel Prize in economics.

According to Professor Lindberg, "prizes have been awarded for a specific contribution (such as new analytical methods in finance and econometrics), two or several specific contributions (such as the prizes to Milton Friedman and Franco Modigliani) and for lifetime contributions (such as the prize to Paul Samuelson, Simon Kuznets and Maurice Allais)." Many economists believe that the Nobel Prize has to be awarded for a specific contribution, but in fact, the award oftentimes seems to be for lifetime achievement, which is a fact acknowledged in the essay by Professor Lindbeck on the Nobel Prize website.[32] An observer of the awards will find this consistent with the Nobel prizes awarded in economic sciences from 1969 to 2017. The committee's written statements on an award are sometimes at odds with what a reasonable person would conclude. In other words, the committee may write that an award was given for a specific contribution, when instead, many economists would believe that it was for a lifetime contribution.

Whether an economist receives the prize for lifetime achievement or a specific contribution, the Nobel Committee must decide what constitutes a contribution of that level. Professor Lindbeck states on the Nobel Prize website that the winner should have made "a 'worthy' contribution." The committee looks at the "*originality* of the contribution, its *scientific and practical importance*, and its *impact* on scientific work." Professor Lindbeck believes that it usually takes longer in economics than

[29] It should also be noted that some economists do not think there is anything novel in this new work.

[30] See https://www.nobelprize.org/nobel_prizes/themes/economic-sciences/lindbeck/ for more information.

[31] The categories described by Alfred Nobel were Chemistry, Literature, Peace, Physics, and Physiology or Medicine, which were first awarded in 1901.

[32] Professor Lindbeck discusses this at length:

> Though the Academy, and its selection committee, has followed the same general principles as applied to the prizes in the natural sciences, that is, to award specific contributions, the degree of "specificity" of the awards has varied considerably. Examples of prizes with high specificity are the awards to Wassily Leontief and to contributions to econometric methods, as well as the prizes to game theory and financial economics. Other prizes are characterized by quite small degrees of specificity, such as the prizes to Paul Samuelson, Milton Friedman, Friedrich von Hayek, Gunnar Myrdal and Amartya Sen. In the case of Paul Samuelson, reference was made to his contribution to "raising the level of analysis in economic science." … Simon Kuznets was awarded for his life-time contributions to the empirical analyses of economic development. Thus, the Academy has awarded not only narrowly defined specific contribution but also clusters of such contributions, including life-time achievements if these consist of major contribution to economic science, widely interpreted.

in the natural sciences for the committee to discover whether a new contribution is valid or just a fad. Ironically, though, Nobel Prize winner Joseph Stiglitz jokingly told one of the authors, upon winning the prize in 2001, "You usually get awarded the prize, just as people realize your theories don't work."

Professor Lindbeck denies that political considerations play into the award of a Nobel Prize in Economic Sciences; however, at times it has seemed as if some awards were politically motivated. In addition, even if the prize was not awarded for a political reason, at times, the written statements by the Nobel Committee about the prize seems to be tailored toward a political angle. For example, when Jean Tirole won the Nobel Prize in Economic Sciences, the committee pointed out his work on "how to regulate industries with a few powerful firms." In fact, many economists believed he won it for a lifetime contribution in the field of industrial organization. When Peter Diamond, Dale T. Mortensen, and Christopher A. Pissarides won it in 2010, the world had been through a large recession in 2008 and 2009, in which unemployment increased and labor participation rates declined, and the Nobel Committee awarded the prize to these men for understanding unemployment.

Because the prize in economics only started in 1969, whereas the other prizes started in 1901, the Nobel Committee has had to play catch up. Professor Lindbeck has stated that two dominant criteria are used to choose the order of winners. One is to give early prizes to particularly important contributions (according to their point of view) and to shift the awards between candidates in different fields. There is also a tendency to give prizes in chronological order of discovery, which would favor older candidates. In recent years, it has not been clear that older candidates have been favored. At times, also, potential winners of the Nobel Prize die before the committee can award them the prize. The most notable in this regard was Fischer Black, co-developer of the Black–Scholes formula. It was not until 1974 that the prize rules specified that the award could not be given posthumously except if the winner died between the award being announced and the ceremony date in December.[33] This exception occurred when William Vickrey was awarded the Nobel Prize on October 9, 1996, and died two days later on October 11.[34]

Why is there a prohibition on awarding the prize posthumously? Some speculate that this is because Nobel's original will is interpreted as indicating that the award should be given to the person who made the greatest contribution in the prior year. Despite the Nobel Committee deviating from this view by awarding prizes for contributions made over the recipient's lifetime, they probably felt that it would be in the spirit of the will that winners should be alive at the time the winner of the award was announced. It could also be for the pleasure of having the recipient present at the formal ceremony in Sweden in front of the King and Queen of Sweden.

Professor Lindbeck claims that the Nobel Committee does not use quantitative indicators, such as number or frequency of citations. This is an admirable quality; academic citations are fraught with problems inside the academic system. From our own observations, this seems to be accurate, because the organizations that use citations to forecast the Nobel Prize winners frequently incorrectly predict the Nobel Prize.[35] Having said that, there is also a high correlation between eventual winners and their citation rank.

Furthermore, it seems as though certain fields and certain schools lobby harder for candidates. This might explain why one particular university has so many Nobel winners. It also seems that those who win the Nobel Prize have their work connected to other Nobel Prize winners, even if the connecting logic is mainly constructed by the Nobel Prize Committee.

Given the criteria laid out by Professor Lindbeck for awarding a Nobel Prize, we believe that Stephen Ross certainly should have qualified. In addition, given that the posthumous restrictions are tenuous at best, it would be fitting if the Nobel Committee awarded

[33] Before 1974, the award was only given posthumously to Erik Axel Karlfeldt (1931 Nobel Prize in Literature) and Dag Hammarskjold (1961 Nobel Peace Prize). Since 1974 there was one exception: Ralph Steinman was awarded the 2011 Nobel Prize for Medicine along with two other biologists. Dr. Steinman passed away several days before the announcement, unbeknownst to the Nobel Foundation. Despite his passing, the Nobel foundation decided to grant the award posthumously because it felt that the decision was made in good faith, based on the assumption that the Nobel laureate was alive.

[34] In 2016, the Nobel Prize was announced on October 11. Had that occurred in 1996, Vickrey might not have been able to receive the prize.

[35] For example, Thomson Reuters, which as of 2017 is Clarivate (https://clarivate.com/2017-citation-laureates/).

Stephen Ross and Fischer Black the Nobel Prize at some future date for their major contributions to asset pricing.

CONCLUSION

Stephen Ross was one of the major contributors to economics and financial economics over the years. He was also a good person, and despite his fame and success, he always took the time to help his students or his colleagues. He contributed greatly as a scholar to the departments of Wharton, Yale, and MIT. Unfortunately, he died suddenly and unexpectedly.

Many economists, including the authors of this paper, believe that he should have won the Nobel Prize in Economics prior to his passing in 2017. If one believes in the ideology of Perelman, the great Russian mathematician, then it is irrelevant whether he won the prize or not. What is relevant is the impact he had on the finance profession, economics, and the world.[36]

In this article, we have attempted to highlight some of Ross's most well-known works, including the APT, the binomial option pricing model, and his early contributions to agency theory. We have also highlighted some of his contributions to interest rate modeling, survivorship bias, and, most recently, the recovery theorem. In our opinion, Stephen Ross met the criteria to win the Nobel Prize and probably should have been awarded the prize many years before he died. We are not the only ones who believe this to be true. We asked some Nobel Prize winners in economics what they thought about Stephen Ross.[37] First, we asked William Sharpe, who won the 1990 Nobel Prize for the CAPM, one of the most well-known models in finance. He would probably be most able to comment on the APT work of Ross.

> I strongly believe that Steve should have been given the Nobel Prize for his body of work. Steve was a fascinating, enthusiastic and wonderful person. He loved the field of finance and made many profound contributions to it. — Bill Sharpe, November 2017

Second, we asked Professor Merton, who won the 1997 Nobel Prize for his contributions to option pricing theory. We thought he could comment most about Ross's contribution of the binomial option pricing model.

> There is a significant element of luck in winning the Nobel Prize. I think Steve should have been recognized with the Prize for his many foundational papers in asset pricing and capital markets. Steve was an extraordinary contributor to economics and finance for nearly a half century and even initiated a new fundamental debate at the tail-end of his career. — Bob Merton, November 2017

Third, we spoke to Bengt Hölmstrom, winner of the 2016 Nobel Prize in Economics for his work with incentives and contract theory. Although Ross's work in agency theory was at the beginning of his career and he did not pursue it, we wanted to get an idea of what it meant for people who had pursued the area for their entire life.

> Steve's paper was seminal. It offered the first formulation of a canonical principal-agent model and was instrumental in getting the research on

[36] Grigori Perelman solved the previously unsolved Poincare conjecture. For this and his other contributions, he was awarded the prestigious Fields Medal in Mathematics (the corresponding equivalent to the Nobel Prize), and he refused it. The offer was for "his contributions to geometry and his revolutionary insights into the analytical and geometric structure of the Ricci flow." He declined the award, stating "I'm not interested in money or fame; I don't want to be on display like an animal in a zoo." (BBC News [2010]). He also said the following: "if the proof is correct then no other recognition is needed." On March 18, 2010, it was announced that he had met the criteria to receive the first Clay Millennium Prize for resolution of the Poincare conjecture (BBC News [2006]). The prize carries with it a $1 million gift. On July 1, 2010, he turned down the prize of $1 million, saying that he considered the decision of the board of CMI and the award very unfair and that his contribution to solving the Poincare conjecture was no greater than that of Richard S. Hamilton, the mathematician who pioneered the Ricci flow with the aim of attacking the conjecture (Interfax [2010]; Ritter [2010]). Mikhail Gromov, the Russian geometer, said that he understood Perelman's logic: "To do great work, you have to have a pure mind. You can think only about the mathematics. Everything else is human weakness. Accepting prizes is showing weakness." Others might view Perelman's refusal to accept a Fields as arrogant, Gromov said, but his principles are admirable. "The ideal scientist does science and cares about nothing else," he said. "He wants to live this ideal. Now, I don't think he really lives on this ideal plane. But he wants to" (*The New Yorker* [2006]).

[37] During the research for this paper, we learned that other economists had suggested Stephen Ross to the Nobel Prize committee.

> agency theory started. Subsequent work led to alternative formulations, more amenable to analysis and interpretation. But the basic structure of Steve's original formulation has survived intact. Steve went on to do seminal work in many areas of finance, too. The hallmark of all his research is elegance, generality and depth. He would have been a most worthy Nobel Prize winner.
> — Bengt Hölmstrom, November 2017

In terms of Stephen Ross's other contributions, we asked Harry Markowitz his opinion, and he told us what he thought about the APT and the CIR model.

> It would have been certainly plausible for Ross to win the Nobel Prize for his work with the APT. In my consulting work for a major firm, I used the CIR model to simulate interest rates, because, at the time, given low interest rates, the CIR model generated the most plausible scenarios as compared with historical interest rate history.
> — Harry Markowitz, November 2017

Ultimately, Professor Ross did not win a Nobel Prize before he died, but his contributions went beyond the recognition of a committee. Like Fischer Black, his work stands taller than any award-giving body. Although he only could see from one eye, his vision through his work was ultimately clearer than someone with multiple sets of eyes. Stephen Ross was one of the people who helped contribute to the field of financial economics so that the rest of us had a better understanding of it. He used to say to colleagues that "Finance was in a class by itself." What he meant, we think, is that the link between theory and practice was so fruitful. What he did not say is that he helped it get that way.

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