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Tracking spot oil: The elusive quest☆

Ludwig Chincarini [a,b]

[a] University of San Francisco, School of Management, 101 Howard Street, Suite 500, San Francisco, CA 94105, USA
[b] USCF Investments, 1999 Harrison Street #1530, Oakland, CA 94612, USA

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ABSTRACT

It has been well documented that oil investing vehicles do a poor job of tracking spot oil over long periods. We create optimized investment vehicles that attempt to use backward- and forward-looking indicators in the oil market in order to better track spot oil. Nevertheless, we find that it is extremely difficult to track spot oil using a combination of oil futures, oil stocks, and oil ETFs. However, some contango drag from investing in oil futures can be eliminated, although at the cost of a higher tracking error, by using a forward-looking dynamic strategy that invests in both oil futures and oil stocks.

1. Introduction

Managing the tracking error of an oil portfolio is very different from managing the tracking error of an equity or a bond portfolio. The primary reason is that investing in the underlying asset is impractical for an oil portfolio, while it is very easily done for both equities and bonds. When managing a liquid oil investment vehicle, a portfolio manager can invest in oil futures contracts or in public companies that are engaged in the oil or energy business. This is how most oil ETFs and mutual funds invest in oil.[1]

Just as S&P 500 index managers are measured against S&P 500 returns, crude oil managers are often measured against spot crude oil returns, usually those of West Texas Intermediate (WTI) or Brent crude oil. Although ETFs do not claim to match the spot commodity return in their prospectus, this is an implicit benchmark adopted by many practitioners and investors.[2] Oil investment

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E-mail addresses: chincarinil@hotmail.com, lbchincarini@usfca.edu.

[1] Although individual investors typically choose to invest in an oil ETF or mutual fund, they can also directly purchase oil futures or oil company stocks.

[2] For example, the prospectus of the largest oil ETF, the United States Oil Fund (USO), states that USO is "an exchange-traded security designed to track the daily price movements of West Texas Intermediate light, sweet crude oil" as measured by the near-month crude oil futures contract traded on the NYMEX. More specifically, the investment objective of USO is for the "daily changes in percentage terms of its shares' per share net asset value ('NAV') to reflect the daily changes in percentage terms of the spot price of light, sweet crude oil delivered to Cushing, Oklahoma, as measured by the daily changes in the price of a specified short-term futures contract on light, sweet crude oil, which is the futures contract on light, sweet crude oil as traded on the New York Mercantile Exchange (the 'NYMEX') that is the near month contract to expire, except when the near month contract is within two weeks of expiration, in which case it will be measured by the futures contract that is the next month contract to expire."

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vehicles' failure to move in line with spot oil has been criticized by investors (Blas, 2008; Burton and Karsh, 2009; Constable, 2016; Eisen and Josephs, 2016; HedgeWise, 2014). However, while this failure is well documented, the cause of the tracking error remains a source of disagreement (Bessembinder et al., 2016; Fattouh et al., 2013; Irwin and Sanders, 2011; Main et al., 2013; Singleton, 2014; Stoll and Whaley, 2010).

We know that neither investing in oil futures contracts nor investing in oil companies does a good job of tracking spot oil. This paper examines whether a static or dynamic optimization strategy might be able to track oil better than these fairly simple indexing techniques.[3] To our knowledge, there is no published research on this topic. We therefore seek to optimize strategies for creating portfolios of oil stocks and oil futures with the explicit objective of minimizing the ex-ante tracking error with respect to WTI spot oil.

We find that it is extremely difficult to significantly reduce the tracking error, even when using forward-looking information about the oil futures curve in order to create optimized portfolios across stocks and oil futures. On the positive side, we find that the average excess return (i.e., a portfolio's outperformance) can be improved using these techniques, but this incurs a higher tracking error. Finally, in addition to our main purpose, we also document many of the problems faced by managers of oil investment vehicles.

The rest of the paper is organized as follows. Section 2 discusses the data. Section 3 discusses the tracking issues surrounding oil futures, oil ETFs, and oil mutual funds. Section 4 describes several optimization techniques that might be used to track spot oil. Section 5 reports the tracking performance of the optimization models. Finally, Section 6 concludes the paper.

2. The data

For our empirical analysis of investing in oil, we obtain daily oil futures data from Bloomberg along with daily stock return data from CRSP. For our risk-free rate of return, we use the daily return series of the 1-month Treasury bill, obtained from the Fama-French factor database.[4] For the spot price of oil, we use the Cushing WTI Spot Price FOB, obtained from the Federal Reserve Bank of St. Louis.[5] For each oil futures contract (i.e. the first twelve contracts), we construct a continuous return series by rolling each futures contract on the day of expiration of the nearest contract (a.k.a. the last day of trading).[6]

We select oil companies from the CRSP based on their standard industrial classification (SIC) codes.[7] The group of companies that we consider as candidates for our historical optimizations are listed in Table 1. We classify oil and gas companies into three categories: upstream, midstream, and downstream companies. The upstream companies are involved in exploration and production, while the midstream companies are involved in processing, storing, marketing, and transporting oil and gas products. The downstream companies include oil refineries, petrochemical plants, petroleum product distributors, retail outlets, and natural gas distribution businesses. Each stock was assigned to one of these categories based on its SIC code.

Table 2 shows the average correlation of the stock returns of the companies in each industry with spot oil returns. We also compute the average beta for companies in each industry with respect to spot oil. The statistics in the table are computed using monthly return data. The table illustrates that some oil-related companies have a high correlation with spot oil, while others, such as the midstream companies, have a lower correlation with spot oil.

We split our sample into two time periods, from 1994 to 2005 and from 2006 to 2017. The reason for the two sample periods is the absence of oil futures ETFs prior to 2006; the first oil futures ETF was launched back in 2006. The companies that have the highest correlation with spot oil are the upstream oil companies. For the first period, the upstream company correlations are all 24% or higher, and they are all 46% or higher for the second period. These correlations are also supported by the betas for each industry versus spot oil. The betas provide a more intuitive indication of how the return in an industry is related to the return in spot oil. For example, over the period 2006 to 2017, a 1% change in spot oil prices led to a 0.56% change in the return of the average Oil and Gas Field Exploration Service company, while it only led to a 0.11% change for the average Crude Petroleum Pipeline company.[8] While these correlations are not incredibly high, they do offer some support for the idea that the "right" basket of oil companies might be able to track spot oil.

To make the analysis more tractable, we ranked the companies by market capitalization and used only the top 100 of these for the optimizations.[9] The characteristics of the top 100 companies over the two sample periods (although these could change on any given day) are shown in Table 3.[10] The table illustrates that most of our companies came from two industries, the Crude Petroleum

[3] One might argue that traders and investors would be more interested in a strategy that outperforms a passive strategy of investing in spot oil rather than mimicking spot oil. This is in fact a by-product of our analysis, as discussed later in this paper.

[4] Although the benchmark for the oil industry is the 3-month Treasury bill, we use the 1-month bill since total return data were readily available.

[5] This is the series DCIWTICO at https://research.stlouisfed.org/fred2. The same data can be found at http://www.eia.gov/petroleum/data.cfm.

[6] That is, we sell the current contract and buy the next maturity contract. We found the same general results when we rolled the futures contracts at 3, 5, 10, and 15 days before expiration of the nearest contract.

[7] Although not reported in this paper, we also performed the same analysis using NAICS codes and found similar results to those we present in this paper.

[8] The low (0.11%) correlation for the Crude Petroleum Pipeline companies, which are often Master Limited Partnerships (MLPs), is one of the reasons they are often attractive to retirees.

[9] Shares outstanding were lagged by 3 months to avoid a look-ahead bias. We also performed our analysis on the entire set of oil companies, but the results did not change in any qualitative way.

[10] In some cases, we have stocks that were delisted on day $t + 1$, which may not have been known on day t (although we usually know this information prior to the day of delisting due to announcements from the stock exchange or the company itself). To facilitate our analysis, we eliminated these stocks from the optimization on day t. Most of these securities are likely to be smaller companies. We checked forward every single day to see whether any securities failed to be traded.

Table 1
SIC classification codes for oil companies.

SIC	Industry	SIC Name	Business Description
1311	Mining	Crude Petroleum and Natural Gas	Establishments primarily engaged in operating oil and gas field properties.
1381	Mining	Drilling Oil and Gas Wells	Establishments primarily engaged in drilling wells for oil or gas field operations for others on a contract or fee basis.
1382	Mining	Oil and Gas Field Exploration Services	Establishments primarily engaged in performing geophysical, geological, and other exploration services for oil and gas on a contract or fee basis.
1389	Mining	Oil and Gas Field Services	Establishments primarily engaged in performing oil and gas field services, not elsewhere classified, for others on a contract or fee basis.
2911	Manufacturing	Petroleum Refining	Establishments primarily engaged in producing gasoline, kerosene, distillate fuel oils, residual fuel oils, and lubricants
3533	Manufacturing	Oil and Gas Field Machinery and Equipment	Establishments primarily engaged in manufacturing machinery and equipment for use in oil and gas fields or for drilling water wells
4612	Transportation & Utilities	Crude Petroleum Pipelines	Establishments primarily engaged in the pipeline transportation of crude petroleum.
4613	Transportation & Utilities	Refined Petroleum Pipelines	Establishments primarily engaged in the pipeline transportation of refined products of petroleum, such as gasoline and fuel oil.
5171	Wholesale Trade	Petroleum Bulk Stations and Terminals	Establishments primarily engaged in the wholesale distribution of petroleum products
5172	Wholesale Trade	Petroleum and Petroleum Products Wholesalers	Establishments primarily engaged in the wholesale distribution of petroleum products

Note: This table contains the SIC codes associated with oil companies that were chosen for the historical optimizations. These codes are used to identify oil companies in the stock universe. This oil company list was compiled by manually inspecting the SIC Names for words like oil, petroleum, or anything else that seemed related to oil production.

Table 2
Average correlations of oil company stocks with spot oil returns by SIC classification.

SIC Industry	SIC Code	Avg. Stocks	1994–2005		2006–2017		Entire Sample	
			Corr.	β_s	Corr.	β_s	Corr.	β_s
Upstream								
Crude Petroleum and Natural Gas	1311	91.66	0.26	0.41	0.46	0.44	0.40	0.43
Drilling Oil and Gas Wells	1381	15.54	0.31	0.51	0.55	0.51	0.47	0.51
Oil and Gas Field Exploration Services	1382	7.68	0.26	0.46	0.47	0.56	0.41	0.54
Oil and Gas Field Services	1389	12.06	0.24	0.40	0.49	0.43	0.42	0.43
Oil and Gas Field Machinery and Equipment	3533	7.97	0.26	0.53	0.54	0.42	0.45	0.46
Midstream								
Crude Petroleum Pipelines	4612	6.07	0.18	0.10	0.27	0.11	0.25	0.11
Refined Petroleum Pipelines	4613	1.29	0.07	0.08	0.39	0.41	0.25	0.21
Downstream								
Petroleum Refining	2911	21.48	0.20	0.26	0.42	0.25	0.35	0.24
Petroleum Bulk Stations and Terminals	5171	2.12	0.23	0.28	0.22	0.13	0.23	0.17
Petroleum and Petroleum Products Wholesalers	5172	2.74	0.11	0.23	0.23	0.19	0.20	0.21

Note: The table shows the correlation and beta of Oil & Gas related companies' returns based on SIC code with spot oil returns. Corr. represents the average of correlations relative to spot oil across all stocks within each category using monthly returns. β_s represents the average exposure to oil spot. The betas are constructed by estimating regressions of individual monthly stock returns over a 3-year rolling period on monthly returns of spot oil using the equation $r^i_{t,t+1} = a + br^S_{t,t+1} + \epsilon$, where $r^S_{t,t+1}$ is the monthly return of spot oil from t to $t+1$ and $r^i_{t,t+1}$ is the monthly return of the underlying stock. The average of all stock betas for each industry is reported in the table.

and Natural Gas sector (49% of the stock universe during the first period and 44% during the second period) and the Petroleum Refining sector (17.45% and 17.79%, respectively, during the first and second periods). Columns 8, 9, and 10 reveal that some industries outperformed spot oil in both periods, while others underperformed in both periods. All of the industries have a higher tracking error with respect to spot oil than they do with respect to the individual futures contracts. For example, the Petroleum and Petroleum Products Wholesalers sector has an annualized tracking error of 49% with respect to spot oil in the first period and 40% in the second period. The number of stocks from any industry that were used in our sample varied over time, with a high of 58 stocks from the Crude Petroleum and Natural Gas industry and a low of zero stocks for several other industries, including Oil and Gas Field Exploration Services.

In the oil futures data, we limit ourselves to the first 12 months of the futures contracts. Over different time periods, their tracking error with respect to spot oil varied. Table 4 shows the characteristics of these futures contracts over the two time periods.

Table 3
Summary statistics of sample of 100 largest stocks.

SIC Industry	SIC Code	Avg. Stocks	Max	Min	Avg. Return	S.D.	Excess Risk-Free	Excess Spot Oil	T.E.
Sample Period: 1994–2005									
Crude Petroleum and Natural Gas	1311	48.90	58.00	37.00	13.44	20.56	9.64	1.17	37.39
Drilling Oil and Gas Wells	1381	11.38	15.00	7.00	14.64	36.17	10.84	2.37	43.03
Oil and Gas Field Exploration Services	1382	4.65	10.00	0.00	−7.57	32.00	−11.37	−19.85	43.36
Oil and Gas Field Services	1389	7.95	11.00	5.00	12.48	34.45	8.68	0.21	43.10
Oil and Gas Field Machinery and Equipment	3533	5.11	10.00	1.00	17.30	41.42	13.50	5.03	47.05
Petroleum Refining	2911	17.45	22.00	12.00	14.33	20.62	10.54	2.06	39.16
Crude Petroleum Pipelines	4612	1.60	3.00	1.00	12.52	19.23	8.72	0.25	42.09
Refined Petroleum Pipelines	4613	1.71	4.00	0.00	11.81	16.46	8.02	−0.46	40.78
Petroleum Bulk Stations and Terminals	5171	0.44	1.00	0.00	9.21	26.07	5.42	−3.06	45.75
Petroleum and Petroleum Products Wholesalers	5172	0.81	2.00	0.00	16.33	34.94	12.54	4.06	49.40
Sample Period: 2006–2017									
Crude Petroleum and Natural Gas	1311	44.20	58.00	18.00	3.21	33.09	2.20	3.66	31.82
Drilling Oil and Gas Wells	1381	10.55	15.00	7.00	−9.43	40.21	−10.44	−8.98	35.61
Oil and Gas Field Exploration Services	1382	3.87	31.00	0.00	−0.79	45.04	−1.79	−0.34	41.60
Oil and Gas Field Services	1389	7.90	12.00	4.00	3.72	34.79	2.72	4.18	35.12
Oil and Gas Field Machinery and Equipment	3533	6.58	9.00	4.00	3.16	37.78	2.15	3.61	36.22
Petroleum Refining	2911	17.79	26.00	11.00	8.11	24.44	7.10	8.56	33.26
Crude Petroleum Pipelines	4612	6.08	11.00	2.00	13.27	21.46	12.26	13.72	34.27
Refined Petroleum Pipelines	4613	0.84	5.00	0.00	1.75	23.37	0.74	2.20	38.28
Petroleum Bulk Stations and Terminals	5171	0.78	2.00	0.00	−5.98	30.34	−6.99	−5.53	41.07
Petroleum and Petroleum Products Wholesalers	5172	1.41	5.00	0.00	−4.10	23.97	−5.11	−3.65	39.60

Note: The table shows the summary statistics of the top 100 oil stocks in our investment universe by market capitalization. Avg. Stocks indicates the average number of stocks belonged to each SIC industry over the sample period. Min and Max represent the minimum and maximum of number of stocks belonging to each SIC industry over the sample period. Avg. Return represents the compounded geometric return over the sample period of the market capitalized weighting of stocks in the industry. S.D. represents the annualized standard deviation of the returns computed as the daily standard deviation multiplied by $\sqrt{250}$. Excess Risk-free is the difference between the annualized compounded return of the industry and the risk-free rate. Excess Spot Oil is the difference between the annualized compounded return of the industry and the spot oil return. T.E. is the tracking error of the industry returns with respect to spot oil, which is computed as the annualized standard deviation of the daily differences between the industry returns and spot oil returns.

The summary statistics reaffirm what has been documented in other research and in the press: Individual futures contracts do not track spot oil well. During the first time period (1994–2005), short-term maturity oil futures outperformed spot oil more than 50% of the time, while longer maturity contracts did not. The geometric annualized excess return was positive for all of the first 12 futures contracts, with the lowest being 5.04% for the 12-month futures contract. Over this period, the oil futures market was generally in backwardation, with the average annualized contango being more than −5% for many contracts.

During the second time period (2006–2017), the tracking error for oil futures contracts was still high. However, due to the presence of contango and for other reasons, the oil futures contracts underperformed spot oil by as much as −10.50% for the near-term futures contract and −1.17% for the 12-month futures contract. Finally, the annualized returns of spot oil and cash over the first sample period were 12.27% and 3.79%, respectively, while they were −0.45% and 1.01% for the second sample period.

In this paper, we use data beginning in January 1994 since futures data were more reliable and accurate after 1994, and we exclude the dates on which no oil futures were traded.[11]

3. Tracking performance of oil investment vehicles

The first challenge for oil investing is that the perceived benchmark for oil investors is the spot oil return. Since investing in oil futures or oil companies is not necessarily the same as investing in spot oil, this creates perception problems. In fact, a strategy of investing in individual oil futures contracts will not replicate the returns of spot oil, nor will a strategy of investing in oil companies. All of the exchange-traded funds that invest in oil companies or oil futures have similar tracking errors with respect to spot oil, which has led to complaints and criticisms in the press (Blas, 2008; Burton and Karsh, 2009; Constable, 2016; Eisen and Josephs, 2016).

Moreover, a simple spot oil benchmark is not realistic because it is not possible to possess oil without significant costs. To invest directly in spot oil, an investor would have to buy physical oil and store it somewhere.[12] Oil storage has been estimated to cost

[11] There were 23 such observations in our sample. The dates were 1994/11/25, 1995/07/03, 1995/11/24, 1996/07/05, 1996/11/29, 1997/11/28, 1998/11/27, 1999/11/26, 1999/12/31, 2000/01/03, 2000/07/03, 2000/11/24, 2001/11/23, 2001/12/24, 2002/07/05, 2002/11/29, 2003/11/28, 2003/12/26, 2004/01/02, 2004/11/26, 2004/12/31, 2005/11/25, and 2007/01/03. Most of these dates coincided with missing spot oil returns. In addition, there were three instances when only the first two or first three futures contracts were active. The 23 dates normally fell around U.S. holidays. For example, on July 5, 1996, right after Independence Day, the stock market closed early at 1 p.m. and the commodities exchanges in New York were closed all day. These events were announced ahead of time. We also excluded days on which the stock market was closed. We examined two ways to deal with the problem of excluded or deleted trading days. First, we do not rebalance the portfolio until the next valid trading day for oil futures. This is equivalent to not trading on days when the futures market did not trade. Second, we assume that oil futures' returns on days with no futures data were equal to 0.

[12] Cushing, Oklahoma, for example, is a common place to store oil in the United States. Oil can also be stored in ships on the ocean, as well as in salt dome caverns, large above-ground tanks, small above-ground tanks, and floating storage tanks (these are listed in increasing order of storage costs).

Table 4
Summary statistics of first 12 oil futures contracts.

Contract	Mean	S.D.	Excess	% Beat	β_s	Contango	T.E.	Avg.Volume
Sample Period: 1994–2005								
Fut1Roll0	23.01	34.45	10.73	51.72	0.79	−2.13	18.62	64.99
Fut2Roll0	21.77	31.58	9.49	50.85	0.71	−4.51	19.82	46.80
Fut3Roll0	22.29	29.12	10.02	51.08	0.64	−5.05	20.79	14.89
Fut4Roll0	21.74	27.29	9.47	49.98	0.59	−5.38	21.66	6.45
Fut5Roll0	21.31	25.97	9.04	49.38	0.56	−5.53	22.44	3.55
Fut6Roll0	20.88	24.91	8.60	48.98	0.53	−5.59	23.17	2.53
Fut7Roll0	20.15	24.08	7.88	49.08	0.51	−5.61	23.80	1.88
Fut8Roll0	19.76	23.40	7.49	49.13	0.48	−5.58	24.40	1.51
Fut9Roll0	18.83	22.81	6.55	49.08	0.47	−5.53	24.95	1.15
Fut10Roll0	18.39	22.30	6.12	49.15	0.45	−5.46	25.44	0.89
Fut11Roll0	17.88	21.87	5.61	49.03	0.43	−5.37	25.93	0.70
Fut12Roll0	17.31	21.49	5.04	48.88	0.42	−5.28	26.37	0.64
Spot	12.27	38.42	.	.	1.00	.	.	
Cash	3.79	0.11	.	.	.	.	.	.
Sample Period: 2006–2017								
Fut1Roll0	−10.95	36.84	−10.50	48.64	0.93	0.02	11.04	309.36
Fut2Roll0	−10.52	34.69	−10.07	48.64	0.85	0.07	13.96	156.42
Fut3Roll0	−7.93	33.57	−7.48	48.59	0.81	0.07	15.14	58.08
Fut4Roll0	−6.35	32.62	−5.90	50.15	0.78	0.06	15.83	32.60
Fut5Roll0	−4.97	31.79	−4.52	50.35	0.76	0.05	16.38	21.73
Fut6Roll0	−3.86	31.06	−3.41	50.51	0.73	0.05	16.84	16.14
Fut7Roll0	−3.18	30.39	−2.72	51.05	0.72	0.04	17.26	12.15
Fut8Roll0	−2.69	29.78	−2.24	50.98	0.70	0.04	17.67	9.45
Fut9Roll0	−2.31	29.19	−1.86	51.34	0.68	0.04	18.08	7.77
Fut10Roll0	−2.05	28.64	−1.60	51.31	0.66	0.03	18.48	6.63
Fut11Roll0	−1.82	28.13	−1.37	51.21	0.65	0.03	18.88	4.81
Fut12Roll0	−1.62	27.66	−1.17	51.11	0.64	0.03	19.25	4.65
Spot	−0.45	38.10	.	.	1.00	.	.	.
Cash	1.01	0.11	.	.	.	.	.	.

Note: The table shows the summary statistics of the first 12 oil futures contracts rolled at maturity. For each futures contract, the first number indicates the specific futures contract, either 1, 2, 3, …12, depending on whether the nearest-term, 2nd, 3rd, or any other until the 12th contract is used. The second number represents the roll date. Thus, a "0" indicates the contract was theoretically rolled on the expiration date of the nearest contract. Mean represents the compounded geometric return over the sample period. S.D. represents the annualized standard deviation of the returns computed as the daily standard deviation multiplied by $\sqrt{250}$. Excess is the difference between the annualized compounded return of each futures contract and the spot oil return. % Beat is the percentage of times in which a given futures contract's return exceeded the spot oil return. β_s represents the average exposure of the futures strategy to spot oil. Beta of futures is estimated by using daily return data over the sample period and estimating the following regression: $r^i_{t,t+1} = a + br^S_{t,t+1} + \epsilon$, where $r^S_{t,t+1}$ is the return of spot oil from t to $t+1$ and $r^i_{t,t+1}$ is the return of the underlying futures contract. Contango is calculated daily as $250 * (F_i - S)/F_i$, where a positive value indicates contango and a negative value indicates backwardation in the market. T.E. is the tracking error of the futures contract measured against spot oil, which is estimated as the annualized standard deviation of the daily differences between the futures contract returns and spot oil returns. Avg. Volume is the daily average of dollar trading volume for each futures contract in 1000s of U.S. dollars over the sample period. Cash represents the Fama-French one-month Treasury bill return.

anywhere between \$0.20 and \$1.20 per barrel per month.[13] This is not a trivial amount. For example, when oil trades at \$36 per barrel, this is equivalent to a cost of 0.56%–3.3% of the trade per month, depending on which value is used. Thus, a more relevant index for oil would be one that subtracts storage costs from the spot price of oil.[14]

In addition to storage costs driving a wedge between oil futures returns and spot oil returns, contango, risk premiums, and crowding can also create a divergence between the two (Bessembinder et al., 2016; Bruno et al., 2018; Chincarini 2012, 2018; Chincarini et al., 2016; Fattouh et al., 2013; Irwin and Sanders, 2011; Main et al., 2013; Singleton, 2014; Stoll and Whaley, 2010). Despite being unrealistic, the spot oil benchmark is nevertheless used as a comparison benchmark for both oil practitioners and

[13] Deutsche Bank estimates that oil storage costs are roughly \$0.40 per barrel per month and that they amounted to costs of 22% per annum between 1989 and 1994 (Brhanavan et al., 2007). In 2008, analysts estimated that oil storage costs on land ran between 40 and 70 cents per barrel per month and that floating and ship storage costs ran as high as \$1.60 per barrel per month (Blas, 2008). Goldman Sachs estimated a monthly cost of 0.80% for oil storage (Greely, 2008).

[14] Chincarini et al. (2016) suggest that the return for a more appropriate oil benchmark might be calculated as.

$$r_{t,t+k} = \left(\frac{S_{t+k}}{S_t} - 1\right) - \frac{u \cdot k}{30}, \tag{1}$$

where u represents the monthly storage costs, k represents the days over which the return is computed, and a typical month is assumed to consist of 30 days. In their paper, they use \$0.40 as their estimate of storage costs, based on the Deutsche Bank research (Brhanavan et al., 2007). Storage costs are not the only costs involved, as there are also insurance and transportation costs. It is extremely difficult to obtain actual storage costs, as most private companies either do not collect or will not disclose this information, nor do public agencies capture this data. One way to create a historical storage cost index might be to combine historical inflation or rent costs in an area with the actual value of storage costs and then adjust this figure based on storage capacity utilization.

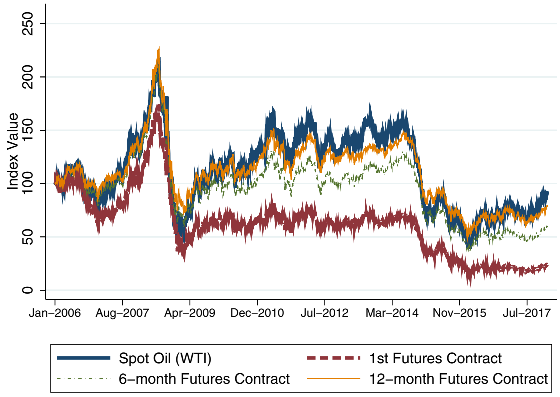

Fig. 1. Indices of Oil Futures and Spot Oil. This figure shows the cumulative returns of investing in spot oil and three rolling futures contracts; the 1-month, 6-month, and 12-month futures contracts. The strategy rolls futures on the expiration date of the nearest-term contract at closing futures prices. All indices start at 100.

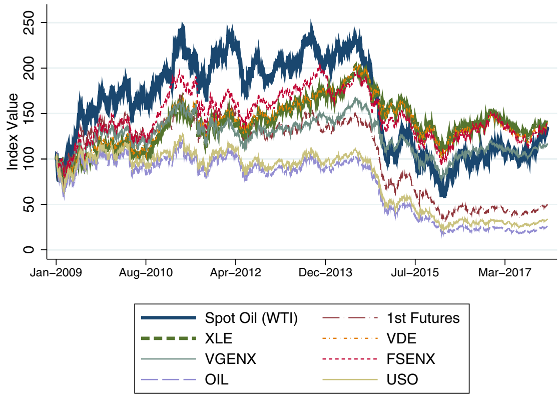

Fig. 2. The Performance of the Largest Oil Investing ETFs and Mutual Funds. The figure shows the net-of-fee performance of the largest ETFs investing in oil companies, XLE and VDE; the largest mutual funds investing in oil companies, FSENX and VGENX, and the largest ETFs investing in oil futures, USO and OIL. The spot oil index and the 1st futures index is also shown. Spot oil was obtained from the Federal Reserve of St. Louis. The series is DCIWTICO, which can be downloaded from https://research.stlouisfed.org/fred2. The same data can be found at the http://www.eia.gov/petroleum/data.cfm. All other data is from Bloomberg. All indices start at 100.

hedgers of oil. Thus, it makes sense to explore whether spot oil can be better tracked.

The divergence between oil futures and spot oil is illustrated in Fig. 1, which shows the growth of $100 from January 2006 to December 2017 from rolling the nearest-term futures contract, the 6-month futures contract, and the 12-month futures contract. All of these futures contracts have a tracking error with regard to spot oil, but the worst tracking error occurs with the 1st futures contract (i.e., in the nearest-term futures contract).

In the ETF and mutual fund world, some managers invest in oil by buying futures contracts, and others invest in oil by buying oil company stocks. These are also not free from the tracking problem. Fig. 2 shows the performance of the two largest oil ETFs that invest in futures (USO and OIL), the two largest oil ETFs that invest in oil companies (XLE and VDE), and the two largest mutual funds that invest in oil companies (VGENX and FSENX).[15] It is evident from the figure that the worst performers against spot oil were USO and OIL, the two ETFs that used futures. Over the entire period (2009–2017), the total returns for USO and OIL were −63.72% and −71.90%, respectively, compared to 38.60% for spot oil.[16] The ETFs that invested in oil companies had the smallest tracking error over the entire period from 2009 to December 2017.[17] Over the entire period, the total returns for XLE and VDE were 51.27% and 46.90%, respectively, compared to 38.60% for spot oil. However, in the interim, even these ETFs had a poor tracking performance with respect to spot oil. The tracking error on an annualized basis was greater than 28% for all ETFs and mutual funds investing in oil companies. Ultimately, then, tracking errors are a problem for all oil investment vehicles.

[15] The "largest" means those with the most assets under management as of the beginning of 2018. Neither of the mutual funds invest solely in futures contracts.

[16] This differs from the raw return over the entire period using just spot oil prices, since we deleted some days because the stock market was not trading.

[17] We began our analysis with 200, since many of the ETFs did not exist prior to that year. USO was the first oil-investing ETF.

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Table 5
Summary statistics from rolling future strategies and oil ETFs and mutual funds.

Strategy	Futures					Cash Mean	Spot			Tracking Error		
	Mean	"Roll"	"Spot"	S.D.	Sharpe		Mean	S.D.	Sharpe	Excess	T.E.	β
Investment Period: 1994–December 2005												
Fut1Roll0	26.66	2.83	23.83	34.45	0.67	3.72	18.98	38.42	0.40	11.40	18.62	0.80
Fut2Roll0	24.70	6.70	17.99	31.58	0.66	3.72	18.98	38.42	0.40	9.44	19.82	0.73
Fut6Roll0	22.07	8.24	13.83	24.91	0.74	3.72	18.98	38.42	0.40	6.82	23.17	0.56
Fut12Roll0	18.28	7.75	10.53	21.49	0.68	3.72	18.98	38.42	0.40	3.02	26.37	0.44
Investment Period: 2006–December 2017												
Fut1Roll0	−4.82	−3.68	−1.14	36.84	−0.16	1.00	6.77	38.10	0.15	−10.59	11.04	0.93
Fut2Roll0	−5.10	−10.54	5.45	34.69	−0.18	1.00	6.77	38.10	0.15	−10.87	13.96	0.87
Fut6Roll0	0.89	−7.28	8.17	31.06	−0.00	1.00	6.77	38.10	0.15	−4.88	16.84	0.76
Fut12Roll0	2.19	−4.30	6.49	27.66	0.04	1.00	6.77	38.10	0.15	−3.58	19.25	0.66
Investment Period: 2009–December 2017												
Fut1Roll0	−1.00	−3.77	2.77	35.32	−0.03	0.15	10.11	36.14	0.28	−10.96	9.24	0.94
Fut2Roll0	−2.74	−10.51	7.78	33.29	−0.09	0.15	10.11	36.14	0.28	−12.70	12.20	0.88
Fut6Roll0	1.89	−7.53	9.42	29.68	0.06	0.15	10.11	36.14	0.28	−8.07	15.70	0.77
Fut12Roll0	2.19	−4.46	6.65	26.11	0.08	0.15	10.11	36.14	0.28	−7.77	18.42	0.66
ETFs using Oil Futures Contracts												
USO	−5.79	.	.	32.88	−0.18	0.15	10.11	36.14	0.28	−15.90	14.73	0.83
OILNF	−7.75	.	.	35.31	−0.22	0.15	10.11	36.14	0.28	−17.86	18.22	0.86
UCO	−14.34	.	.	63.78	−0.23	0.15	10.11	36.14	0.28	−24.45	34.56	1.58
DBO	−2.88	.	.	29.00	−0.10	0.15	10.11	36.14	0.28	−12.99	18.27	0.71
SCO	7.77	.	.	64.23	0.12	0.15	10.11	36.14	0.28	−2.34	98.28	−1.61
DWT	.	.	.	.	.	.	.	.	.	.	.	.
UWT	.	.	.	.	.	.	.	.	.	.	.	.
UWTIF	.	.	.	.	.	.	.	.	.	.	.	.
BNO	.	.	.	.	.	.	.	.	.	.	.	.
USL	−0.12	.	.	27.60	−0.01	0.15	10.11	36.14	0.28	−10.24	18.39	0.67
Top 5 ETFs and Mutual Funds using Oil Stocks												
XLE	7.37	.	.	23.64	0.31	0.15	10.11	36.14	0.28	−2.74	28.86	0.42
VDE	7.19	.	.	24.24	0.29	0.15	10.11	36.14	0.28	−2.92	28.52	0.43
XOP	8.51	.	.	34.64	0.24	0.15	10.11	36.14	0.28	−1.60	30.55	0.62
IXC	4.81	.	.	23.02	0.20	0.15	10.11	36.14	0.28	−5.30	28.65	0.41
IYE	6.47	.	.	23.46	0.27	0.15	10.11	36.14	0.28	−3.64	28.65	0.42
VGENX	5.18	.	.	24.47	0.21	0.15	10.11	36.14	0.28	−4.93	28.37	0.45
FSENX	8.32	.	.	28.05	0.29	0.15	10.11	36.14	0.28	−1.79	29.07	0.50
FAGNX	9.51	.	.	27.84	0.34	0.15	10.11	36.14	0.28	−0.60	28.75	0.50
VENAX	7.17	.	.	24.36	0.29	0.15	10.11	36.14	0.28	−2.94	28.61	0.44
FTSEX	4.30	.	.	28.54	0.15	0.15	10.11	36.14	0.28	−5.81	28.28	0.52

Note: The table presents the various statistics with respect to futures and spot returns of oil. For each futures contract, the first number indicates the specific futures contract, either 1, 2, 6, or 12 depending on whether the nearest-term, 2nd, 6th, or 12th contract is used. The second number represents the roll date. Thus, a "0" indicates the contract was theoretically rolled on the expiration date of the nearest-term contract. Mean represents the annualized average return calculated as the average daily return multiplied by 250, "Roll" represents the annualized return of the expected roll, "Spot" represents the annualized return of the "spot" change according to the following decomposition of futures returns into two components, $r^i_{t,t+k,d} = \underbrace{\frac{F^*_{t,d,m-k} - F^i_{t,d,m}}{F^i_{t,d,m}}}_{\text{"Expected Roll"}} + \underbrace{\frac{F^i_{t+k,d,m-k} - F^*_{t,d,m-k}}{F^i_{t,d,m}}}_{\text{"Spot Return"}}$. Cash represents the annualized average return of a one-month Treasury bill, S.D. represents the annualized standard deviation of the returns computed as the daily standard deviation multiplied by $\sqrt{250}$. Sharpe represents the annualized Sharpe ratio computed as the average return of the respective instrument minus the risk-free rate of return divided by the volatility of that difference. Excess represents the annualized average return difference between the futures investing strategy plus cash and the actual spot oil returns, except in the case of ETFs and mutual funds where it represents their net-of-fee excess returns over spot oil. T.E. represents the annualized tracking error of the futures strategy plus it's cash return on collateral which is assumed to get 100% of the cash return, and the spot return of oil and β is the beta from a regression of the daily spot return in oil against the daily return of the futures contract. All values are multiplied by 100, except for β, so as to represent percentage points.

Table 5 shows the annualized average daily statistics for the various future contracts strategies as well as selected ETFs and mutual funds. Over the first period, 1994 to 2005, the average return for rolling the near-term contract was 26.66%.[18] The high returns were similar for the 2-, 6-, and 12-month futures, at 24.70%, 22.07%, and 18.28%, respectively. A significant portion of the returns came from the "roll" of the contract or the backwardation in the marketplace, with the exception of the near-term futures

[18] These are arithmetic average returns, not geometric average returns.

contract, which moved closely with spot oil.[19] The roll effect is the average estimated loss from rolling futures contracts due to futures prices being higher than spot oil prices on average (i.e., the contango effect).

Over the same period, spot oil had a return of 18.98%. Thus, this was generally a good time for investing in oil futures, which significantly outperformed spot oil. Typically, investors invest in oil on a leveraged basis, since oil futures require a small margin for investing. Thus, in addition to the oil futures return, a strategy of investing in oil futures will secure an extra pick-up from investing the cash in a cash instrument. For our purposes, we assumed full collateralization of the strategy and used the Fama-French 1-month Treasury return as the cash return, which over this period was 3.72% annualized. Thus, the total excess performances of the futures-investing strategy were approximately 11.40%, 9.44%, 6.82%, and 3.02%, respectively, for the 1-month, 2-month, 6-month, and 12-month maturity contracts.

We also measured the daily beta for each strategy with respect to spot oil.[20] This is an indication of how much the futures strategy returns vary with spot oil returns. For example, if $\beta = 1$, this indicates that for a 10% change in the spot oil price, the futures strategy also changes by 10%. Over this period, the strategy that tracked spot oil the best was investing in the 1-month or 2-month futures contract, for which the βs equaled 0.80 and 0.73, respectively, while the βs for the 6-month and 12-month futures were 0.56 and 0.44, respectively. Thus, for a 10% percentage change in the spot oil price, the 6-month and 12-month futures' prices moved on average by 5.6% and 4.4%, respectively. Thus, for a given movement in the underlying spot prices, the 12-month futures price moved by less than the 1-month futures price. Despite the overall good performance of the futures investing strategy during this period, the tracking error was still quite high: On an annualized basis, the futures investing strategy had a tracking error of 18.62%–26.37% with respect to spot oil.[21] Generally, oil futures investments do not track spot oil very well over a long period of time.[22]

From 2006 to 2017, the oil futures investing strategy was entirely different. First and foremost, oil futures investing underperformed the spot oil benchmark. The fact that oil futures investing did so poorly with respect to spot oil returns is probably the reason why the poor tracking received so much attention in the press (Blas, 2008; Burton and Karsh, 2009; Constable, 2016; Eisen and Josephs, 2016). That is, prior to 2006, since oil futures investing outperformed spot oil, investors were less concerned about the tracking issue. In fact, the tracking error was lower in the period 2006 to 2017, and the daily beta estimates show that, on average, the oil futures strategy moved more in line with spot oil. The other reason for the attention to the high tracking error in the latter period (2006–2017) versus the earlier period (1994–2006) might be that only since 2006 have investable ETF products existed to invest in oil. There is a debate regarding what might have caused the underperformance of futures investing relative to spot oil in the latter period. Singleton (2014) and Hamilton and Wu (2015) believe it was due to the increased investor usage of indexed oil products. That is, the "crowding" or increasingly concentrated demand for long oil futures products put pressure on futures returns. Other studies have also attempted to understand the link between long-only investors in oil and the relationship between oil futures returns and spot oil returns (Bessembinder et al., 2016; Burton and Karsh, 2009; Irwin and Sanders, 2011; Main et al., 2013).

Regardless of the reason for the change in the behavior of futures price returns beginning around 2006, the average returns from the futures strategy underperformed spot oil by −10.59%, −10.87%, −4.88%, and −3.58% for the 1-month, 2-month, 6-month, and 12-month contracts, respectively. Over this period, the average annualized spot oil return was 6.44%, while all of the futures' returns were lower and, in some instances, negative. Many of the negative returns can be attributed to the roll effect. During the period 2006 to 2017, the roll effect was most severe for the second futures contract, which suffered an annualized loss of about 10% due to contango in the oil futures market.

The ETFs and ETNs that invest in oil futures have risk and return characteristics that are similar to the underlying oil futures contracts. The behavior of ETFs that purchase oil companies is different. Of course, all of these have management fees or expense ratios that further increase the tracking error.

As mentioned earlier, the largest oil futures ETF is USO, which had almost $2 billion in AUM as of 2018.[23] Next after USO are OIL, UCO, DBO, and SCO, which as a group have another $1.6 billion. The smallest ETF in the top 20 is OILU, which is a leveraged crude oil fund with $11 million under management. XLE and VDE are the largest ETFs that invest in oil companies, with $18 billion and $4 billion, respectively, in assets. The largest mutual funds are VGENX and FSENX, with $9.7 billion and $2 billion, respectively. The smallest mutual fund is BACAX, with just $82 million.

To make the analysis meaningful, we chose the period from 2009 to 2017 because more oil ETFs and mutual funds existed during this period. In Table 5, it can be seen that over this period, the ETFs investing in *oil futures contracts* suffered a fate similar to the oil futures investing strategies. For example, USO had a −5.79% average return during this period, while spot oil had a 10.11% return. This represented an underperformance vis-a-vis spot oil of −15.90%. The underperformances of OIL and UCO were even larger, at −17.86% and −24.45%, respectively. DBO did slightly better, with an underperformance of −12.99%. Presumably, the reason for DBO's better performance was that this ETF uses futures contracts with the least amount of contango and hence is expected to lose the least from futures roll. In fact, over the same period, only the 6- and 12-month futures strategies had positive annualized returns. In all cases, the tracking error of these ETFs was greater than 14% annually.

[19] The roll effect is computed as the portion of the actual futures return that was expected ex-ante had the futures curve stayed constant from one day to the next. Details of the calculation can be found in Chincarini et al. (2016).

[20] We used the regression $r^i_{t,t+1} = a + br^S_{t,t+1} + \epsilon$ to estimate the β exposures of the investment vehicles, where $r^S_{t,t+1}$ is the return for spot oil from t to $t+1$ and $r^i_{t,t+1}$ is the return for the underlying investment vehicle.

[21] This corroborates the existing consensus that tracking spot oil seems to be unachievable.

[22] The tracking error is measured as the daily standard deviation of the returns for the futures contract minus the daily returns for spot oil, annualized by multiplying by $\sqrt{250}$.

[23] Tables 13 and 14 in Appendix B list the largest ETFs that invest in oil futures contracts and oil companies, respectively, by their AUM as of 2018.

In contrast, the ETFs and mutual funds that invested in *oil companies* did better, but in most cases they still underperformed vis-a-vis spot oil over the entire period. For example, the two leading ETFs, XLE and VDE, underperformed spot oil by an average of −2.74% and −2.92%, respectively, and the leading mutual funds, VGENX and FSENX, underperformed spot oil by −4.93% and −1.79%, respectively. Over the entire period, the investment vehicles that invested in oil companies performed better with respect to spot oil, despite a much higher tracking error: Most of the ETFs and mutual funds investing directly in oil companies had annualized tracking errors that were greater than 28%, much higher than the ETFs investing in oil futures.

Altogether, investing in oil creates substantial tracking errors with respect to spot oil. Part of the errors are due to the fact that spot oil is not an appropriate benchmark, since it fails to account for storage and other related costs. Another problem is that contango has increased in futures markets, which makes the tracking error of oil futures investing with respect to spot oil much larger. On the other hand, investing directly in oil companies creates a host of other risks that cause even larger tracking errors.

4. An optimized strategy to track spot oil

We have established that mimicking the returns of spot oil is complex for a variety of reasons. When futures are used to replicate spot oil, the divergence will be greater or smaller depending on factors such as the costs of oil storage and contango and backwardation in the futures market. Investing in oil companies avoids the problem of contango and backwardation but introduces a new set of risk exposures that make the tracking error with respect to spot oil even larger.

One might wonder, however, whether it is possible to mimic spot oil using all of the potential oil investments, by choosing the optimal weights to reduce the tracking error as much as possible. If this were possible, this investment strategy might appeal to a group of investors who are concerned with minimizing the tracking error with respect to spot oil, storage costs notwithstanding. In this section, we examine the effectiveness of such a strategy.

Our approach is to use historical data to attempt to construct a portfolio from oil company stocks, oil futures contracts, and cash that reduces the tracking error of the investment strategy with respect to spot oil. Our basic optimization uses daily data to optimize portfolios in a way that minimizes the tracking error of the portfolio with respect to spot oil while fully investing the portfolio. We use a look-back period of 1000 days to form the portfolio and then compute the strategy's performance on the next day.[24]

Many types of strategies can be tried for replicating spot oil performance. For example, even with a mean-variance approach to minimizing the tracking error, different look-back periods can be chosen for the variance–covariance matrix, and an exponentially weighted moving average can be used to compute the variance–covariance matrix with different rebalancing periods ranging from 1 to n days[25] and different subsets of oil futures contracts and oil companies can be chosen for the optimization. Because many optimizations with different parameters produced qualitatively similar results, we only report the results for some of the optimizations in this paper.

First, we chose both a passive and a dynamic strategy to mimic oil. The passive strategy always optimized the portfolio by considering the top 100 oil stocks in our selected universe and the first 12 oil future contracts.[26] We call this *Static Mean Variance*.

The dynamic strategy examined the nature of the futures curve on the day of re-optimization or rebalancing.[27] We computed the contango of the first 12 oil futures contracts and eliminated the contracts with contango from the optimization for that particular day. If three or more oil futures contracts were available that were not in contango on that day, the optimization was performed with only those oil futures contracts. If there were less than three oil futures contracts in backwardation, then we performed the optimization with only oil stocks.[28] We call this strategy *Dynamic Optimization 1*. We also created another dynamic strategy in which we allowed the optimizer to simultaneously consider all of the backwardated futures contracts and oil stocks in every optimization. We call this strategy *Dynamic Optimization 2*.

We considered two alternatives for our rebalancing period. The first alternative used daily historical return data and rebalanced every day, and the second also used daily return data but re-optimized or rebalanced only once each month, to limit transaction costs.[29] When the portfolio was not rebalanced every day (i.e., when the daily return data was re-optimized/rebalanced once a month), we updated the weights for the portfolio using the formula in Appendix A (Chincarini and Kim, 2006).

[24] Use of other look-back periods does not change the main results of this paper.

[25] By rebalancing periods, we mean how often the portfolio is re-optimized and the optimal weights altered.

[26] The available oil companies could change every day based on the market capitalization ranking. In addition, we also included cash as a portfolio choice in every optimization.

[27] One of the drawbacks of mean-variance optimization and regression analysis that is not discussed often is that these techniques take the "best" static weights over the optimization period. It is often the case that static historically optimized weights are suboptimal, especially for investment strategies that might have very useful information at the time of rebalancing that is omitted. A simple example can illustrate this. Suppose we were given the 500 stocks of the S&P 500 and asked to minimize the tracking error with respect to the S&P 500. The optimal weights would be the averages of the best static weights over the time period. However, there is a much better solution, both in-sample and for forecasting. That would be the market capitalization (or float-weighted) weighting that grows by the relative return over time. Although this would be much better, mean-variance and regression analysis would never give us these superior and forward-looking weights. While we are still looking for better techniques, we created our own dynamic optimization that takes the contango of the futures market into account, which provides a heavy drag for futures investing vis-a-vis spot oil returns.

[28] It should be noted that frequently, with a small number of oil futures contracts, the optimization could not be solved given the optimization constraints. In such cases, we chose to equally weight all of the backwardated oil futures contracts on that day.

[29] We also ran backtests using monthly return data, but this severely limited the sample. We do not report the results, although they were qualitatively similar to the results presented in this paper.

Table 6
Optimization parameters for historical backtest of oil tracking portfolio.

Number	Objective	Constraints	
1	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	No Constraints	Minimize the tracking error of the portfolio with spot oil.
2	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$\tilde{\mathbf{w}}'\boldsymbol{\iota} = 0$	Same as #1 and the weights of the portfolio should sum to 1.
3	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$\tilde{\mathbf{w}}'\boldsymbol{\iota} = 0,\ 0 \le \mathbf{w}$	Same as #2 and no shorting allowed.
4	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$\tilde{\mathbf{w}}'\boldsymbol{\iota} = 0,\ \tilde{\mathbf{w}}'\tilde{\boldsymbol{\mu}} \ge 0$	Same as #3 and the mean of the portfolio minus the mean return of spot oil should be greater or equal to zero.
5	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$-4 \le \tilde{\mathbf{w}}'\boldsymbol{\iota} \le 4,\ \tilde{\mathbf{w}}'\tilde{\boldsymbol{\mu}} \ge 0$	Same as #4, but the net exposure of the portfolio should be in the range of −4 to 4.
6	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$-4 \le \tilde{\mathbf{w}}'\boldsymbol{\iota} \le 4,\ \tilde{\mathbf{w}}'\tilde{\boldsymbol{\mu}} \ge 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta} = 0$	Same as #5 and that the weighted average beta of the portfolio with respect to historic spot oil should be equal to 0.
7	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$-4 \le \tilde{\mathbf{w}}'\boldsymbol{\iota} \le 4,\ \tilde{\mathbf{w}}'\tilde{\boldsymbol{\mu}} \ge 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta} = 0,$ $0 \le \mathbf{w} \le 0.30$	Same as #6 and no shorting allowed and maximum weight in any individual security of 0.30.
8	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$-4 \le \tilde{\mathbf{w}}'\boldsymbol{\iota} \le 4,\ \tilde{\mathbf{w}}'\tilde{\boldsymbol{\mu}} \ge 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta} = 0,$ $-0.10 \le \mathbf{w} \le 0.10$	Same as #6 and the minimum and maximum weight of the portfolio securities should be between −10% and 10%.
9	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$0 \le \tilde{\mathbf{w}}'\boldsymbol{\iota} \le 4,\ \tilde{\mathbf{w}}'\boldsymbol{\beta} = 0, \tilde{\mathbf{w}}'\boldsymbol{\beta}_{\boldsymbol{RMRF}} = 0,$ $\tilde{\mathbf{w}}'\boldsymbol{\beta}_{\boldsymbol{SMB}} = 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta}_{HML} = 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta}_{\boldsymbol{MOM}} = 0,$ $-0.10 \le \mathbf{w} \le 0.30$	Similar to #7 and that the exposures of the portfolio to the Fama-French factors should be zero, net exposure should be limited to being levered long, the average portfolio return minus the return of spot oil is not constrained to be positive, and the maximum weight can be as high as 0.30.
10	$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}'\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{w}}$	$0 \le \tilde{\mathbf{w}}'\boldsymbol{\iota} \le 4\ \tilde{\mathbf{w}}'\tilde{\boldsymbol{\mu}} \ge 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta} = 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta}_{\boldsymbol{RMRF}} = 0,$ $\tilde{\mathbf{w}}'\boldsymbol{\beta}_{\boldsymbol{SMB}} = 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta}_{HML} = 0,\ \tilde{\mathbf{w}}'\boldsymbol{\beta}_{\boldsymbol{MOM}} = 0,$ $-0.10 \le \mathbf{w} \le 0.30$	Same as #9 except the mean of the portfolio minus the mean return of spot oil should be greater or equal to zero.

Note: This table represents the optimization parameters used in the historical out-of-sample backtests to replicate the historical daily returns of spot oil. $\mathbf{w}$ represents the actual vector of weights of the futures, oil stocks, and cash in the portfolio, while $\tilde{\mathbf{w}}$ represents the actual vector of weights of the futures, oil stocks, and cash of the portfolio and −1 as the last entry for spot oil.

As mentioned earlier, we considered only the top 100 oil companies by market capitalization for any given date.[30] A company was classified as an oil company if it was classified with one of the SIC codes 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, or 5172.

In addition to the optimizations, we had to compute the historical out-of-sample performance for the strategies. To make our investment strategies more realistic, we incorporated risk-free returns from the collateralization of futures investments. We also created a penalty for leverage using the risk-free rate. We used the same interest rate—the Fama-French Treasury bill rate—for leverage costs and futures collateralization. Thus, if the total sum of absolute portfolio weights was greater than 1, we charged r_f for the amount in excess of 1 (leveraging costs). We also credited individual futures in our portfolio on either the long or the short side by the risk-free rate. The actual calculations are shown in Appendix A.

In all of the backtesting, once the stock and futures universe were chosen, we selected the type of optimization for finding the optimal weights that minimized the historical tracking error of our portfolio with respect to spot oil returns. We considered 10 different optimization parameter sets, which are listed in Table 6. Optimization 1 is an unconstrained optimization that uses oil futures contracts and oil company stocks to construct a portfolio that minimizes the historical tracking error with respect to spot oil returns. Optimizations 2 through 4 are similar but have either the constraint that the portfolio weights must sum to 1, the constraint that no short-selling is permitted, or the constraint that the portfolio's average return must be greater than or equal to spot oil's average return. Optimizations 5 and 6 have constraints similar to optimization 4 and also constrain the portfolio's net exposure or "leverage" to a maximum of 4, with optimization 6 adding the additional constraint that the β exposure to spot oil is 1—that is, the weighted average of the βs for all securities in the portfolio vis-a-vis spot oil is forced to equal 1.[31] Optimizations 7 and 8 have constraints similar to optimization 6 but also add weight constraints to the portfolio so that no security has too large a positive or negative weight, in addition to some other constraints. Optimizations 9 and 10 are similar to optimization 7, but in addition attempt to set all exposures to the Fama-French factors equal to the exposures to spot oil and also to make the portfolio's β exposure to spot oil be equal to 1, with the leverage constrained to be long-only weight constraints with a minimum of −10% and a maximum of 30% for each security.

5. Empirical evaluation of spot oil tracking methods

5.1. Static mean-variance optimizations

For the static mean-variance optimizations, we considered daily and monthly (every 30 days) rebalancing strategies. Thus, for each business day, we used a 500-day window to estimate the beta exposures of the stocks and futures to spot oil and a 1000-day window to estimate the variance–covariance matrix of asset returns. After optimizing, we held the portfolio until the next business

[30] We also performed optimizations with an unlimited number of oil companies and with sizes ranging from 20 to 100. None of these changes altered the main qualitative results of this paper.

[31] For those unfamiliar with oil investing, the typical β of an oil stock vis-a-vis spot oil is about 0.25.

Table 7
Optimization results of tracking portfolios. Static mean-variance: Daily rebalance.

Portfolio Strategy	Sample Period: 1997–2005†						Sample Period: 2006–2017					
	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max
Optimization												
Version 1	16.23	15.27	0.96	21.29	−15.61	16.43	−12.77	−0.45	−12.32	12.33	−18.13	9.77
Version 2	16.23	15.27	0.96	21.29	−15.61	16.43	−12.63	−0.45	−12.18	12.32	−18.00	9.77
Version 3	24.01	15.27	8.74	20.17	−15.55	16.32	−9.32	−0.45	−8.87	11.12	−17.90	9.56
Version 4	24.07	15.27	8.80	20.32	−15.57	16.31	−7.75	−0.45	−7.30	12.58	−17.02	9.81
Version 5	24.06	15.27	8.79	20.32	−15.57	16.32	−7.77	−0.45	−7.32	12.58	−17.02	9.81
Version 6	20.58	15.27	5.31	24.23	−16.14	17.10	−13.57	−0.45	−13.12	13.31	−18.38	9.76
Version 7	36.41	15.27	21.14	26.11	−17.73	15.61	−9.78	−0.45	−9.32	15.41	−19.20	9.15
Version 8	30.31	15.27	15.04	36.76	−17.41	14.71	−9.32	−0.45	−8.86	19.07	−19.04	8.85
Version 9	25.74	15.27	10.47	26.23	−17.62	15.46	−14.48	−0.45	−14.03	15.96	−20.05	8.98
Version 10	25.03	15.27	9.76	26.20	−17.63	15.46	−13.83	−0.45	−13.37	16.16	−19.90	9.24
Rolling Futures												
Fut1Roll0	22.99	15.27	7.72	20.07	−15.65	16.46	−10.06	−0.45	−9.60	11.04	−18.10	9.52
Fut2Roll0	25.50	15.27	10.23	21.21	−17.17	15.50	−9.62	−0.45	−9.17	13.96	−17.83	11.35
Fut3Roll0	28.64	15.27	13.37	22.20	−16.94	14.75	−7.01	−0.45	−6.55	15.14	−17.45	11.43
Equal-Weight	18.86	15.27	3.59	37.11	−17.62	13.50	3.18	−0.45	3.63	29.31	−17.69	14.59
Market-Cap	21.17	15.27	5.90	30.02	−15.93	14.60	0.90	−0.45	1.36	21.46	−14.81	9.62

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. † The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios that minimize the tracking error relative to spot oil. The portfolios are rebalanced on every trading day, with a few exceptions due to missing data. μ_p is the compounded geometric return of a particular strategy over the given period. μ_s is the same metric, but for spot oil. $\mu_p - \mu_s$ is the difference between the two. T.E. is the tracking error of a strategy measured against spot oil, which is formulated as the annualized standard deviation of the daily difference between the strategy and spot oil returns. Min and Max indicate the minimum and maximum daily return differences. Fut1Roll0, Fut2Roll0, and Fut3Roll0 show the same statistics for rolling the 1st, 2nd, and 3rd futures contracts, Equal-Weight represents the return of a portfolio that equally weights the top 100 stocks by market capitalization and the first 12 futures contracts in every period (thus, all weights are positive and their sum is 1), and Market-Cap is a portfolio that weights all 100 stocks by market capitalization in every period and equally weights the first 12 futures contracts. Futures and oil stocks are combined with an arbitrary weight of 0.50 in each group (thus, all weights are positive and their sum is 1). Versions 1 to 10 represent the specific optimization parameters described in Table 6.

day or 30 business days later, in the case of monthly rebalancing, and then again re-optimized the portfolio.[32]

The out-of-sample optimization results for the daily rebalancing are shown in Table 7. This table illustrates the theme from our general discussion in Section 3. While a portfolio of oil stocks and oil futures outperforms the returns of spot oil over the period from 1997 to 2005, it underperforms over the period from 2006 to 2017, and this is true for every variation on the optimization (optimization 1 to optimization 10). In fact, despite the elaborate optimizations, the resulting portfolios don't do much better than investing in single oil futures contracts; see Fut1Roll0 (nearest-term futures contract), Fut2Roll0 (second nearest), and Fut3Roll0 (third nearest). As a matter of interest, we also show the performance of a market-cap-weighted and an equal-weighted portfolio of the top 100 oil stocks and 12 oil futures. Both of these portfolios actually perform better vis-a-vis spot oil, albeit with a higher tracking error.[33]

We also present statistics regarding the optimal weights for the various portfolios in Table 8. The statistics include the average weights for each investment category (oil futures, oil stocks, and cash), the minimum and maximum weights of any security, the average portfolio leverage, the long, short, and aggregate weights, and the ex-post exposures of the portfolios to the Fama-French factors. For example, for the unconstrained optimization (version 1) over the sample period 1994 to 2005, the average holding of futures contracts was 97%, the average cash holding was 14%, and on average, oil stocks were shorted by −8%. The largest weight in futures contracts was 642% (leverage) and the smallest weight was −1249% (short leverage). The average leverage of the portfolios was 25.34 (this is given by the sum of the average absolute value of short and long positions). The beta exposures show that this portfolio had a 0.78 beta with respect to spot oil and a zero beta for exposures to the Fama-French factors. The Fama-French exposures

[32] During our work, we encountered numerous issues with respect to historical backtesting. First, during certain periods, stock returns and spot oil returns were so vastly different that it was impossible to create a long-only portfolio with just stocks and futures contracts that would satisfy the constraints and minimize the tracking error. A particularly interesting case occurred in 2008, when stocks had very high historical returns compared to spot oil. Second, a frustrating feature of mean-variance optimization is that it is backward-looking and static and thus ignores important information in the futures market, such as contango. To address this problem, we created the dynamic strategies. Third, oil stocks' exposures to spot oil are typically low, on the order of 0.25. Thus, without leveraging an oil stock portfolio, at times it was impossible to create a portfolio with an average beta equal to the spot oil beta. Fourth, optimizing with several futures contracts can produce weights on many contracts, since there is a high degree of collinearity among the futures contracts. Fifth, when doing daily rebalancing over a period of 24 years, there will almost inevitably be days for which the optimizer fails to find an optimal set of weights that satisfy the problem constraints. To alleviate this, we increased the allowed tolerance for the optimization from 0.0000001 to 0.001. Sixth, although we initially wanted to create portfolios with no short constraints for all cases, this was not always possible, and hence we expanded the individual security limits to broader ranges.

[33] USL, an oil-futures ETF, actually gives equal weights to the first 12-month futures contracts to provide a more stable return given contango.

Table 8
Optimization weights and ex-post beta exposure statistics. Static mean-variance: Daily rebalance.

Optimization	Average Weights			Min. Weight			Max. Weight			Leverage			Ex-post Beta Exposure					nobs	Fail.
	Futures	Stocks	Cash	Futures	Stocks	Cash	Futures	Stocks	Cash	Total	Long	Short	β_{spot}	β_{Mkt}	β_{SMB}	β_{HML}	β_{UMD}		
Sample Period: 1997–2005†																			
Version 1	0.97	−0.08	0.14	−12.49	−0.20	0.00	6.42	0.15	1.00	25.34	13.28	−12.06	0.78	0.00	0.04	0.01	0.01	2002	0.00
Version 2	0.97	−0.08	0.11	−12.49	−0.20	0.02	6.42	0.15	0.27	25.11	13.06	−12.06	0.78	0.00	0.04	0.01	0.01	2002	0.00
Version 3	0.97	0.03	0.00	0.00	0.00	0.00	0.97	0.02	0.00	1.00	1.00	0.00	0.76	0.03	0.08	0.06	0.00	2002	0.00
Version 4	0.96	0.04	0.00	0.00	0.00	0.00	0.98	0.11	0.00	1.00	1.00	0.00	0.75	0.05	0.09	0.07	0.01	2002	0.00
Version 5	0.96	0.04	0.00	0.00	0.00	0.00	0.98	0.11	0.00	1.00	1.00	0.00	0.75	0.05	0.09	0.07	0.01	2002	0.00
Version 6	1.20	−0.09	0.26	−17.01	−0.24	0.00	10.58	0.23	1.00	29.62	15.57	−14.05	0.97	0.01	0.05	0.01	0.02	2002	0.00
Version 7	1.50	0.17	0.07	0.00	0.00	0.00	0.30	0.15	0.30	1.66	1.66	0.00	1.05	0.15	0.17	0.21	−0.00	2002	0.00
Version 8	1.20	0.33	0.03	0.10	−0.10	0.00	0.10	0.10	0.10	6.61	4.21	−2.40	0.90	0.20	0.16	0.21	0.06	2002	0.00
Version 9	1.37	−0.05	0.06	−0.10	−0.10	0.00	0.30	0.30	0.30	5.43	3.44	−1.99	0.98	−0.00	0.03	0.04	0.01	2002	0.00
Version 10	1.37	−0.05	0.09	−0.10	−0.10	0.00	0.30	0.30	0.30	5.44	3.44	−1.99	0.98	−0.01	0.03	0.03	0.00	2002	0.00
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.00	0.07	0.05	0.01	2002	
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.72	0.01	0.06	0.06	0.01	2002	
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.66	0.01	0.06	0.06	0.01	2002	
Sample Period: 2006–2017																			
Version 1	0.97	0.01	0.26	−17.71	−0.20	0.00	13.84	0.16	1.00	17.81	9.41	−8.40	0.89	0.07	−0.00	0.05	0.01	3014	0.00
Version 2	0.97	0.01	0.02	−17.52	−0.20	0.00	13.78	0.16	0.17	17.77	9.39	−8.39	0.89	0.07	−0.00	0.05	0.01	3014	0.00
Version 3	0.96	0.04	0.00	0.00	0.00	0.00	0.99	0.03	0.00	1.00	1.00	0.00	0.88	0.08	0.01	0.04	0.02	3014	0.00
Version 4	0.84	0.16	0.00	0.00	0.00	0.00	0.96	0.20	0.00	1.00	1.00	0.00	0.80	0.19	0.03	0.04	0.03	3014	0.00
Version 5	0.84	0.16	0.00	0.00	0.00	0.00	0.96	0.20	0.00	1.00	1.00	0.00	0.80	0.19	0.03	0.04	0.03	3014	0.00
Version 6	1.06	−0.00	0.68	−24.18	−0.23	0.01	19.87	0.21	1.00	22.69	12.12	−10.57	0.97	0.06	0.01	0.04	0.08	3014	0.00
Version 7	1.20	0.06	0.12	0.00	0.00	0.00	0.30	0.30	0.30	1.29	1.29	0.00	1.01	0.13	0.04	0.08	0.05	3014	0.00
Version 8	1.17	0.02	0.07	−0.10	−0.10	0.01	0.10	0.10	0.10	4.31	2.76	−1.55	0.93	0.13	0.06	0.12	0.13	3014	0.00
Version 9	1.10	−0.04	0.15	−0.10	−0.10	0.00	0.30	0.30	0.30	4.41	2.76	−1.65	0.95	0.05	0.02	0.04	0.01	3014	0.00
Version 10	1.10	−0.03	0.25	−0.10	−0.10	0.00	0.30	0.30	0.30	4.71	2.98	−1.73	0.95	0.05	0.03	0.02	0.05	3014	0.00
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.91	0.04	0.00	0.04	0.02	3014	
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.84	0.07	0.01	0.04	0.03	3014	
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.08	0.01	0.04	0.03	3014	

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. † The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, and returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios for minimizing the tracking error relative to spot oil. The table shows the average weights for the selected optimized portfolios. The table also shows the minimum and maximum weights for stocks, futures contracts, and cash over the entire period. The average sum of the long weights over time, the average sum of the short weights over time, and the average of the absolute value of each of these are also shown. Finally, the table shows the ex-post exposures of the optimized portfolio returns to spot oil and the Fama-French factors. These are the exposures of the portfolios based on the regression $r^i_{t,t+1} = a + \beta_{spot} r^S_{t,t+1} + \beta_{Mkt}(r^{Mkt}_{t,t+1} - r^f_{t,t+1}) + \beta_{SMB} r^{SMB}_{t,t+1} + \beta_{HML} r^{HML}_{t,t+1} + \beta_{UMD} r^{UMD}_{t,t+1} + \epsilon_{t,t+1}$, where $r^S_{t,t+1}$ is the return of spot oil from t to $t+1$, $r^{Mkt}_{t,t+1}$ is the return of the Fama-French market factor, $r^f_{t,t+1}$ is the return of the 1-month Treasury bill, $r^{SMB}_{t,t+1}$ is the return of the Fama-French size factor, $r^{HML}_{t,t+1}$ is the return of the Fama-French value factor, $r^{UMD}_{t,t+1}$ is the return of the Fama-French momentum factor, and $r^i_{t,t+1}$ is the return of either the optimized portfolio or the futures contract. "Fail." indicates the percentage of the optimizations that failed to find a solution. In these cases, the program weighted the stocks and futures for that investment period equally. Versions 1 to 10 represent the specific optimization parameters described in Table 6.

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Table 9
Optimization results of tracking portfolios. Dynamic 1: Daily rebalance.

Portfolio Strategy	Sample Period: 1997–2005						Sample Period: 2006–2017					
	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max
Optimization												
Version 1	37.57	15.27	22.30	27.83	−16.02	16.79	−1.61	−0.45	−1.15	27.53	−17.75	12.37
Version 2	37.41	15.27	22.14	27.83	−16.03	16.79	−1.59	−0.45	−1.14	27.53	−17.75	12.38
Version 3	43.59	15.27	28.32	27.72	−17.17	16.42	7.81	−0.45	8.27	26.44	−14.87	12.24
Version 4	40.93	15.27	25.66	28.12	−17.18	16.42	8.33	−0.45	8.78	26.60	−14.86	12.24
Version 5	40.93	15.27	25.66	28.12	−17.18	16.42	8.33	−0.45	8.78	26.60	−14.86	12.24
Version 6	19.28	15.27	4.01	46.05	−16.18	18.11	−12.02	−0.45	−11.57	41.46	−22.63	15.48
Version 7	20.04	15.27	4.76	64.87	−28.47	27.70	−10.48	−0.45	−10.03	65.51	−30.90	44.31
Version 8	15.73	15.27	0.46	56.14	−22.55	24.56	−7.55	−0.45	−7.10	51.48	−23.97	31.14
Version 9	28.09	15.27	12.82	32.50	−17.91	14.97	10.94	−0.45	11.39	43.48	−24.85	16.38
Version 10	28.09	15.27	12.82	32.50	−17.91	14.97	9.43	−0.45	9.88	42.42	−23.73	16.38
Rolling Futures												
Fut1Roll0	22.99	15.27	7.72	20.07	−15.65	16.46	−10.06	−0.45	−9.60	11.04	−18.10	9.52
Fut2Roll0	25.50	15.27	10.23	21.21	−17.17	15.50	−9.62	−0.45	−9.17	13.96	−17.83	11.35
Fut3Roll0	28.64	15.27	13.37	22.20	−16.94	14.75	−7.01	−0.45	−6.55	15.14	−17.45	11.43
Equal-Weight	31.98	15.27	16.71	30.40	−17.91	14.91	10.36	−0.45	10.82	29.21	−16.08	16.38
Market-Cap	38.64	15.27	23.37	30.00	−16.65	14.97	11.08	−0.45	11.53	28.92	−14.40	13.47

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. † The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios that minimize the tracking error relative to spot oil. The portfolios are rebalanced on every trading day, with a few exceptions due to missing data. μ_p is the compounded geometric return of a particular strategy over the given period. μ_s is the same metric, but for spot oil. $\mu_p - \mu_s$ is the difference between the two. T.E. is the tracking error of a strategy measured against spot oil, which is formulated as the annualized standard deviation of the daily difference between the strategy and spot oil returns. Min and Max indicate the minimum and maximum daily return differences. Fut1Roll0, Fut2Roll0, Fut3Roll0 show the same statistics for rolling the 1st, 2nd, and 3rd futures, Equal-Weight represents the return of a portfolio that equally weights the top 100 stocks by market capitalization and the first 12 futures contracts in every period (thus, all weights are positive and their sum is 1), and Market-Cap is a portfolio that weights all 100 stocks by market capitalization in every period and equally weights the first 12 futures contracts. Futures and oil stocks are combined with an arbitrary weight of 0.50 in each group (thus, all weights are positive and their sum is 1). Versions 1 to 10 represent the specific optimization parameters described in Table 6.

are close to zero because most of the portfolios' returns are explained by the returns of spot oil.[34] Compare this to the version 9 optimization, which controls for the weights of individual securities, total leverage, and net exposures to the Fama-French factors. The average futures weight is higher, at 137%, while the average oil stock and cash exposures are slightly lower. The minimum and maximum weights are −0.10 and 0.30, due to the constraints. The leverage on longs and shorts is also constrained, and thus, the average long leverage is 3.44 and the average short leverage is −1.99, with a total average leverage of 5.44. The average spot oil beta is much closer to 1, while the net Fama-French exposures are similar.

The key numbers are the annualized performance of each strategy versus spot oil (Table 7, Columns 4 and 10), the tracking error of each strategy (Table 7, Columns 5 and 11), and the ex-post β_{spot} of each strategy with respect to spot oil (Table 8, Column 14). Ideally, we would like to see a β_{spot} equal to 1 and a tracking error of zero. For the first period, 1997 to 2005, all of the optimizations had a tracking error that was comparable to investing in single oil futures contracts. However, some of the optimization strategies (versions 6 through 10) had better ex-post spot oil betas than did the single futures contracts. They also outperformed spot oil just as the single oil futures contracts did. During the second period, from 2006 to 2017, the tracking error was slightly higher than investing directly in single futures contracts, although the spot oil betas were comparable or better (i.e., closer to 1) than the single stock futures contracts. However, the underperformance with respect to spot oil was larger than for individual futures contracts. Thus, it is safe to conclude that the standard mean-variance optimization procedure did not improve the tracking of spot oil.

The optimization results with monthly rebalancing are very similar to the daily rebalancing results (see Tables 15 and 16 in Appendix B). One advantage of monthly rebalancing is that it leads to lower transaction costs, but unfortunately, the success in tracking spot oil is still very poor.

5.2. Dynamic optimizations

The dynamic strategies performed better than the static mean-variance optimizations. As might be expected, the Dynamic 1 strategy improved on the excess performance of spot oil (see Tables 9 and 10). While this is very encouraging, it came at the cost of a much larger tracking error. In some cases, like versions 7 and 8, the annualized tracking error was more than 5 times the error of

[34] Although not shown in these tables, during the period 1997 to 2005, both spot oil and oil futures had zero exposure to the Fama-French factors; however, from 2006 to 2017, they both had a large and significant exposures to the Fama-French market factors of around 0.60. This could be related to the financialization of the oil market.

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Table 10
Optimization weights and ex-post beta exposure statistics. Dynamic 1: Daily rebalance.

Optimization	Average Weights			Min. Weight			Max. Weight			Leverage			Ex-post Beta Exposure					nobs	Fail.
	Futures	Stocks	Cash	Futures	Stocks	Cash	Futures	Stocks	Cash	Total	Long	Short	β_{spot}	β_{Mkt}	β_{SMB}	β_{HML}	β_{UMD}		
Sample Period: 1997–2005[†]																			
Version 1	0.98	0.22	0.09	−13.15	−0.38	0.00	11.92	0.32	1.00	18.20	9.59	−8.61	0.66	0.02	0.09	0.05	−0.04	2002	0.00
Version 2	0.98	0.23	0.24	−13.15	−0.38	0.00	12.55	0.32	1.00	18.24	9.62	−8.62	0.66	0.02	0.09	0.05	−0.04	2002	0.00
Version 3	1.00	0.71	0.09	0.00	0.00	0.00	1.00	0.24	0.48	1.00	1.00	0.00	0.61	0.08	0.12	0.09	−0.04	2002	0.00
Version 4	0.99	0.72	0.00	0.00	0.00	0.00	1.00	0.25	0.00	0.96	0.96	0.00	0.59	0.09	0.12	0.10	−0.03	2002	0.09
Version 5	0.99	0.72	0.00	0.00	0.00	0.00	1.00	0.25	0.00	0.96	0.96	0.00	0.59	0.09	0.12	0.10	−0.03	2002	0.09
Version 6	1.29	0.76	0.21	−18.77	−0.76	0.00	14.73	1.18	1.00	25.27	13.38	−11.89	0.96	0.12	0.13	−0.04	−0.12	2002	0.00
Version 7	1.65	3.59	0.00	0.00	0.00	0.00	0.33	0.30	0.00	1.99	1.99	0.00	1.02	0.60	0.42	0.13	−0.39	2002	0.05
Version 8	1.00	2.28	0.00	0.08	−0.10	0.00	0.33	0.10	0.00	2.76	2.05	−0.71	0.64	0.36	0.19	−0.12	−0.23	2002	0.72
Version 9	1.00	1.18	0.00	0.08	−0.10	0.00	0.33	0.30	0.00	1.45	1.25	−0.20	0.51	0.14	0.11	−0.01	−0.04	2002	0.95
Version 10	1.00	1.18	0.00	0.08	−0.10	0.00	0.33	0.30	0.00	1.45	1.25	−0.20	0.51	0.14	0.11	−0.01	−0.04	2002	0.95
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.00	0.07	0.05	0.01	2002	.
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.72	0.01	0.06	0.06	0.01	2002	.
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.66	0.01	0.06	0.06	0.01	2002	.
Sample Period: 2006–2017																			
Version 1	0.98	0.56	0.34	−19.42	−0.48	0.00	24.62	0.36	1.00	6.16	3.52	−2.64	0.45	0.34	−0.00	0.12	−0.02	3014	0.00
Version 2	0.98	0.56	0.33	−19.12	−0.48	0.00	24.44	0.36	0.58	6.27	3.63	−2.63	0.45	0.34	−0.00	0.12	−0.02	3014	0.00
Version 3	1.00	0.79	0.16	0.00	0.00	0.00	1.00	0.33	0.39	1.00	1.00	0.00	0.39	0.48	0.03	0.10	0.01	3014	0.00
Version 4	0.97	0.79	0.21	0.00	0.00	0.00	1.00	0.33	0.59	0.92	0.92	0.00	0.37	0.50	0.03	0.10	−0.01	3014	0.15
Version 5	0.97	0.79	0.21	0.00	0.00	0.00	1.00	0.33	0.59	0.92	0.92	0.00	0.37	0.50	0.03	0.10	−0.01	3014	0.15
Version 6	1.15	0.70	0.58	−99.67	−0.94	0.00	123.38	0.93	1.00	22.11	11.68	−10.43	0.70	0.46	0.07	0.26	0.08	3014	0.00
Version 7	1.17	2.64	0.08	0.00	0.00	0.00	0.33	0.30	0.12	2.28	2.28	0.00	0.93	1.88	0.19	0.24	−0.17	3014	0.01
Version 8	1.00	1.41	0.00	0.08	−0.10	0.00	0.33	0.10	0.00	6.88	4.07	−2.81	0.78	1.28	0.13	0.11	0.07	3014	0.25
Version 9	1.00	0.75	0.00	0.08	−0.10	0.00	0.33	0.30	0.00	8.24	4.49	−3.75	0.57	0.54	−0.00	−0.09	−0.06	3014	0.44
Version 10	1.00	0.76	0.00	0.08	−0.10	0.00	0.33	0.30	0.00	8.27	4.51	−3.75	0.56	0.53	−0.04	−0.06	−0.00	3014	0.44
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.91	0.04	0.00	0.04	0.02	3014	.
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.84	0.07	0.01	0.04	0.03	3014	.
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.08	0.01	0.04	0.03	3014	.

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. [†] The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, and returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios for minimizing the tracking error relative to spot oil. The table shows the average weights for the selected optimized portfolios. The table also shows the minimum and maximum weights for stocks, futures contracts, and cash over the entire period. The average sum of the long weights over time, the average sum of the short weights over time, and the average of the absolute value of each of these are also shown. Finally, the table shows the ex-post exposures of the optimized portfolio returns to spot oil and the Fama-French factors. These are the exposures of the portfolios based on the regression $r^i_{t,t+1} = a + \beta_{spot} r^S_{t,t+1} + \beta_{Mkt}(r^{Mkt}_{t,t+1} - r^f_{t,t+1}) + \beta_{SMB} r^{SMB}_{t,t+1} + \beta_{HML} r^{HML}_{t,t+1} + \beta_{UMD} r^{UMD}_{t,t+1} + \epsilon_{t,t+1}$, where $r^S_{t,t+1}$ is the return of spot oil from t to $t+1$, $r^{Mkt}_{t,t+1}$ is the return of the Fama-French market factor, $r^f_{t,t+1}$ is the return of the 1-month Treasury bill, $r^{SMB}_{t,t+1}$ is the return of the Fama-French size factor, $r^{HML}_{t,t+1}$ is the return of the Fama-French value factor, $r^{UMD}_{t,t+1}$ is the return of the Fama-French momentum factor, and $r^i_{t,t+1}$ is the return of either the optimized portfolio or the futures contract. "Fail." indicates the percentage of the optimizations that failed to find a solution. In these cases, the program weighted the stocks and futures for that investment period equally. Versions 1 to 10 represent the specific optimization parameters described in Table 6.

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Table 11
Optimization results of tracking portfolios. Dynamic 2: Daily rebalance.

Portfolio Strategy	Sample Period: 1997–2005						Sample Period: 2006–2017					
	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max
Optimization												
Version 1	25.73	15.27	10.46	24.70	−15.98	16.43	−5.37	−0.45	−4.91	23.32	−17.75	9.77
Version 2	25.69	15.27	10.41	24.69	−15.97	16.43	−5.18	−0.45	−4.72	23.32	−17.75	9.77
Version 3	40.38	15.27	25.11	23.98	−17.15	16.32	2.69	−0.45	3.14	22.12	−14.87	9.56
Version 4	39.92	15.27	24.65	24.07	−17.15	16.31	1.79	−0.45	2.25	22.41	−14.86	9.81
Version 5	39.93	15.27	24.66	24.06	−17.15	16.32	1.80	−0.45	2.25	22.42	−14.86	9.81
Version 6	16.88	15.27	1.61	34.58	−16.14	18.05	−11.60	−0.45	−11.15	34.26	−22.63	14.27
Version 7	29.45	15.27	14.18	58.38	−28.47	27.07	−6.43	−0.45	−5.98	59.69	−30.90	44.31
Version 8	42.99	15.27	27.72	60.49	−22.54	22.49	−10.09	−0.45	−9.64	49.50	−23.97	31.14
Version 9	39.21	15.27	23.94	35.12	−17.91	15.46	3.48	−0.45	3.93	40.42	−24.85	16.38
Version 10	38.99	15.27	23.72	35.13	−17.91	15.46	5.28	−0.45	5.74	39.68	−23.73	16.38
Rolling Futures												
Fut1Roll0	22.99	15.27	7.72	20.07	−15.65	16.46	−10.06	−0.45	−9.60	11.04	−18.10	9.52
Fut2Roll0	25.50	15.27	10.23	21.21	−17.17	15.50	−9.62	−0.45	−9.17	13.96	−17.83	11.35
Fut3Roll0	28.64	15.27	13.37	22.20	−16.94	14.75	−7.01	−0.45	−6.55	15.14	−17.45	11.43
Equal-Weight	19.25	15.27	3.98	37.99	−17.91	13.50	4.25	−0.45	4.70	31.24	−17.83	16.38
Market-Cap	35.84	15.27	20.56	26.16	−16.65	14.97	8.05	−0.45	8.50	24.06	−14.40	13.47

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. † The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios that minimize the tracking error relative to spot oil. The portfolios are rebalanced on every trading day, with a few exceptions due to missing data. μ_p is the compounded geometric return of a particular strategy over the given period. μ_s is the same metric, but for spot oil. $\mu_p - \mu_s$ is the difference between the two. T.E. is the tracking error of a strategy measured against spot oil, which is formulated as the annualized standard deviation of the daily difference between the strategy and spot oil returns. Min and Max indicate the minimum and maximum daily return differences. Fut1Roll0, Fut2Roll0, and Fut3Roll0 show the same statistics for rolling the 1st, 2nd, and 3rd futures contracts, Equal-Weight represents the return of a portfolio that equally weights the top 100 stocks by market capitalization and the first 12 futures contracts in every period (thus, all weights are positive and their sum is 1), and Market-Cap is a portfolio that weights all 100 stocks by market capitalization in every period and equally weights the first 12 futures contracts. Futures and oil stocks are combined with an arbitrary weight of 0.50 in each group (thus, all weights are positive and their sum is 1). Versions 1 to 10 represent the specific optimization parameters described in Table 6.

the nearest-term oil futures contract (65.51 and 51.48 versus 11.04 for the period 2006 to 2017).

This larger tracking error can also be seen in the ex-post spot beta. The spot oil beta for most of the optimizations was smaller than the single stock futures' betas, suggesting that these portfolios tracked the movements in spot oil less closely. The optimizations that also attempt to control the risk from the Fama-French factor exposures to be the same as those for spot oil (versions 9 and 10) outperformed spot oil on an annualized basis in both periods.[35] Most impressively, during the period 2006 to 2017, which plagued the oil investing world, optimization versions 9 and 10 outperformed spot oil by 11.39% and 9.88%, respectively, compared to the nearest term futures contract, which underperformed by −9.60%. This is a very encouraging result for oil managers.

The results for the Dynamic 2 strategy are less impressive than those for Dynamic 1 (see Tables 11 and 12).[36] Despite the lower tracking error, the excess return with respect to spot oil is also lower. However, just as we saw with Dynamic 1, these dynamic optimizations do better at reducing the underperformance of oil investing with respect to spot oil. In the case of Dynamic 2, we also find that the ex-post betas are closer to those of single stock futures.

5.3. *Robustness analysis*

In addition to the basic optimizations, we executed many other variants in order to test the robustness of our results. This included using year-on-year returns rather than daily returns in an attempt to smooth the movements of spot oil. The results from these backtests were very similar to the results reported in this paper.

We attempted to create portfolios that tracked spot oil returns one period in the future. This was based on the idea that perhaps, oil futures' prices and oil stock prices react more quickly to news than spot oil does. Thus, we used the returns of spot oil at $t+1$ and returns from the assets at time t to construct our average returns and our variance–covariance matrices. The results were similar to the results reported in this paper.

We also executed the optimizations with a smaller set of oil companies, using only SIC codes 1311, 1381, and 1382, and also using only 1311, 1381, 1382, 1389, and 3533. These two groups represent upstream oil companies, since the comovement of their returns

[35] It should be noted that the failure rates for these optimizations were quite high. They failed 95% of the time during the first sample period. Thus, in effect, these were just equal-weighted portfolios of backwardated futures contracts.

[36] Figs. 3 and 4 in Appendix B show the performance of the Dynamic 1 portfolios, as well as the performance of spot oil and a strategy of rolling the first futures contract.

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Table 12
Optimization weights and ex-post beta exposure statistics. Dynamic 2: Daily rebalance.

Optimization	Average Weights			Min. Weight			Max. Weight			Leverage			Ex-post Beta Exposure					nobs	Fail.
	Futures	Stocks	Cash	Futures	Stocks	Cash	Futures	Stocks	Cash	Total	Long	Short	β_{spot}	β_{Mkt}	β_{SMB}	β_{HML}	β_{UMD}		
Sample Period: 1997–2005†																			
Version 1	0.97	−0.04	0.11	−15.44	−0.38	0.00	12.03	0.32	1.00	22.16	11.62	−10.54	0.72	0.01	0.04	0.03	−0.02	2002	0.00
Version 2	0.97	−0.04	0.17	−15.43	−0.38	0.00	12.12	0.32	1.00	22.09	11.54	−10.54	0.72	0.01	0.04	0.03	−0.02	2002	0.00
Version 3	0.97	0.11	0.03	0.00	0.00	0.00	1.00	0.24	0.48	1.00	1.00	0.00	0.68	0.06	0.08	0.10	−0.02	2002	0.00
Version 4	0.96	0.12	0.03	0.00	0.00	0.00	1.00	0.24	0.48	1.00	1.00	0.00	0.67	0.07	0.09	0.11	−0.01	2002	0.00
Version 5	0.96	0.12	0.03	0.00	0.00	0.00	1.00	0.24	0.48	1.00	1.00	0.00	0.67	0.07	0.09	0.11	−0.01	2002	0.00
Version 6	1.22	0.02	0.21	−22.08	−0.74	0.00	17.12	1.14	1.00	28.67	15.05	−13.62	0.96	0.05	0.04	−0.02	−0.04	2002	0.00
Version 7	1.34	1.11	0.07	0.00	0.00	0.00	0.30	0.30	0.30	2.17	2.17	0.00	1.05	0.64	0.41	0.25	−0.27	2002	0.00
Version 8	0.86	1.05	0.03	0.01	−0.10	0.00	0.10	0.10	0.10	7.91	5.04	−2.88	0.90	0.55	0.30	0.18	0.00	2002	0.01
Version 9	1.21	0.25	0.00	−0.10	−0.10	0.00	0.30	0.30	0.00	6.01	3.73	−2.28	0.87	0.08	0.09	−0.02	−0.03	2002	0.16
Version 10	1.21	0.25	0.00	−0.10	−0.10	0.00	0.30	0.30	0.00	6.02	3.73	−2.29	0.87	0.07	0.08	−0.02	−0.03	2002	0.16
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.00	0.07	0.05	0.01	2002	.
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.72	0.01	0.06	0.06	0.01	2002	.
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.66	0.01	0.06	0.06	0.01	2002	.
Sample Period: 2006–2017																			
Version 1	0.97	0.28	0.34	−17.10	−0.48	0.00	21.22	0.36	1.00	5.73	3.33	−2.39	0.61	0.27	0.01	0.11	−0.03	3014	0.00
Version 2	0.97	0.28	0.22	−16.86	−0.48	0.00	21.20	0.36	0.58	5.78	3.39	−2.39	0.61	0.27	0.01	0.11	−0.03	3014	0.00
Version 3	0.97	0.40	0.10	0.00	0.00	0.00	1.00	0.32	0.39	1.00	1.00	0.00	0.57	0.36	0.03	0.10	−0.00	3014	0.00
Version 4	0.86	0.46	0.10	0.00	0.00	0.00	0.99	0.33	0.39	1.00	1.00	0.00	0.54	0.42	0.04	0.10	0.00	3014	0.00
Version 5	0.86	0.46	0.10	0.00	0.00	0.00	0.99	0.33	0.39	1.00	1.00	0.00	0.54	0.42	0.04	0.10	0.00	3014	0.00
Version 6	1.05	0.34	0.54	−23.95	−0.94	0.00	27.15	0.93	1.00	10.42	5.82	−4.60	0.79	0.33	0.02	0.18	0.05	3014	0.00
Version 7	0.72	1.81	0.16	0.00	0.00	0.00	0.30	0.30	0.30	2.32	2.32	0.00	0.93	1.72	0.19	0.22	−0.17	3014	0.00
Version 8	0.54	1.02	0.06	−0.05	−0.10	0.00	0.10	0.10	0.10	7.66	4.47	−3.19	0.83	1.28	0.17	0.19	0.15	3014	0.00
Version 9	0.69	0.46	0.13	−0.10	−0.10	0.00	0.30	0.30	0.30	8.61	4.71	−3.90	0.67	0.42	0.03	0.01	0.01	3014	0.12
Version 10	0.69	0.47	0.13	−0.10	−0.10	0.00	0.30	0.30	0.30	8.66	4.74	−3.93	0.67	0.41	0.02	0.03	0.06	3014	0.12
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.91	0.04	0.00	0.04	0.02	3014	.
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.84	0.07	0.01	0.04	0.03	3014	.
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.08	0.01	0.04	0.03	3014	.

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. † The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, and returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios for minimizing the tracking error relative to spot oil. The table shows the average weights for the selected optimized portfolios. The table also shows the minimum and maximum weights for stocks, futures contracts, and cash over the entire period. The average sum of the long weights over time, the average sum of the short weights over time, and the average of the absolute value of each of these are also shown. Finally, the table shows the ex-post exposures of the optimized portfolio returns to spot oil and the Fama-French factors. These are the exposures of the portfolios based on the regression $r^i_{t,t+1} = a + \beta_{spot} r^S_{t,t+1} + \beta_{Mkt}(r^{Mkt}_{t,t+1} - r^f_{t,t+1}) + \beta_{SMB} r^{SMB}_{t,t+1} + \beta_{HML} r^{HML}_{t,t+1} + \beta_{UMD} r^{UMD}_{t,t+1} + \epsilon_{t,t+1}$, where $r^S_{t,t+1}$ is the return of spot oil from t to $t + 1$, $r^{Mkt}_{t,t+1}$ is the return of the Fama-French market factor, $r^f_{t,t+1}$ is the return of the 1-month Treasury bill, $r^{SMB}_{t,t+1}$ is the return of the Fama-French size factor, $r^{HML}_{t,t+1}$ is the return of the Fama-French value factor, $r^{UMD}_{t,t+1}$ is the return of the Fama-French momentum factor, and $r^i_{t,t+1}$ is the return of either the optimized portfolio or the futures contract. “Fail.” indicates the percentage of the optimizations that failed to find a solution. In these cases, the program weighted the stocks and futures for that investment period equally. Versions 1 to 10 represent the specific optimization parameters described in Table 6.

with spot oil returns is higher. We found that these portfolios could produce higher average returns than spot oil with a minimal increase in the tracking error compared to the larger universe of stocks. Once again, however, the tracking errors were larger than the tracking errors of single futures contracts, on the order of 42% per annum compared to 11% for the first-month futures contract.

In addition, we considered an optimization that relied less on historical data. In this case, we used only futures contracts and constructed an optimized portfolio that attempted to achieve an absolute value of current contango that was as close to zero as possible, given certain weight constraints, as well as a beta with respect to spot oil that was as close to 1 as possible. These optimizations produced excess returns with respect to spot oil in the same range as the individual futures contracts, but with higher tracking errors.

We also considered a very simple strategy in which an investor uses the top 5 ETFs that invest in oil companies (XLE, VDE, XOP, IXC, and IYE) and/or the top 12 oil futures contracts. When oil futures were in backwardation, the investor would choose an equally weighted portfolio of backwardated oil futures contracts. When the oil market was not in backwardation, the investor would choose an equally weighted portfolio of the top 5 oil company ETFs. We found that the tracking error of this portfolio and its variants was much higher than the tracking error when simply investing in the first futures contract: The tracking error was typically on the order of 24%, compared with about 9% for the first futures contract over the period 2009 to 2017. The one benefit, as we also noted in the dynamic optimizations that use oil companies, was that due to avoiding contango-affected futures contracts, the average return over the period was closer to that of spot oil.

6. Conclusion

In order to be exposed to oil, investors can buy either a portfolio of oil futures, a portfolio of oil stocks, or a combination of the two. In recent years, investing in oil futures has underperformed the hypothetical benchmark of spot oil, leading to frustration for some oil investors. Oil stocks have fared better, but they do not track spot oil closely over a short investment horizon. Although many studies have tried to understand this problem—including studies that question the relevance of a spot oil benchmark, studies that examine the role that contango plays in the tracking error, and studies that examine the effect of index manager crowding—the source of the tracking error is still not entirely explained.

Our paper contributes to the literature by asking whether an optimization framework might lead to portfolios that track spot oil better than simple indexing strategies. We constructed both static mean-variance portfolios of oil stocks and oil futures and dynamic mean-variance portfolios that account for contemporaneous contango, as well as other simple buy-and-hold strategies. We found that it was not possible to track spot oil much better compared to passive buy-and-hold investment strategies. On the positive side, we found that a dynamic optimization strategy that avoids futures' contracts in contango can provide annualized excess returns over spot oil, although this comes at the cost of an increased tracking error.

We believe that our work adds to the current understanding of the limitations of actual oil investing and its relation to spot oil. However, much remains to be understood, and future research should continue to examine how storage costs, macroeconomic effects, and crowding contribute to the behavior of oil stocks, oil futures, and oil investing. We also believe that new techniques will be needed to address this problem, if it can be addressed at all. These would include new estimation techniques and new investment approaches, such as total return swaps for some predefined period on a predefined notional amount.

In summary, oil investing with the objective of tracking spot oil is difficult, no matter whether one uses oil futures, oil stocks, or a combination of the two. We find that static mean-variance optimized portfolios do not solve this problem. Dynamic optimization techniques are able to improve the average annualized excess return over spot oil returns, but at the cost of higher tracking errors. Ultimately, our results imply that investors are left to choose between an oil-investing product that tracks short-term movements of spot oil very well but may underperform, or a product that may do better with respect to spot oil in terms of average performance but that may also move very differently over a short-term horizon. Overall, tracking spot oil is extremely difficult.

Appendix A. Mathematics of long and short returns

For each month, our optimizer chooses the weights of equities and futures contracts in a way that minimizes the tracking error with respect to spot oil. When the portfolio is not rebalanced daily, but at some other frequency, the weights of each security need to be updated along with the returns of each security to compute the return for the short portfolio, the long portfolio, and, most importantly, the entire portfolio. The weights are updated according to the following algorithm:

$$w_{i,t+jk} = w_{i,t,t+(j-1)k} \frac{(1 + r^L_{i,t+(j-1)k,t+jk})}{(1 + \sum_{i=1}^{N} w_{i,t} r^L_{i,t+(j-1)k,t+jk})}, \tag{2}$$

where k is the rebalancing interval, j is an index that equals 1 for the first period after a rebalancing date, $r^L_{i,t,t+jk}$ represents the actual long return of a security from $t + (j - 1)k$ to $t + jk$, and $w_{i,t+jk}$ is the weight of a security (long or short) at time $t + jk$. A short security will have a negative weight, while a long security will have a positive weight.

The return of the portfolio over each time interval (e.g., daily) is given by

$$r_{P,t,t+jk} = \sum_{i=1}^{N} w_{i,t+(j-1)k} r^{L}_{i,t+(j-1)k,t+jk}. \tag{3}$$

This computes all of the portfolio returns as if all leveraging is done without any collateral constraints. The total leverage of the portfolio is $l_{P,t} = \sum_{i=1}^{N} |w_{it}|$.[37]

We also modified the returns by a leverage penalty and futures collateralization gains. Thus, for each return interval, the total return of the optimized portfolio is given by

$$\tilde{r}_{P,t,t+jk} = r_{P,t,t+jk} - \underbrace{r_{f,t+(j-1)k,t+jk}\left(\sum_{i=1}^{N} |w_{i,t+(j-1)k}| - 1\right)}_{\text{Leverage Cost}} + \underbrace{r_{f,t+(j-1)k,t+jk}\left(\sum_{i=1}^{N_f} |w^{f}_{i,t+(j-1)k}|\right)}_{\text{Futures Collateral Return}}, \tag{4}$$

where $r_{f,t+(j-1)k,t+jk}$ is the risk-free rate of return from period $t + (j - 1)k$ to period $t + jk$, N_f is the number of futures contracts in the portfolio, $w^{f}_{i,t+(j-1)k}$ is the weight of a particular futures contract, $w_{i,t+(j-1)k}$ is the weight of a stock or futures contract in the optimal portfolio, and $\tilde{r}_{P,t,t+jk}$ is the total portfolio return, including leverage costs and futures collateral gains.

Appendix B. Additional tables and figures

Table 13
Largest 20 ETFs that invest in Oil Futures.

ETF Ticker	Fund Type	Security Name	Inception Date	AUM	E.R.	T.E.
USO	ETF	United States Oil Fund LP	04/10/2006	1973	0.72	N.A.
OIL	ETN	iPath S&P GSCI Crude Oil TR	08/15/2006	629	0.75	7.41
UCO	ETF	ProShares Ultra Bloomberg CR	11/25/2008	437	0.95	26.89
DBO	ETF	PowerShares DB Oil Fund	01/05/2007	313	0.78	1.94
SCO	ETF	ProShares UltraShort Bloomberg Crude Oil	11/25/2008	207	0.95	82.57
DWT	ETN	VelocityShares 3x Inverse Crude Oil	12/09/2016	207	1.5	110.69
UWT	ETN	VelocityShares 3x Long Crude Oil	12/09/2016	118	1.5	54.93
UWTIF	ETN	VelocityShares 3x Long Crude ETN	02/07/2012	103	1.35	45.30
BNO	ETF	United States Brent Oil Fund	06/02/2010	92	0.9	N.A.
USL	ETF	United States 12 Month Oil Fund	12/06/2007	82	0.79	N.A.
OILB	ETN	iPath Series B S&P GSCI Crude Oil	11/18/2016	63	0.45	6.74
DTO	ETN	DB Crude Oil Double Short ETN	06/16/2008	59	0.75	78.73
UBRT	ETN	AxelaTrader 3x Long Brent Crude Oil	09/15/2017	42	1.35	N.A.
OIIL	ETN	Credit Suisse X-Links WTI Crude Index	02/08/2016	38	0.55	7.15
OLEM	ETN	iPath Pure Beta Crude Oil ETN	04/20/2011	30	0.85	5.01
OILD	ETF	ProShares UltraPro 3x Short Crude Oil	03/27/2017	23	0.95	N.A.
WTIU	ETN	ProShares Daily 3x Long Crude Oil	01/05/2017	22	1.45	55.17
WTID	ETN	ProShares Daily 3x Inverse Crude Oil	01/05/2017	16	1.85	110.85
DBRT	ETN	AxelaTrader 3x Inverse Brent Crude Oil	09/15/2017	14	1.65	N.A.
OILU	ETF	ProShares UltraPro 3x Crude Oil	03/27/2017	12	0.95	N.A.

Note: This table shows various statistics of the top 20 ETFs trading on U.S. exchanges ranked by AUM that invest only in oil futures. Inception Date indicates the date the ETF or ETN was launched. AUM represents the assets under management as of February 19, 2018, expressed in millions of U.S. dollars. E.R. is the expense ratio of the fund as of February 19, 2018. Tracking Error is the standard deviation of the difference between the ETF and spot oil daily returns, estimated over the period February 19, 2017 to February 19, 2018. N.A. indicates that Bloomberg did not return a tracking error value for this particular investment vehicle. Source: Bloomberg.

[37] Although we did not do this, a researcher can alter the returns by the leverage if the researcher wants to assume that a greater allocation of capital is required for any position; this is done by simply creating a new set of weights $\tilde{w}_{it} = \frac{1}{l^*} w_{it}$, where $1 \le l^* \le l_{P,t}$.

Table 14
Largest 20 ETFs and Mutual Funds that invest directly in Oil Companies.

Equity Fund	Fund Type	Security Name	Inception Date	AUM	E.R.	T.E.
XLE	ETF	Energy Select Sector SPDR Fund	12/22/1998	17936	0.13	0.46
VDE	ETF	Vanguard Energy ETF	09/29/2004	3949	0.1	0.30
XOP	ETF	SPDR S&P Oil & Gas Exploration & Production	06/22/2006	2510	0.35	0.58
IXC	ETF	iShares Global Energy ETF	11/16/2001	1174	0.47	1.45
IYE	ETF	iShares U.S. Energy ETF	06/16/2000	1105	0.43	0.46
FENY	ETF	Fidelity MSCI Energy Index ETF	10/24/2013	508	0.08	0.55
IEO	ETF	iShares U.S. Oil & Gas Exploration & Production	05/05/2006	332	0.43	0.80
RYE	ETF	Guggenheim S&P 500 Equal Weight Energy	11/07/2006	265	0.4	0.78
FXN	ETF	First Trust Energy AlphaDEX Fund	05/10/2007	208	0.62	0.60
FCG	ETF	First Trust Natural Gas ETF	05/11/2007	163	0.6	1.01
VGENX	Open-End Fund	Vanguard Energy Fund	05/23/1984	9764	0.37	3.38
FSENX	Open-End Fund	Fidelity Select Energy Portfolio	07/14/1981	2002	0.88	17.07
FAGNX	Open-End Fund	Fidelity Advisor Energy Fund	12/29/1987	949	1.35	17.12
VENAX	Open-End Fund	Vanguard Energy Index Fund	09/29/2004	606	0.10	17.97
FSTEX	Open-End Fund	Invesco Energy Fund/United States	01/19/1984	576	1.26	18.58
BGR	Closed-End Fund	BlackRock Energy and Resources Trust	12/23/2004	437	1.24	N.A.
NDP	Closed-End Fund	Tortoise Energy Independence Fund Inc	07/27/2012	316	1.58	15.91
ICENX	Open-End Fund	ICON Energy Fund	11/05/1997	191	1.42	8.48
SSGRX	Open-End Fund	BlackRock Energy & Resources Portfolio	03/02/1990	149	1.36	10.78
BACAX	Open-End Fund	BlackRock All-Cap Energy & Resources Portfolio	02/16/2005	83	1.36	3.76

Note: This table shows various statistics of the top 10 ETFs and top 10 mutual funds ranked by AUM that invest in oil companies. Inception Date indicates the date the ETF, ETN, or mutual fund was launched. AUM represents the assets under management as of February 19, 2018, expressed in millions of U.S. dollars. E.R. is the expense ratio of the fund as of February 19, 2018. Tracking Error is the standard deviation of the difference between the investment vehicle and spot oil daily returns, estimated over the period February 19, 2017 to February 19, 2018. N.A. indicates that Bloomberg did not return a tracking error value for this particular investment vehicle. Source: Bloomberg.

Table 15
Optimization Results of Tracking Portfolios. Standard Mean Variance: Monthly Rebalance.

Portfolio Strategy	Sample Period: 1997–2005						Sample Period: 2006–2017					
	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max	μ_p	μ_s	$\mu_p - \mu_s$	T.E.	Min	Max
Optimization												
Version 1	19.04	15.27	3.77	21.27	−15.51	16.43	−11.76	−0.45	−11.30	12.69	−19.46	9.78
Version 2	19.05	15.27	3.78	21.27	−15.51	16.43	−11.72	−0.45	−11.27	12.67	−19.25	9.78
Version 3	24.30	15.27	9.03	20.16	−15.51	16.28	−9.39	−0.45	−8.94	11.08	−17.71	9.59
Version 4	24.58	15.27	9.31	20.32	−15.53	16.30	−7.82	−0.45	−7.37	12.56	−16.29	9.61
Version 5	24.58	15.27	9.30	20.32	−15.53	16.30	−7.82	−0.45	−7.37	12.56	−16.29	9.61
Version 6	24.10	15.27	8.83	24.16	−16.25	17.09	−12.38	−0.45	−11.93	13.78	−20.31	9.71
Version 7	36.71	15.27	21.44	26.43	−18.03	15.62	−8.91	−0.45	−8.46	15.35	−19.69	9.39
Version 8	33.34	15.27	18.06	36.77	−17.86	15.07	−7.36	−0.45	−6.90	19.19	−19.84	8.65
Version 9	29.74	15.27	14.47	26.14	−17.92	15.56	−13.52	−0.45	−13.07	16.14	−21.82	9.42
Version 10	29.68	15.27	14.41	26.14	−17.92	15.55	−12.73	−0.45	−12.28	16.34	−21.59	9.59
Rolling Futures												
Fut1Roll0	22.99	15.27	7.72	20.07	−15.65	16.46	−10.06	−0.45	−9.60	11.04	−18.10	9.52
Fut2Roll0	25.50	15.27	10.23	21.21	−17.17	15.50	−9.62	−0.45	−9.17	13.96	−17.83	11.35
Fut3Roll0	28.64	15.27	13.37	22.20	−16.94	14.75	−7.01	−0.45	−6.55	15.14	−17.45	11.43
Equal-Weight	20.32	15.27	5.05	37.14	−17.62	13.54	3.36	−0.45	3.81	29.30	−17.68	14.63
Market-Cap	21.03	15.27	5.76	30.02	−15.93	14.60	1.05	−0.45	1.50	21.47	−14.81	9.62

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. † The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios that minimize the tracking error relative to spot oil. The portfolios are rebalanced on a monthly interval. μ_p is the compounded geometric return of a particular strategy over the given period. μ_s is the same metric, but for spot oil. $\mu_p - \mu_s$ is the difference between the two. T.E. is the tracking error of a strategy measured against spot oil, which is formulated as the annualized standard deviation of the daily difference between the strategy and spot oil returns. Min and Max indicate the minimum and maximum daily return differences. Fut1Roll0, Fut2Roll0, Fut3Roll0 show the same statistics for rolling the 1st, 2nd, and 3rd futures, Equal-Weight represents the return of a portfolio that equally weights the top 100 stocks by market capitalization and the first 12 futures contracts in every period (thus, all weights are positive and their sum is 1), and Market-Cap is a portfolio that weights all 100 stocks by market capitalization in every period and equally weights the first 12 futures contracts. Futures and oil stocks are combined with an arbitrary weight of 0.50 in each group (thus, all weights are positive and their sum is 1). Versions 1 to 10 represent the specific optimization parameters described in Table 6.

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Table 16
Optimization Weights and Ex-post Beta Exposure Statistics. Standard Mean Variance: Monthly Rebalance.

Optimization	Average Weights			Min. Weight			Max. Weight			Leverage			Ex-post Beta Exposure					nobs	Fail.
	Futures	Stocks	Cash	Futures	Stocks	Cash	Futures	Stocks	Cash	Total	Long	Short	β_{spot}	β_{Mkt}	β_{SMB}	β_{HML}	β_{UMD}		
Sample Period: 1997–2005[†]																			
Version 1	0.97	−0.08	0.13	−13.44	−0.22	0.00	6.78	0.18	1.18	25.42	13.31	−12.11	0.78	0.01	0.05	0.01	−0.00	2002	0.00
Version 2	0.97	−0.08	0.11	−13.44	−0.22	0.02	6.77	0.18	0.28	25.23	13.11	−12.11	0.78	0.01	0.05	0.01	−0.00	2002	0.00
Version 3	0.97	0.03	0.00	0.00	0.00	0.00	0.98	0.02	0.00	1.00	1.00	0.00	0.76	0.03	0.08	0.06	0.00	2002	0.00
Version 4	0.96	0.04	0.00	0.00	0.00	0.00	0.97	0.07	0.00	1.00	1.00	0.00	0.75	0.05	0.10	0.07	0.01	2002	0.00
Version 5	0.96	0.04	0.00	0.00	0.00	0.00	0.97	0.07	0.00	1.00	1.00	0.00	0.75	0.05	0.10	0.07	0.01	2002	0.00
Version 6	1.19	−0.08	0.24	−19.93	−0.26	0.00	10.86	0.28	1.20	29.57	15.54	−14.03	0.97	0.02	0.04	0.01	0.00	2002	0.00
Version 7	1.50	0.17	0.07	0.00	0.00	0.00	0.42	0.17	0.34	1.65	1.65	0.00	1.05	0.15	0.16	0.19	−0.00	2002	0.00
Version 8	1.18	0.34	0.03	0.07	−0.21	0.00	0.14	0.15	0.11	6.43	4.10	−2.33	0.90	0.18	0.15	0.19	0.04	2002	0.00
Version 9	1.36	−0.03	0.06	−0.14	−0.15	0.00	0.40	0.36	0.34	5.35	3.40	−1.95	0.98	−0.02	0.03	0.02	−0.01	2002	0.00
Version 10	1.36	−0.03	0.09	−0.14	−0.15	0.00	0.40	0.36	0.33	5.36	3.41	−1.95	0.98	−0.02	0.03	0.02	−0.01	2002	0.00
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.00	0.07	0.05	0.01	2002	.
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.72	0.01	0.06	0.06	0.01	2002	.
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.66	0.01	0.06	0.06	0.01	2002	.
Sample Period: 2006–2017																			
Version 1	0.97	0.01	0.27	−17.19	−0.22	0.00	15.92	0.14	2.18	17.98	9.49	−8.49	0.89	0.07	−0.00	0.05	0.01	3014	0.00
Version 2	0.97	0.01	0.02	−17.00	−0.22	0.00	16.08	0.14	0.18	17.95	9.47	−8.47	0.89	0.07	−0.00	0.05	0.01	3014	0.00
Version 3	0.96	0.04	0.00	0.00	0.00	0.00	0.98	0.03	0.00	1.00	1.00	0.00	0.88	0.08	0.01	0.04	0.02	3014	0.00
Version 4	0.84	0.16	0.00	0.00	0.00	0.00	0.96	0.20	0.00	1.00	1.00	0.00	0.80	0.19	0.02	0.04	0.03	3014	0.00
Version 5	0.84	0.16	0.00	0.00	0.00	0.00	0.96	0.20	0.00	1.00	1.00	0.00	0.80	0.19	0.02	0.04	0.03	3014	0.00
Version 6	1.06	−0.00	0.68	−22.93	−0.24	0.02	18.02	0.20	2.31	22.93	12.24	−10.69	0.97	0.06	0.01	0.04	0.08	3014	0.00
Version 7	1.21	0.06	0.12	0.00	0.00	0.00	0.35	0.11	0.47	1.29	1.29	0.00	1.01	0.13	0.04	0.08	0.05	3014	0.00
Version 8	1.17	0.02	0.08	−0.08	−0.17	0.02	0.13	0.17	0.17	4.34	2.78	−1.56	0.93	0.13	0.06	0.13	0.13	3014	0.00
Version 9	1.10	−0.04	0.15	−0.12	−0.16	0.00	0.41	0.25	0.56	4.43	2.77	−1.66	0.95	0.04	0.03	0.04	−0.00	3014	0.00
Version 10	1.10	−0.03	0.25	−0.11	−0.15	0.00	0.38	0.25	0.58	4.72	2.99	−1.73	0.95	0.04	0.03	0.02	0.04	3014	0.00
Fut1Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.91	0.04	0.00	0.04	0.02	3014	.
Fut2Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.84	0.07	0.01	0.04	0.03	3014	.
Fut3Roll0	.	.	.	.	.	.	.	.	.	.	.	.	0.80	0.08	0.01	0.04	0.03	3014	.

Note: This table shows the statistics for varying optimized portfolios and oil futures indices for the period from December 1997 to December 2017, representing a total of 5016 observations. [†] The actual data start in 1994, but since we use a 1000 trading day look-back period for the variance–covariance matrix, the first out-of-sample values start in December 1997. The parameters for this out-of-sample backtest are as follows. Daily data are used, a risk-free asset is included in the optimization, and returns are computed with collateralized futures and also penalized by a risk-free rate for leverage. This particular optimization chooses from the top 100 oil companies in the SIC sectors 1311, 1381, 1382, 1389, 2911, 3533, 4612, 4613, 5171, and 5172 along with all oil futures to construct the optimal portfolios for minimizing the tracking error relative to spot oil. The table shows the average weights for the selected optimized portfolios. The table also shows the minimum and maximum weights for stocks, futures contracts, and cash over the entire period. The average sum of the long weights over time, the average sum of the short weights over time, and the average of the absolute value of each of these are also shown. Finally, the table shows the ex-post exposures of the optimized portfolio returns to spot oil and the Fama-French factors. These are the exposures of the portfolios based on the regression $r^i_{t,t+1} = a + \beta_{spot} r^S_{t,t+1} + \beta_{Mkt}(r^{Mkt}_{t,t+1} - r^f_{t,t+1}) + \beta_{SMB} r^{SMB}_{t,t+1} + \beta_{HML} r^{HML}_{t,t+1} + \beta_{UMD} r^{UMD}_{t,t+1} + \epsilon_{t,t+1}$, where $r^S_{t,t+1}$ is the return of spot oil from t to $t+1$, $r^{Mkt}_{t,t+1}$ is the return of the Fama-French market factor, $r^f_{t,t+1}$ is the return of the 1-month Treasury bill, $r^{SMB}_{t,t+1}$ is the return of the Fama-French size factor, $r^{HML}_{t,t+1}$ is the return of the Fama-French value factor, $r^{UMD}_{t,t+1}$ is the return of the Fama-French momentum factor, and $r^i_{t,t+1}$ is the return of either the optimized portfolio or the futures contract. "Fail." indicates the percentage of the optimizations that failed to find a solution. In these cases, the program weighted the stocks and futures for that investment period equally. Versions 1 to 10 represent the specific optimization parameters described in Table 6.

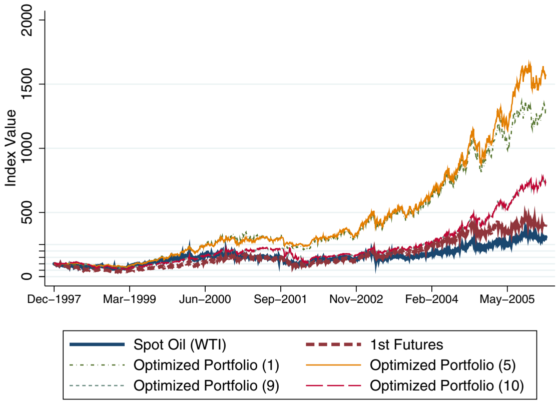

Fig. 3 Selected Optimized Portfolios with Dynamic 1 and Daily Rebalance, 1997–2005. Top 100 oil stocks and all backwardated oil futures are used to construct the optimized portfolio with objective of minimizing tracking errors relative to spot oil. Portfolio is rebalanced on every trading day.

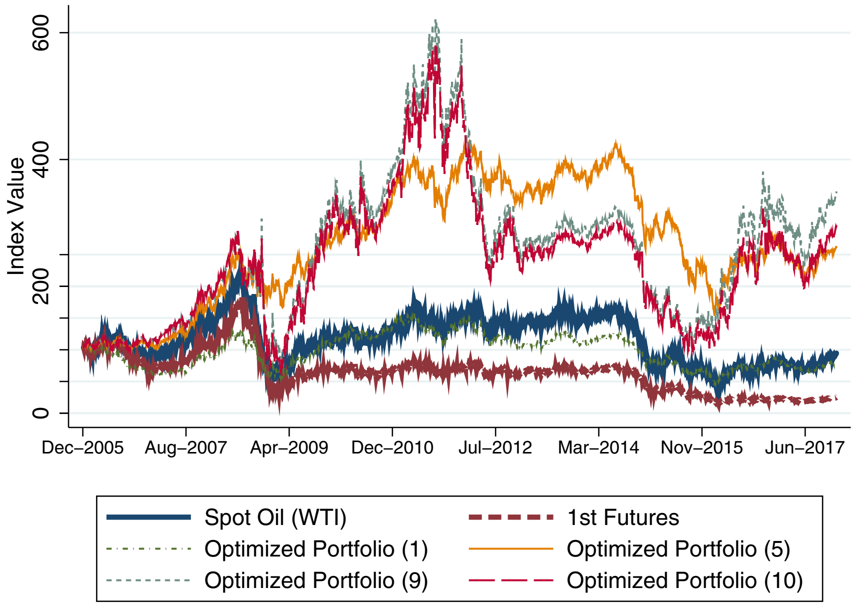

Fig. 4 Selected Optimized Portfolios with Dynamic 1 and Daily Rebalance, 2006–2017. Top 100 oil stocks and all backwardated oil futures are used to construct the optimized portfolio with objective of minimizing tracking errors relative to spot oil. Portfolio is rebalanced on every trading day.

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