

# An Alternative Method to Construct Levered Indexes

SALVATORE BRUNO, LUDWIG CHINCARINI,  
AND ROBERT F. WHITELAW

**SALVATORE BRUNO** is chief investment officer at IndexIQ in Rye Brook, NY.  
sbruno@indexiq.com

**LUDWIG CHINCARINI** is an associate professor of finance at the University of San Francisco School of Management and academic counsel at IndexIQ in Rye Brook, NY.  
chincarinil@hotmail.com

**ROBERT WHITELAW** is the Edward C. Johnson III Professor of Entrepreneurial Finance at New York University Stern School of Business and chief investment strategist at IndexIQ in Rye Brook, NY.  
rwhitela@stern.nyu.edu

The introduction of ETFs has ushered in an era of incredible product development and innovation. Whereas the first ETF introduced in 1993 aimed to deliver the return of the S&P 500 (net of fees and expenses), ETF products evolved to the point where they offered to deliver multiples and inverses of the market returns. In 2006, ProShares introduced ETFs designed to deliver just that: two times the daily return of the S&P 500 (SSO) and negative one times the daily return of the S&P 500 (SH). SSO has had cumulative inflows of more than \$3 billion since its inception in June 2006<sup>1</sup> and stands at nearly \$2.5 billion in assets under management as of April 30, 2013. Likewise, SH has also attracted more than \$3 billion in inflows and has more than \$1.8 billion in assets as of April 30, 2013. Despite a great deal of scrutiny by the press<sup>2</sup> and regulators,<sup>3</sup> investors have demonstrated a desire for more levered investments.

These products were originally designed to provide a predetermined leverage over a one-day holding period and were primarily intended for market-timing investment advisors. However, if an investor has a holding period greater than one day, the objectives of these products (and thus their results) may be misaligned and therefore may be ill-suited for investors who want to capture multiple and inverse exposure over a longer horizon.

For example, in 2011, the S&P 500 Index had a total return of 2.11%.<sup>4</sup> A strategy that delivered exactly two times the daily return of the S&P 500 each day would have had a total return of -1.30% for the full year.<sup>5</sup> An investor looking for a multiple of the S&P 500 return might have been expecting a return closer to 4.22% (or  $2 \times 2.11\%$ ). A strategy that delivered exactly negative one times the daily return of the S&P 500 each day would have had a total return of -7.82% over the same year. An investor looking for the inverse of the S&P 500 return might have been expecting a return closer to -2.11% (or  $-1 \times 2.11\%$ ).

Thus, while a product may fulfill its investment goal of delivering a multiple or inverse of the S&P 500 return on a *daily* basis, it will not necessarily deliver the multiple or inverse of the S&P 500 return for longer holding periods.

## WHY DAILY MULTIPLE AND INVERSE PRODUCTS MAY NOT WORK FOR BUY-AND-HOLD INVESTORS

Numerous academic articles have been written documenting the problems with multiple and inverse ETFs that rebalance daily (Avellaneda and Zhang [2009]; Cheng and Madhavan [2009]; Shum and Kang [2012]; and Tang and Xu [2013]). We will

not attempt to restate all of these arguments here, and curious readers are encouraged to read these papers. Rather, we would like to provide some intuition of the problem, first in the context of a simple example and then using some real-world data.

The basic result can be seen in an example that covers only two days. The cumulative return ( $cr$ ) for a long-only portfolio over two days is:

$$cr = (1 + r_t)(1 + r_{t+1}) - 1 = r_t + r_{t+1} + r_t r_{t+1} \quad (1)$$

where  $r_t$  is the return of the index on day  $t$ , and  $r_{t+1}$  is the return on day  $t + 1$ .

Now consider a product that delivers exactly two times the daily return each day ( $cr_{2X}$ ). (We will explain shortly why this is not really possible, but for the moment, assume that it is.) The cumulative two-day return on this multiple product is:

$$\begin{aligned} cr_{2X} &= (1 + 2r_t)(1 + 2r_{t+1}) - 1 = 2r_t + 2r_{t+1} + 4r_t r_{t+1} \\ &= 2(r_t + r_{t+1} + r_t r_{t+1}) + 2r_t r_{t+1} \end{aligned} \quad (2)$$

So the differential between twice the return on the underlying portfolio ( $2cr$ ) and the return on the multiple product ( $cr_{2X}$ ), which is due to a compounding effect, is:

$$\begin{aligned} 2cr - cr_{2X} &= 2(r_t + r_{t+1} + r_t r_{t+1}) \\ &\quad - [2(r_t + r_{t+1} + r_t r_{t+1}) + 2r_t r_{t+1}] = -2r_t r_{t+1} \end{aligned} \quad (3)$$

A numeric example might make things clearer. Suppose the S&P 500 increases by 5% on Monday and then declines by 3% on Tuesday. An investment of \$10,000 in the index increases by \$185 over these two days. An investor invested in the 2X product might expect a dollar increase of \$370 (or  $2 \times \$185$ ). However, the daily rebalanced 2X product provides only \$340. Thus, \$30 of the expected return, or 8.11% of an investor's expected increase, disappears.

Similarly, consider an inverse fund that delivers the negative of the return on the underlying portfolio on a daily basis ( $cr_{-1X}$ ). Its cumulative two-day return is:

$$\begin{aligned} cr_{-1X} &= (1 - r_t)(1 - r_{t+1}) - 1 = -r_t - r_{t+1} + r_t r_{t+1} \\ &= -1(r_t + r_{t+1} + r_t r_{t+1}) + 2r_t r_{t+1} \end{aligned} \quad (4)$$

So the return differential due to compounding for the inverse product ( $cr_{-1X}$ ) relative to the negative of the cumulative return on the underlying portfolio ( $-1cr$ ) is:

$$\begin{aligned} -1cr - cr_{-1X} &= -1(r_t + r_{t+1} + r_t r_{t+1}) \\ &\quad - [-1(r_t + r_{t+1} + r_t r_{t+1}) + 2r_t r_{t+1}] = -2r_t r_{t+1} \end{aligned} \quad (5)$$

A numeric example for the inverse might also help. Suppose the S&P 500 decreases by 5% on Monday and then increases by 3% on Tuesday. A short investment of \$10,000 in the index increases by \$215 over these two days. An investor invested in the inverse product might expect a dollar increase of \$215. However, the daily rebalanced inverse product increases by only \$185. Thus, \$30 of the expected return, or 13.95% of an investor's expected increase, disappears.

It is clear that the return differentials for both the multiple and inverse products depend on the serial correlation properties of returns in the same way. For longer holding periods, the same result holds, except that it involves functions of serial correlations at longer lags. Basically, if returns exhibit reversals, i.e., positive returns followed by negative returns or vice versa, then daily rebalanced multiple and inverse products underperform their naïve benchmarks, whereas if they exhibit continuations, then these products outperform.

Alternatively, one can view this property of returns in terms of volatility. Holding constant the cumulative return of a portfolio over a year, higher volatility implies greater underperformance of the multiple and inverse products relative to the multiple of the cumulative return on the underlying portfolio. The idea is that higher volatility implies larger returns in both directions, and thus larger reversals at some horizon, given a fixed cumulative return.

Finally, this compounding or volatility effect can also be thought of as a rebalancing effect. To achieve a multiple or inverse return on a daily basis, the portfolio needs to be rebalanced daily. For example, a 2X return is achieved by borrowing an amount equal to the value of the fund and investing the value plus the borrowing in the underlying portfolio. However, if the portfolio goes up on any given day, the value of the fund goes up, and the borrowing and investment must be increased to preserve the leverage. Thus, a reversal the next day will hurt this rebalanced fund more than a fund that does not rebalance because of the increased exposure. Similarly, a daily inverse fund must increase its short position after a negative return on the underlying portfolio as the value of the fund increases, and it will be hurt more from a subsequent positive return than a fund that does not rebalance.

In contrast, the naïve benchmark returns discussed above are achieved by establishing the initial leveraged or inverse position and then holding it, without rebalancing. Therefore, what we label a compounding effect above is due solely to the fact that the daily multiple or inverse products rebalance, while the naïve benchmarks do not.

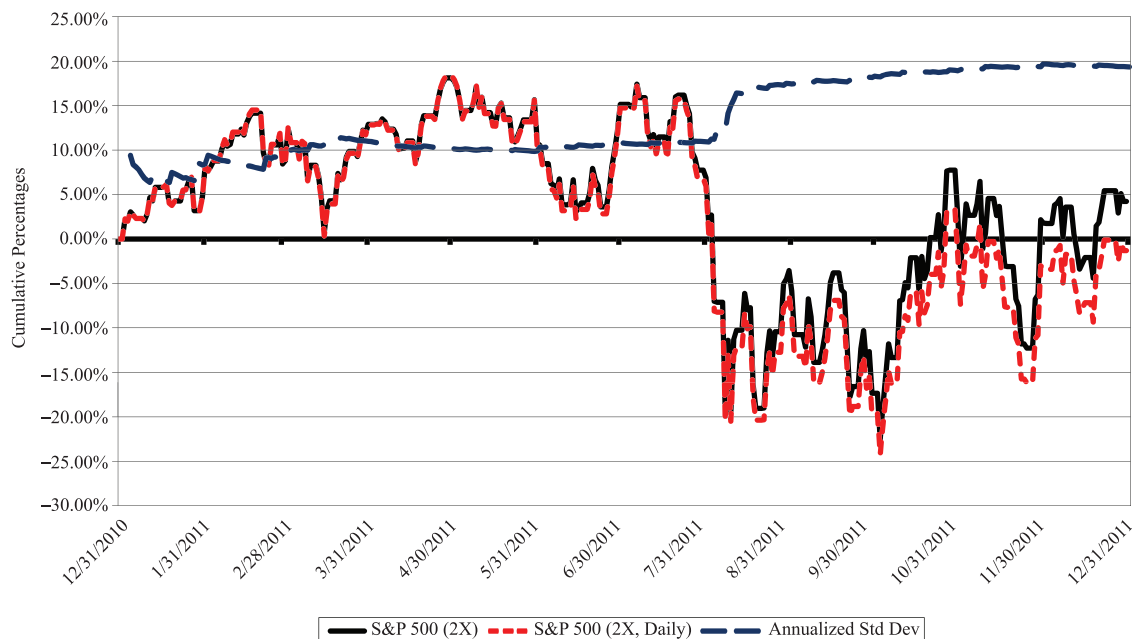
Before moving on to some examples using real data, it is worth expanding on the comment above that even “perfect” multiple or inverse products do not generate exactly the multiple or inverse return on a daily basis. For multiple products, the issue is the financing or borrowing cost implicit in the strategy. A 2X multiple product needs to generate an exposure to the underlying portfolio equal to twice the fund’s assets; i.e., the fund needs to borrow and invest an additional amount equal to fund value. Thus, the return on any given day is twice the return on the underlying portfolio, less the borrowing rate. More generally, for a multiple product designed to generate  $M$  times the return on the underlying portfolio, the true target return is  $Mr_t - (M - 1)r_{fi}$ , where  $r_{fi}$  is the financing (borrowing) rate. Of course, multiple products may not borrow to achieve their return

targets, but instead may enter into total return swaps or use futures contracts. The precise method of implementation does not affect the result above—futures contracts do not require financing, ignoring margin requirements, but they generate the excess return on the underlying asset, not the total return; and total return swaps pay the total return on the underlying asset but require a payment of the short-term rate in exchange.

For inverse products, the formula above also applies, but the multiple,  $M$ , is  $-1$ . These products generate income rather than paying for financing. An inverse product invests the value of the fund’s assets plus an equal amount in proceeds from short selling the underlying portfolio in order to generate the short exposure. Thus, twice the value of the assets can be invested at the short-term rate. Again, implementation with futures or total return swaps generates a similar result. Although this effect is potentially important in some environments, short-term rates are currently close to zero, so we ignore it in the examples that follow.

To illustrate the intuition above with real data, we plot two series in Exhibit 1: two times the cumulative

**EXHIBIT 1**  
2X Cumulative S&P 500 vs. Cumulative 2X S&P 500



Past performance is not a guarantee of future results.

Source: IndexIQ research, FactSet.

return of the S&P 500 in solid (S&P 500 (2X)) and the cumulative return of two times the daily return of the S&P 500 with a dashed line (S&P 500 (2X, Daily)) for 2011. We also plot the cumulative annualized standard deviation of the S&P 500 index. One can see from this chart that the daily multiple strategy (S&P 500 (2X, Daily)) tracked the multiple of the cumulative index (S&P 500 (2X)) quite tightly, while volatility (as measured by the standard deviation) was low. This persisted until July 2011, when volatility climbed from around 10% to closer to 20%. The rise in volatility is associated with an increase in the return differential between the multiple of the cumulative return and the cumulative return of the daily multiple series. In other words, knowing in hindsight that the S&P 500 was basically flat though 2011, any large returns, i.e., volatility, must be reversed over the course of the year, generating under-performance for the daily multiple product.

The same is true for the difference between the negative of the cumulative return of the index and the cumulative return of a daily inverse index.

To illustrate this point, we plot the inverse of the cumulative return of the S&P 500 (S&P 500 Inverse) for

2011 in Exhibit 2. We then plot the cumulative return of the inverse index (S&P 500 (-1X, Daily)). Again, we also plot the cumulative annualized standard deviation of the S&P 500 index.

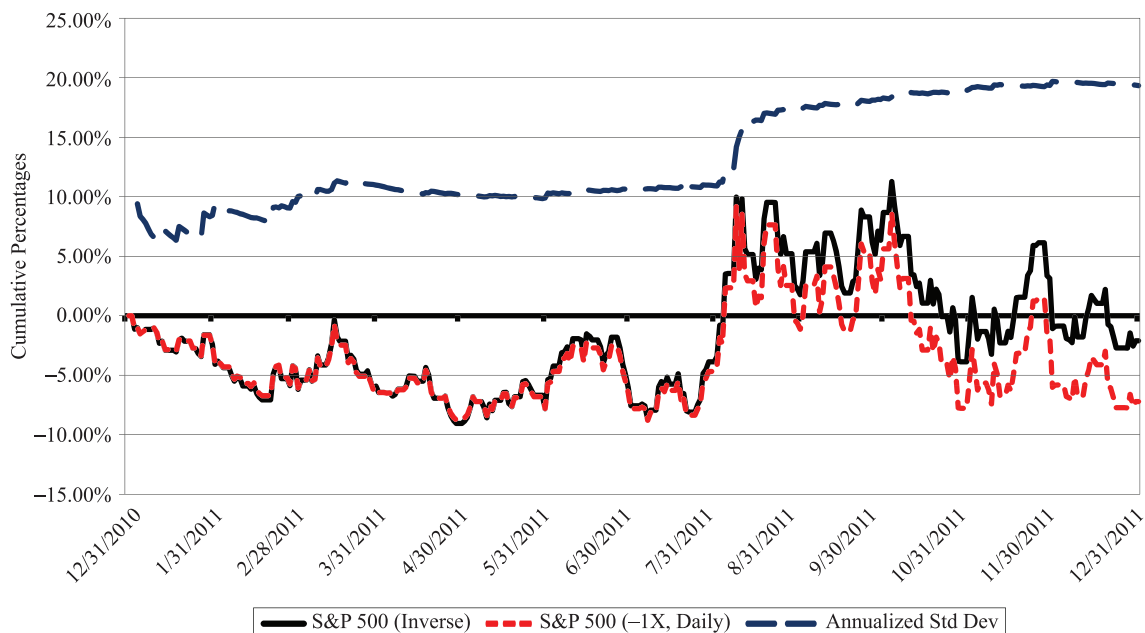
As we saw in Exhibit 1, the daily inverse strategy (S&P 500 (-1X, Daily)) tracks the inverse of the cumulative index (S&P 500 (Inverse)) quite tightly while volatility is low, i.e., through July 2011, but exhibits significant tracking error thereafter.

Given these observations, it is worthwhile to look more closely at how the specific path of returns generated divergence between the two series.

### Compounding Effect

Exhibit 3 shows the returns on three strategies over a four-day period in August 2011: 1) the simple buy-and-hold of the index; 2) the buy-and-hold levered strategy that levers up 2:1 initially, i.e., at the beginning of the year, but never rebalances, which is equivalent to two times the cumulative return on the index; and 3) the strategy that levers up 2:1 but rebalances back to this leverage level every day, which is equivalent to the

**EXHIBIT 2**  
Inverse Cumulative S&P 500 vs. Cumulative Inverse S&P 500



*Past performance is not a guarantee of future results.*

Sources: IndexIQ research, FactSet.

## EXHIBIT 3

### Illustration of the Expectation Errors Created from a Daily Rebalanced Levered Strategy

Position	August 22		August 23		August 24		August 25	
	Cumulative Return	Daily Return	Cumulative Return	Daily Return	Cumulative Return	Daily Return	Cumulative Return	
S&P 500	-9.51	3.43	-6.40	1.33	-5.16	-1.55	-6.64	
S&P 500 (2X)	-19.01		-12.80		-10.32		-13.27	
S&P 500 (2X, daily)	-20.33	6.86	-14.87	2.65	-12.61	-3.11	-15.33	
Expectation Error	-1.32		-2.06		-2.29		-2.05	

Notes: The exhibit represents the cumulative returns of the S&P 500 Index and two other indexes related to the S&P 500. One is the S&P 500 (2X), which does not rebalance and simply reflects the 2X levered value of the S&P 500 for any given horizon since the beginning of the year. The other index, the S&P 500 (2X, daily), is the value of a 2X levered fund that rebalances daily. Expectation Error is not defined here as the traditional Tracking Error (which is the standard deviation of excess returns) but rather as the arithmetic difference between the cumulative returns of the S&P 500 (2X) and S&P 500 (2X, daily). Return is in percentage terms.

Past performance is not a guarantee of future results.

Sources: IndexIQ research, FactSet.

daily multiple strategy. Each strategy starts at the beginning of 2011.

As shown in the first column, through August 22, the S&P 500 was down -9.51% since the beginning of the year. By construction, the buy-and-hold 2X levered strategy is down twice this return, -19.01% over the same time period. The daily rebalanced 2X levered strategy has a slightly lower return, -20.33%, with a cumulative difference of -1.32% over almost eight months.

On August 23, the S&P 500 rose 3.43%. In the context of our simple example above, this is negative serial correlation—the return was negative up to this point in the year and the new daily return is positive—so it should increase the expectation error of the daily multiple strategy. The cumulative returns on the 1X and 2X buy-and-hold strategies are  $(1 - 9.51\%) \times (1 + 3.43\%) - 1 = -6.40\%$  and  $2 \times (-6.40\%) = -12.80\%$ , respectively.<sup>6</sup> However, the new cumulative return on the daily rebalance 2X strategy is  $(1 - 20.33\%) \times (1 + 2 \times [3.43\%]) - 1 = -14.87\%$ , for a new, higher, expectation error of -2.06%. With another positive return on August 24 of 1.33%, the expectation error increases for a second time, to -2.29%. The key is that the positive daily return is reversing a cumulative negative return up to that point, not that we see a sequence of two positive daily returns. Finally, on August 25, the S&P 500 falls to -1.55%, a positive correlation between the new return and the cumulative return up to that point, and the expectation error narrows to -2.05%. This process continues for the rest of the year, yielding the results in Exhibit 1.

Exhibit 4 shows a similar effect of volatility on the difference in returns for the daily inverse strategy, where again we track the returns on three strategies, in this case 1) the buy-and-hold of the index, 2) the buy-and-hold inverse strategy that shorts the index at the beginning of the year and does not rebalance, and 3) the daily inverse strategy that earns the negative of the index return each day.

As shown in the first column, on August 1, the S&P 500 was up, at 3.44% for the year, giving a -3.44% return for the buy-and-hold inverse strategy. In contrast, the daily rebalanced inverse strategy was down, to -4.29%, for an expectation error of -0.85%. On August 2, the S&P 500 dropped -2.56%—a reversal of the positive return up to that point. The new cumulative returns on the S&P 500 and buy-and-hold inverse are  $(1 + 3.44\%) \times (1 - 2.56\%) - 1 = 0.80\%$ , and -0.80%, respectively. However, the cumulative return on the daily inverse diverges further to  $(1 - 4.29\%) \times (1 + 2.56\%) - 1 = -1.84\%$ , for a new expectation error of -1.05%. The positive S&P 500 return of 0.54% on August 3 reverses this divergence slightly, but the further decline to -4.78% on August 4 again increases the expectation error. As with the multiple product, daily returns in the opposite direction to the cumulative return at that point in time hurt the relative performance of the daily rebalanced product.

This trend continued (and accelerated) through the rest of the year as volatility continued to increase, yielding the results in Exhibit 2.



## EXHIBIT 4

### Illustration of the Expectation Errors Created from a Daily Rebalanced Inverse Strategy

Position	August 1		August 2		August 3		August 4	
	Cumulative Return	Daily Return	Cumulative Return	Daily Return	Cumulative Return	Daily Return	Cumulative Return	
S&P 500	3.44	-2.56	0.80	0.54	1.34	-4.78	-3.50	
S&P 500 (-1X)	-3.44		-0.80		-1.34		3.50	
S&P 500 (-1X, daily)	-4.29	2.56	-1.84	-0.54	-2.37	4.78	2.29	
Expectation Error	-0.85		-1.05		-1.03		-1.21	

Notes: The exhibit represents the cumulative returns of the S&P 500 Index and two other indexes related to the S&P 500. One is the -1X S&P 500 (no rebalance), which simply reflects the inverse value of the S&P 500 for any given horizon since the beginning of the year. The other index, the -1X S&P 500 (daily rebalance), is the value of an inverse fund that rebalances daily. Expectation Error is not defined here as the traditional Tracking Error (which is the standard deviation of excess returns) but rather as the arithmetic difference between the cumulative returns of the S&P 500 (-1X) and S&P 500 (-1X, daily). Return is in percentage terms.

Past performance is not a guarantee of future results.

Sources: IndexIQ research, FactSet.

For both the multiple and inverse strategies, the cause of the differences is the path-dependent nature that comes from the compounding effect. That is, targeting daily returns can require trading at exactly the most inopportune times—buying high and selling low.

### AN ALTERNATIVE LEVERED INDEX

Given the evidence documented in the previous section, it's clear that getting precise multiple or inverse exposure from a daily rebalanced strategy can be a very difficult challenge. Of perhaps even greater concern, however, is that the daily rebalanced multiple and inverse strategies can underperform their desired targets over extended holding periods. This can be the result of less-than-full upside participation (downside protection) or more downside capture (upside drag) for the multiple (inverse) solutions. Investors concerned with the performance impact of mistracking may be wondering what, if anything, they can do to get multiple or inverse exposure to the market that might not suffer from these performance drags.

#### Rebalance Frequency

For investors who want more efficient multiple or inverse market exposure for longer than one day, one potential improvement to the daily target products would be to extend the target horizon. So, for example, a strategy might seek to deliver two times (negative

one times) the *monthly* return of the S&P 500 instead of two times (negative one times) the *daily* return. This approach might better align the strategy holding period with the investor's holding period. Columns 2–7 of Exhibit 5 show the impact of switching from targeting a daily exposure to a monthly exposure.<sup>7</sup>

Although by definition the monthly rebalance tracks perfectly on a monthly basis, it may not improve upon the tracking on an annual basis relative to the daily rebalance from a statistical standpoint (i.e., lower annual tracking error). For the monthly rebalance, serial correlation of monthly returns within the year causes this tracking error. However, in the presence of negative serial correlation in the daily returns, less frequent (i.e., monthly) rebalancing may reduce the performance drag and potentially create higher returns. In fact, for the time period from April 2002 through April 2013 in Exhibit 5, “Excess (1)” and “Excess (2)” (i.e., the differences between the annual returns on the monthly and daily rebalance for the 2X and inverse strategies, respectively) show that the monthly rebalance adds value over the daily rebalance for both strategies. The average annual and annualized returns for the monthly rebalance are higher than returns for the daily rebalance and are closer to the targeted market exposure. Further, these results are very consistent with hit rates (monthly rebalances outperforming daily rebalances) in the 70% range for monthly returns, and in the 85%–90% range for annual returns. On an annualized basis, the monthly rebalanced multiple (inverse) strategy outper-

## EXHIBIT 5

### Impact of Switching Exposure Duration, April 2002 through April 2013

	Effect of Changing Rebalance Frequency from Daily to Monthly					Effect of Changing Weighting from Capitalization to Equal		Effect of Tactical Weighting Based on Model					
	S&P 500 (2X, Monthly)	S&P 500 (2X, Daily)	S&P 500 (-1X, Monthly)	S&P 500 (-1X, Daily)	Excess (1)	Excess (2)	S&P 500 (2X, Monthly, EW)	Excess (3)	Alternatively Weighted U.S. Bull Model (4)	Alternatively Weighted U.S. Bear Model (5)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)			
2002	-17.30%	-34.40%	-35.47%	1.06%	16.43%	14.03%	2.40%	-32.58%	1.83%	-29.92%	2.65%	19.76%	3.33%
2003	28.68%	63.01%	60.85%	2.16%	-23.65%	-24.54%	0.89%	56.64%	-6.37%	61.89%	5.25%	-21.71%	1.94%
2004	10.88%	22.25%	21.44%	0.81%	-10.34%	-10.92%	0.58%	31.83%	9.57%	31.20%	-0.63%	-9.43%	0.91%
2005	4.90%	9.41%	8.89%	0.52%	-5.24%	-5.67%	0.43%	14.39%	4.98%	14.78%	0.39%	-2.32%	2.92%
2006	15.79%	33.46%	32.74%	0.72%	-14.05%	-14.51%	0.46%	39.07%	5.61%	36.49%	-2.58%	-15.48%	-1.43%
2007	5.49%	10.32%	8.47%	1.85%	-6.04%	-7.59%	1.54%	18.04%	7.71%	22.39%	4.36%	-4.56%	1.48%
2008	-37.00%	-63.13%	-66.54%	3.42%	50.01%	34.08%	15.93%	-61.64%	1.48%	-59.52%	2.13%	56.67%	6.66%
2009	26.46%	51.95%	48.49%	3.46%	-24.88%	-26.60%	1.72%	56.70%	4.75%	54.62%	-2.08%	-25.75%	-0.87%
2010	15.06%	27.85%	28.14%	-0.29%	-16.18%	-15.88%	-0.30%	31.26%	3.41%	30.10%	-1.16%	-16.42%	-0.24%
2011	2.11%	1.98%	-1.30%	3.28%	-4.34%	-7.22%	2.88%	3.04%	1.06%	3.23%	0.20%	-3.30%	1.04%
2012	16.00%	32.96%	32.42%	0.54%	-14.84%	-15.18%	0.34%	30.94%	-2.01%	36.79%	5.85%	-13.48%	1.36%
2013*	12.74%	26.55%	26.55%	0.01%	-11.71%	-11.68%	-0.03%	28.01%	1.45%	28.33%	0.32%	-11.53%	0.18%
Avg Annual Return	7.37%	16.00%	14.44%	1.56%	-5.68%	-8.05%	2.37%	18.90%	2.90%	20.13%	1.24%	-4.22%	1.46%
Annualized Avg Return	5.75%	9.02%	6.88%	2.14%	-7.69%	-9.61%	1.92%	12.23%	3.21%	13.95%	1.72%	-6.42%	1.27%
Annl Std Dev	18.28%	34.97%	35.31%	1.34%	20.46%	16.67%	4.42%	34.65%	4.28%	34.28%	2.82%	22.35%	2.17%

Note: The S&P 500 (2X, Monthly) is 2X the monthly return of the S&P 500, the S&P 500 (2X, Daily) is 2X the daily return of the S&P 500. Excess (1) is the difference between S&P 500 (2X, Monthly) and S&P 500 (2X, Daily). S&P 500 (-1X, Monthly) is -1X the monthly return of the S&P 500, S&P 500 (-1X, Daily) is -1X the daily return of the S&P 500, and Excess (2) is the difference between S&P 500 (-1X, Monthly) and S&P 500 (-1X, Daily). S&P 500 (2X, Monthly, EW) is 2X the monthly return of the S&P 500 where each sector is equal weighted. Excess (3) is the difference between S&P 500 (2X, Monthly, EW) and S&P 500 (2X, Monthly). Alternatively Weighted U.S. Bull Model is a non-market capitalization weighted model with 2X leverage. Excess (4) is the difference between the Alternatively Weighted U.S. Bull Model and S&P 500 (2X, Monthly, EW). Alternatively Weighted U.S. Bear Model is a non-market capitalization weighted model with -1X exposure. Excess (5) is the difference between the Alternatively Weighted U.S. Bear Model and S&P 500 (-1X, Monthly).

Past performance is not a guarantee of future results.

Sources: IndexIQ research, FactSet.

formed the daily rebalanced multiple (inverse) strategy by 2.14% (1.92%) per year. These results do not account for transaction costs associated with rebalancing. A strategy that rebalances daily would likely have higher turnover and thus higher transaction costs than a strategy that rebalances monthly, thereby further increasing the performance differential. The results also ignore financing costs and revenues, as discussed above, which could have been substantial during years with higher short-term rates, but which are similar for the two different rebalance strategies.

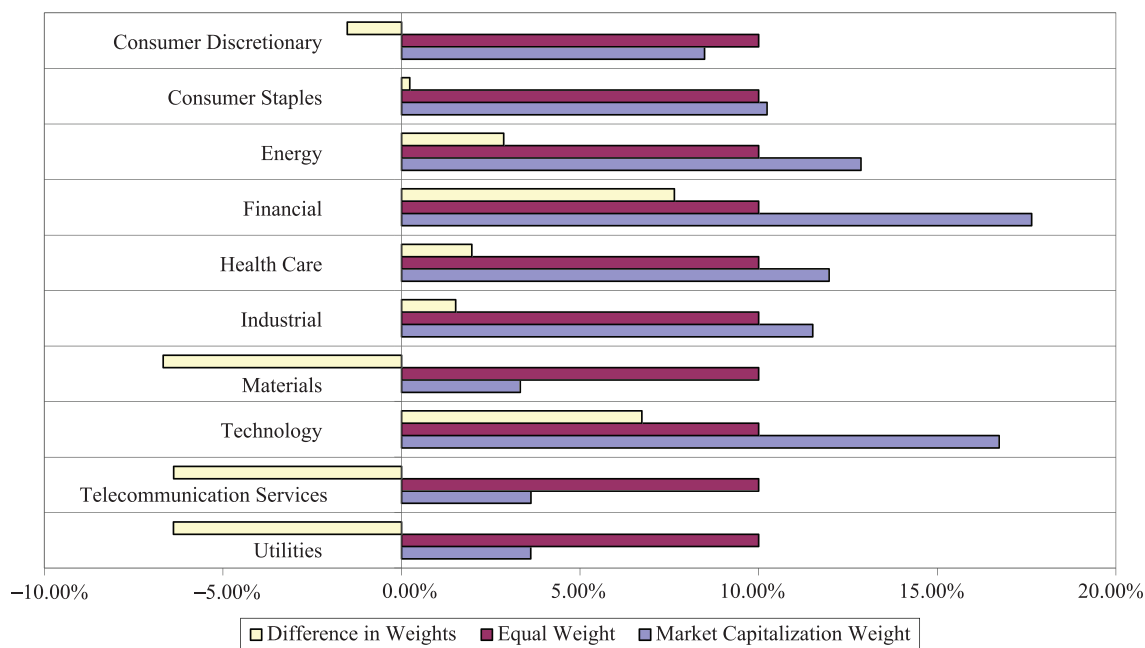
### Weight Basis

The S&P 500 index is a float-adjusted market capitalization-weighted index, whereby each member of the index is weighted proportionately according to its free-floating market cap. Market cap is calculated as the product of the shares outstanding and the market price. However, numerous studies have shown the effectiveness, in terms of performance, of equally weighting each index component (Blitzer et al. [2003]). Other studies

have documented the deleterious effect of including price as a determinant of index weight (Arnott et al. [2005]). Building on this body of research, we explore using non-cap weights to further improve the investment options for buy-and-hold investors.

The equally weighted S&P 500 index is typically constructed at the security level. This has the potential to create trading and liquidity issues when dealing with very small-cap stocks. Another approach to retain the spirit of equal weighting while also avoiding overexposure to individual smaller-cap names is to equally weight the 10 economic sectors that comprise the S&P 500. Stocks *within* the sectors are still cap-weighted; however, the sectors themselves are equally weighted. This can help to avoid overconcentration in any one sector at any point in time. Exhibit 6 shows the cap weights, equal weights, and variance in weights of the 10 economic sectors as of December 31, 2007. With financials and technology at weights of well over 15% based on market cap, they are clearly the most heavily weighted sectors. The return for the financial sector in 2008 was -55%. The relative underweight in Financials of 7.6% in the equal-weight

**EXHIBIT 6**  
Market Capitalization vs. Equal Weighting at the Sector Level



Data as of December 31, 2007.  
Sources: IndexIQ research, FactSet.



index saved more than 4% in performance in 2008. While this is just one example, an equally weighted sector allocation may help to reduce volatility by diversifying the sector exposures.

Columns 8 and 9 of Exhibit 5 show the impact of switching from a cap-weighted multiple monthly target to an equally weighted (EW) sector multiple monthly target.<sup>8</sup>

“Excess (3)” shows that the EW sector adds value over the cap-weighted index for the multiple strategy for the period analyzed. The annualized returns for the EW sector strategy are higher than for the cap-weighted index targets and are closer to the targeted market exposure. Further, these results are very consistent with hit rates (EW sector outperforming cap-weighted) in the 60% range for monthly returns and in the 85% range for annual returns. On an annualized basis, the EW sector strategy outperformed the cap-weighted index by 3.21% per year.

Existing research and the evidence above support the notion that equal weighting has the potential to outperform cap weighting. While this can be used as an advantage for a multiple long exposure, employing equal weighting for inverse exposure would hurt performance. Thus, for inverse exposure, it is preferable to employ the cap-weighted approach.

### Buy Low and Sell High: A Proven Approach to Adding Value

Equally weighting the sectors may represent an improvement over cap-weighting. However, EW is still a rather naïve approach that does not incorporate current information that may indicate the relative attractiveness of the sectors. We follow the broad literature and adjust our portfolio sector weights by focusing on three

important factors: valuation, momentum (Asness et al. [2012]), and earnings opportunity.

**Valuation.** We consider a simple price-to-earnings ratio (P/E) as a representative valuation metric. To avoid biases due to persistent valuation differences, we normalize the P/E for each sector relative to its own history.<sup>9</sup> This makes the standardized score comparable across sectors. Once the standardized scores are calculated, we divide the 10 sectors into halves where sectors with low P/E are ranked in the top half and sectors with high P/E are ranked in the lower half. We calculate the returns for the following month for those sectors in the top half (1) and the bottom half (2) in the current month.<sup>10</sup> The top-ranked sectors had an average annual return of 0.35% greater than those in the bottom half. Thus, sectors with lower prices relative to their earnings had the better returns and should receive a greater long (or less negative short) weight. The return and risk statistics are shown in columns 1 and 2 of Exhibit 7.

**Price Momentum.** We consider a simple 12-month moving average for price momentum. We calculated the returns for those sectors in the top half (1), higher past returns, and bottom half (2), lower past returns. The top sectors had a higher future average annual return by 2.42%. Thus, sectors with more price momentum had better returns and should receive a greater long (or less negative short) weight. The return and risk statistics are shown in columns 3 and 4 of Exhibit 7.

**Earnings Opportunities.** Information embedded in analyst earnings expectations can often be used to identify attractive sectors. We use a combination factor that incorporates both the dispersion (range) of analyst estimates as well as the net balance of analysts raising and lowering estimates. Both factors are included in the composite factor to identify sectors in which the stocks have wide dispersion (i.e., opportunity) and where

## EXHIBIT 7

### Average Return and Risk of Sectors Ranked in the Top Half (1) vs. Sectors Ranked in the Bottom Half (2)

Current Month Rank	Price/Earnings		Price Momentum		Earnings Opportunity		Alternatively Weighted Model	
	Avg Return	Risk	Avg Return	Risk	Avg Return	Risk	Avg Return	Risk
Top Half (1)	6.75%	15.92%	7.75%	14.73%	7.62%	16.07%	8.70%	15.42%
Bottom Half (2)	6.40%	16.02%	5.34%	17.56%	5.31%	15.90%	4.50%	16.33%
Difference	0.35%	-0.10%	2.42%	-2.83%	2.31%	0.17%	4.20%	-0.90%

*Past performance is not a guarantee of future results.*

*Sources: IndexIQ research, FactSet.*

there are also more analysts raising their estimates than lowering estimates (i.e., positive skew). We calculated the returns for those sectors in the top half (1) versus the bottom half (2). The top-ranked sectors had a higher average annual return by 2.31%. Thus, sectors that exhibit both opportunity and a positive skew had better returns and should receive a greater long (or less negative short) weight. The return and risk statistics are shown in columns 5 and 6 of Exhibit 7.

**Overall Model.** We combine all three factors to arrive at an overall score for each sector. We calculated the returns for those sectors that ranked in the top half (1) versus the bottom half (2) of this composite score. These top sectors had a higher average annual return by 4.20%. Interestingly, the average excess return for the overall model exceeded the excess returns for any of the model component factors, indicating that value was added from the interaction of the factors. This is consistent with the findings of Asness et al. [2012], who wrote, “We find consistent and ubiquitous evidence of value and momentum return premia across all the markets we study. We also highlight that studying value and momentum jointly is more powerful than examining each in isolation. The negative correlation between value and momentum strategies and their high positive expected returns implies that a simple combination of the two is much closer to the efficient frontier than either strategy alone, and exhibits less variation across markets and over time.” The return and risk statistics are shown in columns 7–8 of Exhibit 7.

This overall model<sup>11</sup> can be used in conjunction with a proprietary algorithm to rebalance the strategy to maximize the exposure to the most attractive sectors. Specifically, the weights on sectors with a current month rank in the top half (1) are maximized subject to minimum and maximum weights that are determined by their previous month and current month ranks. The weights on sectors with a current month rank in the bottom half (2) are minimized subject to minimum and maximum weights that are determined by their previous month and current month ranks.

Columns 10–13 of Exhibit 5 show the value added from employing the overall model. “Excess (4)” shows that the Alternatively Weighted U.S. Bull Model added value over the EW sector index for the multiple strategy for the period analyzed; i.e., the annualized returns for the Alternatively Weighted U.S. Bull Model are higher than those of the EW sector index. Further, these results

are consistent with hit rates (Alternatively Weighted U.S. Bull Model outperforming the EW sector) above 50% for monthly returns, and near 70% for annual returns. On an annualized basis, the Alternatively Weighted U.S. Bull Model outperformed the EW sector index by 1.72% per year. In total, the Alternatively Weighted U.S. Bull Model outperformed the original daily rebalance 2X strategy by 7.07% per year.

The same set of factors and process were used to create an inverse model, with the obvious difference that the model takes larger short positions in sectors that are expected to underperform. “Excess (5)” in Exhibit 5 shows that the Alternatively Weighted U.S. Bear Model added value over the monthly rebalance inverse sector index for the period analyzed; i.e., the annualized returns for the Alternatively Weighted U.S. Bear Model are higher than those for the monthly inverse sector index. Further, these results are consistent with hit rates (Alternatively Weighted U.S. Bear Model outperforming cap-weighted inverse sector) at 60% for monthly returns and near 75% for annual returns. On an annualized basis, the Alternatively Weighted U.S. Bear Model outperformed the monthly sector index by 1.27% per year. In total, the Alternatively Weighted U.S. Bear Model outperformed the daily rebalance inverse strategy by 3.19% per year.

Looking at the performance profiles of the Alternatively Weighted U.S. Bull and Bear Models can help to provide more insight into their performance in different market conditions. Exhibit 8 shows that the Alternatively Weighted U.S. Bull Model outperformed the 2X S&P 500 Index in 59% of the months. This was split between months where the S&P 500 Index was up (57% hit rate) and months where the S&P 500 Index was down (63% hit rate). By comparison, the S&P 500 (2X, Daily) strategy had an overall hit rate of 27%, with a 34% hit rate in up months and a 15% hit rate in down months. The Alternatively Weighted U.S. Bull Model captured 124% of the upside with only 99% of the downside for a ratio of Upside Participation to Downside Drag of 125%. The S&P 500 (2X, Daily) participated in only 92% of the upside while capturing a full 100% of the downside, for a 92% ratio.

Exhibit 9 shows that the Alternatively Weighted U.S. Bear Model outperformed the –1X S&P 500 Index in 60% of the months. This was split between months where the S&P 500 Index was up (51% hit rate) and months where the S&P 500 Index was down (76% hit

## EXHIBIT 8

### Performance Profile: Alternatively Weighted U.S. Bull Model

Positive Multiple	Overall						Market Up			Market Down		
	#Months	#Successful Months*	Hit Rate	%Upside Participation	%Downside Drag	Upside/Downside	#Months	#Successful Months	Hit Rate	#Months	#Successful Months	Hit Rate
Alternatively Weighted U.S. Bull Model	132	78	59%	124%	99%	125%	86	49	57%	46	29	63%
S&P 500 (2X, Daily)	132	36	27%	92%	100%	92%	86	29	34%	46	7	15%

\*Successful Month for Positive Multiple is where the Return > 2 × Market Return.

Past performance is not a guarantee of future results.

Source: IndexIQ research, FactSet.

## EXHIBIT 9

### Performance Profile: Alternatively Weighted U.S. Bear Model

Inverse	Overall						Market Up			Market Down		
	#Months	#Successful Months*	Hit Rate	%Downside Protection	%Upside Drag	Downside/UpSide	#Months	#Successful Months	Hit Rate	#Months	#Successful Months	Hit Rate
Alternatively Weighted U.S. Bear Model	132	79	60%	118%	100%	118%	86	44	51%	46	35	76%
S&P 500 (-1X, Daily)	132	36	27%	82%	100%	82%	86	29	34%	46	7	15%

\*Successful Month for Inverse is where the Return > Market Return × -1.

Past performance is not a guarantee of future results.

Source: IndexIQ research, FactSet.

rate). By comparison, the S&P 500 (-1X, Daily) strategy had an overall hit rate of 27%, with a 34% hit rate in up months and a 15% hit rate in down months. The Alternatively Weighted U.S. Bear Model protected against 118% of the downside while having a 100% upside drag for a ratio of Downside Protection to Upside Drag of 118%. The S&P 500 (-1X, Daily) protected against only 82% of the downside while also having a full 100% of the upside drag, for an 82% ratio.

**Fees.** Our analysis does not include any adjustment for fees for any of the strategies. Clearly, employing leverage has embedded costs. The effect of fees will vary depending on the cost of leverage. Note that these costs would apply to any strategy that employs leverage, whether rebalanced daily or monthly.<sup>12</sup>

## THE ALTERNATIVE LEVERED INDEX AND ASSET ALLOCATION

There are types of investors who may be attracted to enhanced multiple and inverse strategies. The market-timing investment community may find these instruments useful, since they add a new twist to their market-timing activities—enhanced return. They may also have market-timing needs that require terms longer than one day.

In addition to the market-timing investors, investors looking to obtain efficient long market exposure may want to consider adding the enhanced multiple strategy to a diversified portfolio. Starting with a simple 60% S&P 500 Index and 40% Barclays Aggregate Bond Index, we examine the risk and return properties of including allocations to the enhanced multiple and inverse strategies.<sup>13</sup> There are numerous possible combinations. Exhibits 10 and 11 display the risk and return attributes of four possible portfolios. We use the data underlying Exhibit

## EXHIBIT 10

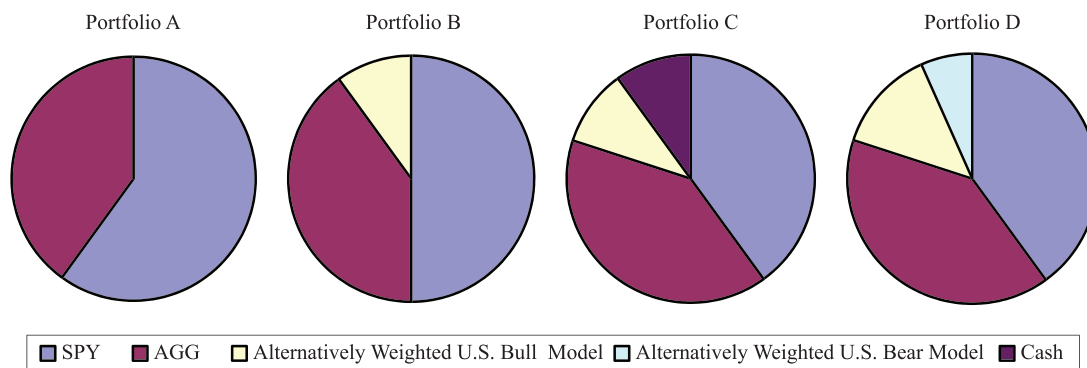
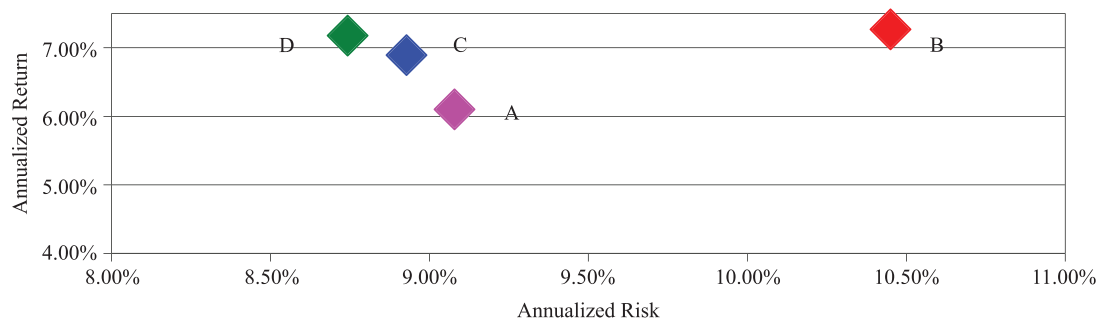
### Adding Allocations to Enhanced and Inverse Strategies

Portfolio	SPY	AGG	Alternatively Weighted U.S. Bull Model	Alternatively Weighted U.S. Bear Model	Cash	Return	Risk	Return/Risk
A	60.00%	40.00%				6.10%	9.08%	0.67
B	50.00%	40.00%	10.00%			7.27%	10.45%	0.70
C	40.00%	40.00%	10.00%		10.00%	6.89%	8.93%	0.77
D	40.00%	40.00%	13.33%	6.67%		7.18%	8.74%	0.82

Past performance is not a guarantee of future results.

Source: IndexIQ research, FactSet.

## EXHIBIT 11



Past performance is not a guarantee of future results.

Source: IndexIQ research, FactSet.

5, and thus our analysis covers the time period from April 2002 through April 2013.

A. Portfolio A might be the current portfolio (60% S&P 500/40% Barclays Aggregate Bond Index) of

an investor or the investment advisor's allocation for his or her client.

B. Move 10% from the S&P 500 to the Alternatively Weighted U.S. Bull Model (for investors who wish to amplify their equity exposure). The portfolio would have an effective exposure to equities of 70%

(50% SPY/20% from the Alternatively Weighted U.S. Bull Model). This resulted in a higher return of 1.17% per year with less increase of the percentage risk, thus creating a higher return-to-risk ratio.

- C. Move 10% to cash to lower volatility and 10% from S&P 500 to the Alternatively Weighted U.S. Bull Model. This allocation may be preferred by investors who wish to keep the same equity-risk profile but would like to benefit from the potential extra returns from the Alternatively Weighted U.S. Bull Model enhancements. This resulted in an increase in returns of 0.79%, with a decrease in risk of 0.15%.
- D. Move 13 $\frac{1}{3}$ % from S&P 500 to the Alternatively Weighted U.S. Bull Model and 6 $\frac{2}{3}$ % to the Alternatively Weighted U.S. Bear Model. This may be for investors who wish to keep a similar equity exposure but also want to benefit from the potential alpha of a long-short combination. By creating a long-short overlay to the portfolio, one may capture the alpha from the enhanced product. Thus, the risk of the portfolio declined while adding extra return, which led to a very attractive return-to-risk trade-off of 0.82.

Although these results are only a snapshot from one sequence of history, they illustrate some of the exciting opportunities offered by the Alternatively Weighted U.S. Bull and Bear Models.

## CONCLUSION

Despite the impressive growth of exchange-traded products that seek to deliver multiple and inverse equity market exposure, investors, regulators, and the media have become more aware of the limitations of these products in trying to achieve target exposure for periods longer than one day. The mathematics behind the problem for longer holding horizons can be complex, but we provide some intuitive answers, supported by simple examples to more easily demonstrate the issues. However, whereas understanding the problem is important, investors may still be left wondering how they can efficiently implement multiple or inverse exposures without the performance drag from mistracking. Building on academic research and an index methodology, there are several steps that one can undertake to potentially reduce

the drag of daily rebalancing and achieve higher returns with a new index. These steps are based on monthly rebalancing instead of daily; using an equally weighted sector approach as the basis for the portfolio weight; and employing a model that includes valuation, momentum, and earnings opportunities. Cumulatively, these steps may have the potential to deliver higher returns than a strategy that seeks to deliver a multiple or inverse of a daily rebalanced cap-weighted benchmark.

## ENDNOTES

The authors would like to thank Julie Abbett and Adam Patti for their comments and feedback.

<sup>1</sup>Monthly flow data calculated as the change in shares outstanding each month times the average price for the month (average of prior month end price and current month end price). Monthly flows are aggregated through time to arrive at the cumulative flows. Because of changes in the price of the ETF over time, cumulative flows do not equal total assets under management.

<sup>2</sup>“UBS AG U.S. brokerage business stopped selling ETFs that use leverage because such products do not conform to its emphasis on long-term investing.” *Bloomberg News*, July 27, 2009.

<sup>3</sup>“Due to the effects of compounding, their performance over longer periods of time can differ significantly from their stated daily objective. Therefore, inverse and leveraged ETFs that are reset daily typically are unsuitable for retail investors who plan to hold them longer than one trading session, particularly in volatile markets.” FINRA Regulatory Notice, June 31, 2009.

<sup>4</sup>Source: FactSet Research Systems. All returns are gross of all transaction costs.

<sup>5</sup>This example uses index returns only and does not represent any actual exchange listed product.

<sup>6</sup>The alert reader will wonder how the 2X buy-and-hold strategy achieves the cumulative return sequence in Exhibit 3. The answer is that the time-varying leverage implicit in such a strategy ensures that it will generate exactly twice the cumulative return on the underlying index. For example, the cumulative return of -19.01% as of August 22 means that the strategy, which started with a leverage ratio of 2 to 1 on January 1 (the position in the S&P 500 relative to its market value), had a leverage ratio of 2.23 to 1 by that time, i.e.,  $2(1 - 9.51\%)/(1 - 19.01\%) = 2.23$ . Given this leverage ratio and a daily return of 3.43% on August 23, the cumulative return at the end of the day was  $(1 - 19.01\%)*(1 + 2.23(3.43\%)) = -12.80\%$ , which is exactly twice the cumulative return on the S&P500.



<sup>7</sup>All returns use index data only and do not include any adjustments for fees or transaction or financing costs.

<sup>8</sup>ETF single-sector returns are used in this analysis. The total returns for the following funds are used for the entire analysis: XLY, XLP, XLE, XLF, XLV, XLI, XLB, XLK, and XLU. The total return for VOX is used starting in October 2004. Sector index returns are used for telecommunication services until September 2004. ETF returns reflect the embedded management fees. No adjustment for transaction costs or other fees have been made. Source: FactSet Research System.

<sup>9</sup>The standardized score for each sector is calculated as the (current P/E – the last 12-month average P/E)/Standard deviation of P/E over the last 12 months. For more information on standardizing factors, see Chincarini et al. [2006].

<sup>10</sup>Returns cover the period from April 2002 reflecting lack of reliable fundamental data prior to that date.

<sup>11</sup>The process described herein is very similar to the process used in the IQ Global Resources Index. That index has been calculating as a live index since October 2007.

<sup>12</sup>We estimate the following fees: financing charge = fed funds rate + 0.5%; transaction and implementation costs: 0.7% annually.

<sup>13</sup>We use SPY (SPDR S&P 500 ETF) total returns for the equity portfolio for the full period. We use AGG (iShares Core Total U.S. Bond Market ETF) total returns for the bond portion from October 2003. Prior to that, we use the Barclays Aggregate Bond Index total returns. We use the 90-day Treasury bill yield as the return for cash.

## REFERENCES

Arnott, R.D., J.C. Hsu, and P. Moore. "Fundamental Indexation." *Financial Analysts Journal*, March/April 2005, pp. 83-99.

Asness, C.S., T.J. Moskowitz, and L.H. Pedersen. "Value and Momentum Everywhere." Chicago Booth Research Paper No. 12-53, June 1, 2012. Available at SSRN: <http://ssrn.com/abstract=2174501> or <http://dx.doi.org/10.2139/ssrn.2174501>.

Avellaneda, M., and S. Zhang. "Path-Dependence of Leveraged ETF Returns." *SIAM Journal on Financial Mathematics*, July 2010, pp. 586-603.

Blitzer, D., L. Chincarini, S. Dash, and M. Dellapa. "The S&P Equal-Weight Index: Uses, Properties, and Historical Simulations." Standard & Poor's publication, January 8, 2003.

Cheng, M., and A. Madhavan. "The Dynamics of Leveraged and Inverse Exchange-Traded Funds." *Journal of Investment Management*, November 2009.

Chincarini, L., and D. Kim. *Quantitative Equity Portfolio Management*. McGraw-Hill, 2006.

Shum, P., and J. Kang. "The Long and Short of Leveraged ETFs: The Financial Crisis and Performance Attribution," June 2012.

Tang, H., and X.E. Xu. "Solving the Return Deviation Conundrum of Leveraged Exchange-Traded Funds." *Journal of Financial and Quantitative Analysis*, February 2013, pp. 309-342.

*To order reprints of this article, please contact Dewey Palmieri at [dpalmieri@ijournals.com](mailto:dpalmieri@ijournals.com) or 212-224-3675.*